Off-Mass-Shell Effects in $\omega \to 3\pi$ Decay

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Off-mass-shell effects in $\omega \to 3\pi$ decay are discussed in the context of hard-pion current-algebra technique. Anomalous Ward identities are used instead of the naive ones. The decay amplitude depends upon ξ, which characterizes the contact term, and δ, the parameter appearing in the $A_1\rho\pi$ system. Results for $\Gamma(\omega \to 3\pi)$ for $\delta=0$ to $\delta=-1$ are presented.

In the preceding paper,[1] we reported the anomalous Ward identities which enter in the hard-pion calculation of ω decays. In particular, we evaluated the amplitude relevant for the process $\omega \to 3\pi$. However, our treatment did not take into account the off-mass-shell effects in the $\omega\rho\pi$ system, which Brown and West[2] have conjectured to be quite significant in $\omega \to 3\pi$ decay. Following GSW,[3] they assumed that the decay $\omega \to 3\pi$ is dominated by the ρ pole and, thus, determined its width by calculating the relevant Feynman graph. This procedure, however, is not in general correct, since it does not admit the possibility of a contact term. Further, it seems unlikely that the decay is completely dominated by the ρ pole. The purpose of this paper is to rederive the results obtained in Paper I, by incorporating the off-mass-shell effects in the $\omega\rho\pi$ system.

Our starting point is the four-point function $\langle AAAV\rangle_0$ defined as

$$\langle AAAV\rangle_0 \equiv \int d^4x d^4y d^4z\, e^{-iq_1\cdot x - iq_2\cdot y - iq_3\cdot z} \times \langle 0|T\{A_\mu{}^a(x)A_\nu{}^b(y)A_\lambda{}^c(z)V_\sigma{}^d(0)\}|0\rangle, \quad (1)$$

with similar definitions for two- and three-point functions. Here A and V are the axial-vector and vector currents, respectively, and a, b, and c are the $SU(3)$ indices. We use all the Ward identities obtained from the function $\langle AAAV\rangle_0$. This gives rise to the so-called abnormal-parity series of Ward identities.[4] The relevant Ward identities, their anomalies, and the explicit pole structure are displayed in Paper I. There will be two sets of equations, depending on the choice of the vector current—i.e., whether V^d is V^0 or V^8. From these two sets of equations we eliminate the φ vertex functions and write the ω vertex function $M^\omega(q_1,q_2,q_3)_\sigma{}^{abc}$ in terms of $M^\omega(q_1,q_2,q_3)_{\mu\nu\lambda\sigma}{}^{abc}$.[5] Thus

$$\begin{aligned} M^\omega(q_1,q_2,q_3)_\sigma{}^{abc} = {} & -C_A{}^3 g_A{}^{-3} F_\pi{}^{-3} q_{1\mu} q_{2\nu} q_{3\lambda} M^\omega(q_1,q_2,q_3)_{\mu\nu\lambda\sigma}{}^{abc} \\ & + f^{abc} F_\pi{}^{-2} g_\rho{}^{-1} (q_1-q_2)_\mu \Delta^V(q_1+q_2)_{\mu\mu'} \Gamma^{(2)\omega}(q_3, q_1+q_2)_{\mu'\sigma}{}^{ce} + (a\leftrightarrow c;\, q_1\leftrightarrow q_3) + (b\leftrightarrow c;\, q_2\leftrightarrow q_3) \\ & + 8if^{abc} F_\pi{}^{-3} m_\omega{}^{-2} \left(\frac{f_B \cos\theta_Y}{\sqrt{6}} - \frac{f_Y \sin\theta_B}{3}\right) \cos^{-1}(\theta_B-\theta_Y) \\ & \times (2b_1-3y)\epsilon_{\mu\sigma'\alpha\beta}(q_1-q_3)_\mu (q_1+q_3)_\alpha q_{4\beta} \Delta_\omega{}^{-1}(q_4)_{\sigma\sigma'} + (b\leftrightarrow a;\, q_2\leftrightarrow q_1) + (c\leftrightarrow b;\, q_3\leftrightarrow q_2) \\ & + \tfrac{1}{3} F_\pi{}^{-3} m_\omega{}^{-2} \left[\frac{f_B \cos\theta_Y}{2} \Delta_{\mu\nu\sigma'}(AA\mathbf{A}V^0) - \frac{f_Y \sin\theta_B}{\sqrt{3}} \Delta_{\mu\nu\sigma'}(AA\mathbf{A}V^8)\right] \cos^{-1}(\theta_B-\theta_Y) \\ & \times q_{1\mu} q_{2\nu} \Delta_\omega{}^{-1}(q_4)_{\sigma\sigma'} + (a\leftrightarrow c;\, q_1\leftrightarrow q_3) + (b\leftrightarrow c;\, q_2\leftrightarrow q_3). \end{aligned} \quad (2)$$

The superscript indicates the number of vector currents entering in the vertex function, and $q_4=-q_1-q_2-q_3$. $\Delta^V(q_1+q_2)_{\mu\mu'}$ is the vector propagator and $\Delta_{\mu\nu\sigma}(\mathbf{A}AAV)$ is the anomaly in the Ward identity obtained by contracting the current in boldface in the four-point function $\langle AAAV\rangle_0$. As has been reported earlier,[4] there are four first-order anomalies in the system: the axial-vector anomalies $\Delta(\mathbf{A}VV)$ and $\Delta(\mathbf{A}AAV)$ and the vector anomalies $\Delta(A\mathbf{V}V)$ and $\Delta(AAA\mathbf{V})$. It has been shown that the anomalous terms can be expressed in terms of two parameters b_1 and b_2.[4] We have chosen

$$b_1 = 1/(16\pi^2 i), \quad b_2 = 0. \quad (3)$$

With this choice of the b_i's the axial-vector anomaly $\Delta(\mathbf{A}VV)$ is zero and so is the vector anomaly $\Delta(AAA\mathbf{V})$. We shall not present here the details of the calculation which we have already given in Paper I. However, it is easily seen that the anomalous terms in Eq. (2) have a

[1] Ahmed Ali and Faheem Hussain, preceding paper, Phys. Rev. D **3**, 1206 (1971), henceforth called Paper I.

[2] S. G. Brown and Geoffrey B. West, Phys. Rev. **174**, 1777 (1968).

[3] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).

[4] R. W. Brown, Chia-Chang Shih, and Bing-Lin-Young, Phys. Rev. **186**, 1491 (1969).

[5] $M(q_1,q_2,q_3)_{\mu\nu\lambda\sigma}{}^{abc}$ involves three axial-vector currents and one vector current. We have also used here the current-field identity:

$$V_\lambda{}^8 = \sqrt{3} f_Y{}^{-1}(\cos\theta_Y\, m_\varphi{}^2 \varphi_\lambda - \sin\theta_Y\, m_\omega{}^2 \omega_\lambda);$$
$$V_\lambda{}^0 = 2 f_B{}^{-1}(\sin\theta_B\, m_\varphi{}^2 \varphi_\lambda + \cos\theta_R\, m_\omega{}^2 \omega_\lambda).$$

zero at $q_4{}^2 = m_\omega{}^2$ and hence will not contribute to the physical $\omega \to 3\pi$ process.[6]

Next, we introduce the contact term $M_C{}^\omega$ in the style of Schnitzer and Weinberg,[7] by subtracting the pole structure from the function $M^\omega(q_1,q_2,q_3)_{\mu\nu\lambda\sigma}{}^{abc}$. Thus

$$M^\omega(q_1,q_2,q_3)_{\mu\nu\lambda\sigma}{}^{abc} \equiv M_C{}^\omega(q_1,q_2,q_3)_{\mu\nu\lambda\sigma}{}^{abc} + i[\Gamma^{(2)\omega}(q_2,q_1,q_3)_{\nu\alpha\sigma}{}^{bc}\Delta^A(q_1+q_3)_{\alpha\alpha'} \times \Gamma^{(1)}(q_1,q_3)_{\mu\lambda\alpha'}{}^{ace} + (q_1 \leftrightarrow q_2;\, \mu \leftrightarrow \nu;\, a \leftrightarrow b) + (q_2 \leftrightarrow q_3;\, \nu \leftrightarrow \lambda;\, b \leftrightarrow c)]. \quad (4)$$

The form of the vertex function $\Gamma^{(2)\omega}(q_1,q_2)_{\mu\nu}$ is constrained by the Ward identities on the $\langle VVA \rangle_0$ vertex function, $\Gamma^{(2)\omega}(q_1,q_2)_{\mu\nu\lambda}$. We write the most general form of the function $\Gamma(q_1,q_2)_{\mu\nu\lambda}$ using Lorentz covariance, Bose statistics, etc., and use the Ward identities to eliminate the various parameters appearing in its decomposition. One then obtains the following form for $\Gamma^{(2)\omega}(q_1,q_2)_{\mu\nu}{}^{ab}$:

$$\Gamma^{(2)\omega}(q_1,q_2)_{\mu\nu}{}^{ab} = -\epsilon_{\mu\nu\lambda\sigma}q_{1\lambda}q_{2\sigma} \times g_{A_1}F_\pi{}^{-1}m_{A_1}{}^{-2}\Bigg\{\Gamma_1 q_1{}^2 + \Gamma_2 q_2{}^2 + \Gamma_3 q_3{}^2 - \frac{g_{A_1}{}^{-2}}{2\pi^2 i}\left(\tfrac{1}{2}f_B d^{ab0}\cos\theta_Y - \frac{f_Y}{\sqrt{3}}d^{ab8}\sin\theta_B\right) \times \cos^{-1}(\theta_B-\theta_Y)(m_{A_1}{}^2-q_1{}^2) \times[(m_\omega{}^2-q_3{}^2)-(m_\rho{}^2-q_2{}^2)]\Bigg\}. \quad (5)$$

The above vertex function enters in the decays $\pi^0 \to 2\gamma$, $\omega \to \pi^0\gamma$, $\rho^0 \to \pi^0\gamma$, etc., if one uses vector-meson dominance. We have crudely estimated the contribution of the anomalous term to these decays. Using $\Gamma(\omega \to \pi^0\gamma) = 1.16$ MeV and the experimental upper limit for $\Gamma(\rho \to \pi^0\gamma) = 0.5$ MeV, it turns out that the anomalous term contributes less than 4% to $\Gamma(\omega \to \pi^0\gamma)$ and less than 1% to $\Gamma(\rho^0 \to \pi^0\gamma)$. Thus we are led to the assumption that if we ignore the anomalous term in Eq. (5), we do not make a serious error. However, we have kept the anomalous terms while determining the contact term in $\omega \to 3\pi$ decay. This is because the expression for the contact term contains an undetermined parameter ξ, whose value can not be guessed *a priori*.

This assumption is necessary if we want to make some educated guess about the $\omega \to 3\pi$ decay. The three parameters Γ_1, Γ_2, and Γ_3 can, in principle, be determined by using $\Gamma(\pi^0 \to 2\gamma)$, $\Gamma(\omega \to \pi^0\gamma)$, and $\Gamma(\rho^0 \to \pi^0\gamma)$ as input. While the first two are known rather accurately, the third one, $\Gamma(\rho^0 \to \pi^0\gamma)$, has only an upper limit (<0.5 MeV). This makes the estimate of $\Gamma(\omega \to 3\pi)$ very ambiguous. On the other hand, if we ignore the anomalous term, Eq. (5) is then reduced to the one obtained from a naive manipulation of Ward identities. Perrin[8] has shown (in the context of hard-pion technique) that the $\omega\rho\pi$ system is then described by only two parameters. Hence we get the following expression for the $\omega\rho\pi$ vertex-function:

$$\Gamma^{(2)\omega}(q_1,q_2)_{\mu\nu}{}^{ab} = \delta^{ab}\epsilon_{\mu\nu\lambda\sigma}q_{1\lambda}q_{2\sigma} \times\left[g_{\omega\rho\pi} - \frac{h_{\omega\rho\pi}}{m^2}(k^2-m_\omega{}^2)\right], \quad (6)$$

where we have redefined the parameters in terms of the coupling constants used by Brown and West.[2] Here $k=-q_1-q_2$, and $m=\frac{1}{2}(m_\omega+m_\rho)$.

For the vertex $\Gamma^{(1)}(q_1,q_2)_{\nu\lambda\sigma}{}^{abc}$, we use the expression for the proper $A_1A_1\rho$ vertex discussed by Schnitzer and Weinberg, which in our normalization reads

$$\Gamma^{(1)}(q_1,q_2)_{\nu\lambda\sigma}{}^{abc} = 2if^{abc}g_\rho{}^{-1}m_\rho{}^2 \times[g_{\nu\lambda}(q_2-q_1)_\sigma + g_{\lambda\sigma}(k-q_2)_\nu + g_{\sigma\nu}(q_1-k)_\lambda - \delta(g_{\nu\sigma}k_\lambda - g_{\lambda\sigma}k_\nu)]. \quad (7)$$

Here q_1, q_2 are the A_1 momenta. Substituting Eqs. (4)–(7) in Eq. (2), assuming the ρ and A_1 dominance of the 1^- and 1^+ spectral function,[9] and using the free propagators for the two-point functions,[10] we can write the proper matrix element for the decay $\omega \to \pi^+\pi^-\pi^0$ by taking the limit as $q_4{}^2 \to m_\omega{}^2$.

$$\lim_{q_4\to m_\omega{}^2} M^\omega(q_1,q_2,q_3)_\sigma{}^{abc}\epsilon_\sigma{}^\omega(q_4) = g_{A_1}{}^3 m_{A_1}{}^{-6}F_\pi{}^{-3}M_C{}^\omega(q_1,q_2,q_3)_{\mu\nu\lambda\sigma}{}^{abc}q_{1\mu}q_{2\nu}q_{3\lambda}\epsilon_\sigma{}^\omega(q_4) - 2f^{abc}\epsilon_{\mu\nu\lambda\sigma}q_{1\mu}q_{2\nu}q_{3\lambda}\epsilon_\sigma{}^\omega$$
$$\times\Bigg\{g_{\omega\rho\pi}\left[m_\rho{}^2 g_\rho{}^{-1}g_{A_1}{}^2 m_{A_1}{}^{-4}F_\pi{}^{-2}(1+\delta)\left(\frac{s}{m_\rho{}^2-s}+\frac{u}{m_\rho{}^2-u}+\frac{t}{m_\rho{}^2-t}\right) - g_\rho F_\pi{}^{-2}\left(\frac{1}{m_\rho{}^2-s}+\frac{1}{m_\rho{}^2-u}+\frac{1}{m_\rho{}^2-t}\right)\right]$$
$$+h_{\omega\rho\pi}m^{-2}[m_\rho{}^2 g_\rho{}^{-1}g_{A_1}{}^2 m_{A_1}{}^{-4}F_\pi{}^{-2}(1+\delta)(m_\omega{}^2+3m_\pi{}^2) - 3g_\rho F_\pi{}^{-2}]\Bigg\}, \quad (8)$$

where $s=(q_1+q_2)^2$, $t=(q_1+q_3)^2$, $u=(q_2+q_3)^2$, and $s+u+t=m_\omega{}^2+3m_\pi{}^2$.

[6] Brown *et al.* have assumed explicitly that there are no vector mesons in their model (free quark model only). If there are vector mesons in their theory, then Ward-identity anomalies entering the hard-pion calculation will in general contain the vector-meson propagators. See B. L. Young, Phys. Rev. D **2**, 606 (1970), for a discussion of this point.

[7] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).

[8] R. Perrin, Phys. Rev. **170**, 1367 (1968).

[9] We have used the normalization and notation of I. S. Gerstein and H. J. Schnitzer, Phys. Rev. **170**, 1638 (1968). We use the pole dominance in the form $C_V = -g_\rho{}^2/m_\rho{}^2$, $C_A = -g_{A_1}{}^2/m_{A_1}{}^2$.

[10] This is a direct consequence of the smoothness postulate on the three-point function (see Ref. 5). The propagators take the form

$$\Delta^V(q)_{\mu\nu} = \frac{C_V m_\rho{}^2}{m_\rho{}^2-q^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_\rho{}^2}\right),$$
$$\Delta^A(v)_{\mu\nu} = \frac{C_A m_{A_1}{}^2}{m_{A_1}{}^2-q^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_{A_1}{}^2}\right).$$

Equation (8) differs from Eq. (2.13) of Paper I in the sense that it includes a considerable off-mass-shell contribution to the $\omega \to 3\pi$ amplitude. We can get back to the old equation if we put $h_{\omega\rho\pi}=0$ in the last equation. Equation (8) can further be simplified if we use (a) Weinberg's first sum rule[11]

$$g_{A_1}{}^2/m_{A_1}{}^2+F_\pi{}^2=g_\rho{}^2/m_\rho{}^2\,, \tag{9}$$

(b) the second spectral-function sum rule[11]

$$g_\rho \simeq g_{A_1}\,, \tag{10}$$

and (c) the KSRF[12] relation

$$g_\rho{}^2=2F_\pi{}^2 m_\rho{}^2\,, \tag{11}$$

which together yield $m_{A_1}{}^2=2m_\rho{}^2$.

Using the above approximate expressions, we obtain

$$\begin{aligned}M^\omega(q_1,q_2,q_3)_\sigma{}^{abc}\epsilon_\sigma{}^\omega(q_4)=&-\tfrac{1}{4}\sqrt{2}m_\rho{}^{-3}M_C{}^\omega(q_1,q_2,q_3)_{\mu\nu\lambda\sigma}{}^{abc}q_{1\mu}q_{2\nu}q_{3\lambda}\epsilon_\sigma{}^\omega(q_4)+2f^{abc}F_\pi{}^{-1}\epsilon_{\mu\nu\lambda\sigma}q_{1\mu}q_{2\nu}q_{3\lambda}\epsilon_\sigma{}^\omega\\ &\times\left\{g_{\omega\rho\pi}\left[\sqrt{2}m_\rho\left(\frac{1}{m_\rho{}^2-s}+\frac{1}{m_\rho{}^2-u}+\frac{1}{m_\rho{}^2-t}\right)-\frac{1+\delta}{2\sqrt{2}}m_\rho{}^{-1}\left(\frac{s}{m_\rho{}^2-s}+\frac{u}{m_\rho{}^2-u}+\frac{t}{m_\rho{}^2-t}\right)\right]\right.\\ &\left.+h_{\omega\rho\pi}m^{-2}\left[3\sqrt{2}m_\rho-\frac{1+\delta}{2\sqrt{2}}m_\rho{}^{-1}(m_\omega{}^2+3m_\pi{}^2)\right]\right\}\,. \end{aligned}\tag{12}$$

We have displayed the evaluation of the contact term in Paper I. We observe that the contact term (containing terms up to second order in pion momenta) has an undetermined parameter, ξ. For details the reader is referred to Sec. III of Paper I. The physical $\omega\to 3\pi$ amplitude then finally becomes

$$\begin{aligned}M^\omega(q_1,q_2,q_3)_\sigma{}^{abc}\epsilon_\sigma{}^\omega(q_4)=&f^{abc}\epsilon_{\mu\nu\lambda\sigma}q_{1\mu}q_{2\nu}q_{3\lambda}\epsilon_\sigma{}^\omega(q_4)\\ &\times\left\{i\xi+2g_{\omega\rho\pi}(F_\pi)^{-1}\left[\sqrt{2}m_\rho\left(\frac{1}{m_\rho{}^2-s}+\frac{1}{m_\rho{}^2-u}+\frac{1}{m_\rho{}^2-t}\right)-\frac{1+\delta}{2\sqrt{2}}m_\rho{}^{-1}\left(\frac{s}{m_\rho{}^2-s}+\frac{u}{m_\rho{}^2-u}+\frac{t}{m_\rho{}^2-t}\right)\right]\right.\\ &\left.+2h_{\omega\rho\pi}F_\pi{}^{-1}m^{-2}\left[3\sqrt{2}m_\rho-\frac{1+\delta}{2\sqrt{2}}m_\rho{}^{-1}(m_\omega{}^2+3m_\pi{}^2)\right]\right\}\,. \end{aligned}\tag{13}$$

We are now left with four parameters: ξ, δ, $g_{\omega\rho\pi}$, and $h_{\omega\rho\pi}$. We evaluate $g_{\omega\rho\pi}$ and $h_{\omega\rho\pi}$ by fitting the experimental width for $\pi^0\to 2\gamma$ and $\omega\to\pi^0\gamma$. The $\pi^0\to 2\gamma$ amplitude will involve only $g_{\omega\rho\pi}$, while the $\omega\to\pi^0\gamma$ amplitude will involve both $g_{\omega\rho\pi}$ and $h_{\omega\rho\pi}$, because of ρ dominance. We have the following expressions[2]:

$$\begin{aligned}\Gamma(\pi^0\to 2\gamma)&=\frac{9m_\pi{}^3}{4\alpha^2m^2}\Gamma_\omega\Gamma_\rho\frac{g_{\omega\rho\pi}{}^2}{4\pi}\,,\\ \Gamma(\omega\to\pi^0\gamma)&=\frac{m^2}{8\alpha}\Gamma_\rho\frac{(g_{\omega\rho\pi}+h_{\omega\rho\pi})^2}{4\pi}\,,\end{aligned}\tag{14}$$

where α is the fine-structure constant ($\sim 1/137$), and Γ_ω, Γ_ρ are the widths for the leptonic decay of the ω and ρ.

$$\begin{aligned}\Gamma_\omega&=(4\pi\, f_\omega{}^2/3m^3)\alpha^2\,,\\ \Gamma_\rho&=(4\pi\, f_\rho{}^2/3m^3)\alpha^2\,.\end{aligned}\tag{15}$$

For Γ_ρ we use $f_\rho{}^2/4\pi=2.66$ obtained by Oakes and Sakurai,[13] which corresponds to a value of 5.1 MeV for Γ_ρ. For Γ_ω we use the result obtained from current field identities

$$\frac{\Gamma(\omega\to e^+e^-)}{\Gamma(\varphi\to e^+e^-)}=\frac{m_\omega}{m_\varphi}\tan^2\theta_Y \tag{16}$$

and use the experimental result for $\Gamma(\varphi\to e^+e^-)$. The experimental numbers[14] we use in our analysis are

$$\begin{aligned}m_\omega&=784 \text{ MeV},\\ m_\rho&=765 \text{ MeV},\\ m_{A_1}&=1019 \text{ MeV},\\ m_{\pi^0}&=135 \text{ MeV},\\ \Gamma(\pi^0\to 2\gamma)&=7.4 \text{ eV},\\ \Gamma(\omega\to\pi^0\gamma)&=1.16 \text{ MeV},\\ \Gamma(\varphi\to e^+e^-)&=1.40 \text{ keV}.\end{aligned}$$

TABLE I. Decay widths for $\omega\to 3\pi$.[a]

δ(MeV)	0	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{3}{4}$	-1
$\Gamma(\omega\to 3\pi)$	6.3	6.7	7.0	7.4	7.9
$\Gamma(\omega\to \omega\pi)$	7.5	7.9	8.4	8.8	9.3

[a] $\Gamma(\omega\to 3\pi)_{\mathrm{GSW}}=3.5$ MeV, $\Gamma(\omega\to 3\pi)_{\mathrm{WB}}=6.2$ MeV, and $\Gamma(\omega\to 3\pi)_{\mathrm{expt}}=10.9\pm 1.1$ MeV.

[11] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).

[12] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

[13] R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters **19**, 1266 (1967).

[14] All the experimental numbers have been taken from the Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970).

Using Eqs. (15) and (16) and the experimental data, we obtain

$$g_{\omega\rho\pi}=1.42\times10^{-2}\ \mathrm{MeV}^{-1},$$
$$h_{\omega\rho\pi}=0.24\times10^{-2}\ \mathrm{MeV}^{-1}.$$

The decay width $\Gamma(\omega\rightarrow 3\pi)$ now depends on two parameters δ and ξ. Table I shows the value of $\Gamma(\omega\rightarrow 3\pi)$ evaluated for the various choice of δ. Entries in the first row are obtained by putting $\xi=0$, while in the second row we have given the result of Paper I, obtained with the neglect of the off-mass-shell effects in the $\omega\rho\pi$ system (again obtained by putting $\xi=0$). Next we determine ξ by fitting the $\omega\rightarrow 3\pi$ amplitude with the decay width $\Gamma(\omega\rightarrow 3\pi)=10.9$ MeV and use this value of ξ for predicting the value of $f_{\gamma-3\pi}$, the form factor which enters in the process $\gamma+\pi\rightarrow\pi+\pi$. Values of $f_{\gamma-3\pi}$ for the various choice of δ are shown in Table II.

We see that our results are an improvement over those of West and Brown, but are still smaller than the experimental ones if we neglect the contact terms. A comparison of the entries in the first two rows shows that off-mass-shell effects tend to decrease the $\omega\rightarrow 3\pi$ width, but are not as startling as predicted by West and Brown. The reason may be traced back to the fact that we have used the improved data and, consequently, our coupling constants $g_{\omega\rho\pi}$ and $h_{\omega\rho\pi}$ differ quite significantly from the ones used by West and Brown.

TABLE II. Form factor for $\gamma+\pi\rightarrow\pi+\pi$.

δ	0	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{3}{4}$	-1	Expt
$f_{\gamma-3\pi}\times10^{-2}$	24.3	23.8	23.3	22.8	22.1	4 ± 15

We observe that it is not possible to determine the contact term from current algebra alone. Pending such a determination, the evaluation of $\Gamma(\omega\rightarrow 3\pi)$ is an open question. However, it seems that the pole term dominates the decay, and the contact term may not be as important as predicted by West and Brown. Finally, we remark that there is considerably uncertainty in the theory because of the variation in the ρ width in different experiments. With more accurate data on the ρ width, the contact terms could be determined to a higher degree of accuracy, and we could have a definite prediction for $f_{\gamma-3\pi}$.

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Regge Model for Inelastic Lepton-Nucleon Scattering*

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Lepton-nucleon scattering into hadrons is analyzed according to the simple Regge-pole model. Assuming $SU(3)$ for the factorized residues, we obtain from a strong-interaction analysis of tensor meson-baryon couplings $\sigma_T{}^{\gamma p}(\nu)-\sigma_T{}^{\gamma n}(\nu)/\sigma_T{}^{\gamma p}(\nu)-\sigma_T{}^{\gamma p}(\infty)=(0.26\pm0.05)$, to be compared with 0.31 obtained from the total photon-nucleon photoproduction data. For the inelastic structure functions, assuming all Regge poles couple in the deep-inelastic region, we find ${}^{\gamma}F_2{}^{p}(\rho)-{}^{\gamma}F_2{}^{n}(\rho)=+0.15\rho^{-1/2}$, $\rho=-\nu/q^2>4$. In the analogous neutrino process we establish from these assumptions ${}^{\nu}F_2{}^{p}(\rho)+{}^{\nu}F_2{}^{n}(\rho)\simeq0.66+(3.5\pm1.7)\rho^{-1/2}$, $\rho>4$. Analyzing the Adler and Gross–Llewellyn Smith sum rules, we obtain the estimates ${}^{\bar{\nu}}F_2{}^{p}(\rho)-{}^{\nu}F_2{}^{p}(\rho)\simeq1.5\rho^{-1/2}$, ${}^{\nu}F_3{}^{p}(\rho)+{}^{\nu}F_3{}^{n}(\rho)\simeq-8.9\rho^{1/2}$, $\rho>4$. We also conclude that if scaling behavior of the neutrino process occurs for $-q^2>1$ GeV^2, then these sum rules are saturated to 90% only for energies $\nu>200$ GeV for $-q^2>1$ GeV^2.

I. INTRODUCTION

IN this paper we analyze high-energy lepton-nucleon scattering into hadrons according to the simple Regge-pole model for high-energy forward scattering. In particular, we examine the photoproduction process for the total photon-nucleon cross sections and the highly inelastic region for photon- and neutrino-induced reactions. We assume that in the highly inelastic region that the inelastic form factors have the scaling behavior conjectured by Bjorken[1] and moreover that all Regge poles with the appropriate quantum numbers couple in this kinematic realm.[2] This later assumption is in accord with the observation that in the deep-inelastic region for electron-nucleon scattering there is a substantial nondiffractive component,[3] and that the observed structure function can be accounted for by trajectories other than the Pomeranchukon coupling.[2]

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[1] J. D. Bjorken, Phys. Rev. 179, 1547 (1969).
[2] H. R. Pagels, Phys. Letters (to be published).
[3] E. Bloom and F. Gilman, Phys. Rev. Letters 25, 1140 (1970).