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Possible Presence of a Scalar Dipion Resonance in ρ-Photoproduction Experiments*

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The scalar resonance $\sigma(740)$ may be present in pion pair production experiments at nonforward angles in the region of the ρ. The consequences of this resonance in the cross section are examined in terms of the t dependence of the mass suppression factor in the ρ shape, $(m_\rho/m)^{n(t)}$, and the possibility of observing ρ-σ interference in the antisymmetric part of the cross section.

I. INTRODUCTION

THE possible existence of a scalar $I=0$ dipion resonance, $\sigma(740)$, has been suspected for some time. It was originally proposed by Durand and Chiu[1] in order to explain the observed forward-backward asymmetry in ρ^0 production in π-p collisions. Since then, most analyses of the $I=0$, S-wave $\pi\pi$ phase shift[2] have given a set of results that rises through 90° at an effective dipion mass of approximately 740 MeV with slopes at the resonance indicating a width of the order of 200 MeV.[3]

What are the consequences of the presence of the σ resonance in photoproduction experiments in which pion pairs are produced, specifically, with an effective mass in the region of the mass of the ρ? Although the σ and ρ overlap both in mass and width, strong interference effects are probably not present, since the σ is not produced in the forward direction. The ρ-σ interference term in the cross section is antisymmetric about the forward direction and also under the interchange of the pions in the charged-pion pair. If an experiment were performed in which the antisymmetric part of the cross section were observed, similar to the antisymmetric experiment performed with electron pairs[4] in order to observe the ρ-amplitude phase relative to quantum electrodynamics, one could then observe the effect of ρ-σ interference. Of course, an analysis of such an experiment would not only involve the uncertainties of the mass and width of the σ, but also the unknown values of the magnitude and phase of the σ-production amplitude.

Even in experiments in which the antisymmetric part of the cross section is not observed, it is still possible to observe the presence of the σ by using the fact that, away from the forward direction, an increasing proportion of the pion pairs are produced via the scalar resonance. If the assumption is made that the dipion-effective-mass dependence of the σ production amplitude differs from that of the ρ-production amplitude, then the predicted mass shape of the cross section in the resonance region is a function of t (aside from the well-known e^{bt} factor presumably present in both ρ and σ production). The two amplitudes differ in the mass dependences of the widths[5] since we are dealing with resonances of different spin. A much stronger difference in mass dependence arises through the following assumption: The mass suppression (MS) factor $(m_\rho/m)^n$, first introduced by Ross and Stodolsky[6] with $n=4$, is present only in ρ production and not in the σ-production amplitude. Under this assumption it is possible to show that the experimentally measured t dependence of the resonance mass shape in the region[7] of the ρ can be fitted with reasonable values of the width of the σ and the magnitude of the σ-production amplitude. The crucial point is that experimentally, the suppression of the resonance shape due to the MS factor for dipion masses above the mass of the ρ becomes less strong as $|t|$ increases. The interpretation given here is that at larger momentum transfers pion pairs are being produced via a resonance that is known to overlap the ρ but which is not suppressed by the factor $(m_\rho/m)^n$.

Justification for the assumption of the presence of an MS factor in the ρ production amplitude but not in the σ amplitude is possible if we examine how the MS factor arises.[6] In ρ production, the photon first materializes into a ρ which then scatters diffractively off the nucleus. A factor m^{-2} appears in the amplitude because one has ρ-nucleus scattering in which the incident ρ is slightly off its energy shell. The mechanism for σ production is undoubtedly different, since the photon cannot materialize into a scalar σ, and therefore diffraction scattering of this resonance slightly off its energy shell is not expected.

The experiment in which the t dependence of the MS factor was observed[7] has been fitted very well using as

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[1] L. Durand and Y. T. Chiu, Phys. Rev. Letters **14**, 329 (1965).
[2] A compilation of these phase shifts is given in P. B. Johnson *et al.*, Phys. Rev. **176**, 1651 (1968).
[3] In a determination of the $I=0$, S-wave $\pi\pi$ phase shift based on S-wave and P-wave interference, there is an ambiguity which arises due to the invariance of the interference term under the transformation $\delta_0^0 \to \frac{1}{2}\pi - (\delta_0^0 - \delta_1^1)$ and which therefore leads to two possible sets of solutions for δ_0^0. The set that rises through 90° is supported by a nonambiguous determination of δ_0^0 by E. Malamud and P. E. Schlein [Phys. Rev. Letters **19**, 1056 (1967)] in which they fit the dipion decay angular distribution in the ρ helicity frame.
[4] J. G. Asbury *et al.*, Phys. Letters **25B**, 565 (1967).
[5] J. D. Jackson, Nuovo Cimento **34**, 1644 (1964).
[6] M. Ross and L. Stodolsky, Phys. Rev. **149**, 1172 (1966).
[7] H. H. Bingham *et al.*, Phys. Rev. Letters **24**, 955 (1970).

a background the Drell-Söding mechanism[8–10] in which two pions are photoproduced via one-pion exchange. In this fit, no MS factor was needed in the ρ-production amplitude, the mass suppression coming solely from interference with the one-pion exchange background. The t dependence of the mass suppression arises due to the propagator of the exchanged pion.[10]

Although a good fit was obtained with this model, there are a number of difficulties with it, among which are possible double counting and an arbitrariness in the amplitude introduced by gauge invariance.[11]

In this paper, the possibility of a σ-production background is examined instead, since such a background probably exists in pion pair photoproduction experiments in which the resonance is produced away from the forward direction. In Sec. II, a fit is made to the t dependence of the exponent of the MS factor. In Sec. III, the expected magnitude of the antisymmetric part of the cross section is calculated. This term is due to ρ-σ interference. No antisymmetry is introduced into the cross section by the Drell-Söding mechanism. The results are summarized in Sec. IV.

II. MASS SUPPRESSION FACTOR AS A FUNCTION OF t

We proceed now to calculate the effective MS factor as a function of t assuming that the ρ amplitude contains a factor $(m_\rho/m)^n$ and the σ amplitude does not. For simplicity we consider the case of a spin-0 target, although in the end we shall compare our results with the experimental results in which a proton target is used. Since at $t=t_{\min}$ there seems to be a need for a MS factor for a large variety of nuclei of various spins,[12] the t dependence of the MS factor is presumably also independent of the spin of the nucleus.

The ρ-production amplitude is proportional to

$$S_\rho=\epsilon^\mu A_{\mu\nu}(m^2-m_\rho^2+im_\rho\Gamma_\rho)^{-1}g_{\rho\pi\pi}(p^+-p^-)^\nu e^{b_\rho t/2}\,, \quad (2.1)$$

where the kinematics are defined in Fig. 1 ($p^2\equiv m^2$) and where, for a spin-0 target, the general photoproduction amplitude is given by

$$A^{\mu\nu}=\left[g^{\mu\nu}-\frac{p^\mu K^\nu}{(K\cdot p)}\right]T_1+\left[\left(\frac{K\cdot P_i}{K\cdot p}\right)p^\mu-P_i{}^\mu\right]$$
$$\times\left\{\left[\left(\frac{p\cdot P_i}{K\cdot p}\right)K^\nu-P_i{}^\nu\right]T_2+\left[\left(\frac{p^2}{K\cdot p}\right)K^\nu-p^\nu\right]T_3\right\}. \quad (2.2)$$

The terms T_2 and T_3 contribute only to nonforward ρ production and shall be ignored in what follows. We shall discuss to what extent these terms modify the results at the end of this section. The σ-production amplitude is proportional to

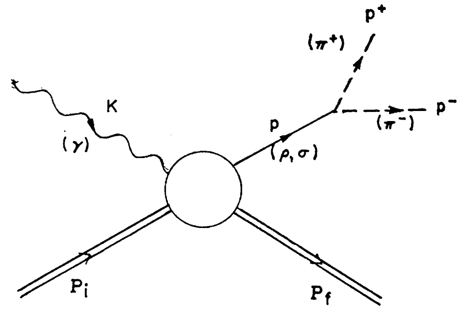

FIG. 1. Diagram for pion pair photoproduction via a ρ or σ resonance.

$$S_\sigma=\epsilon^\mu B_\mu(m^2-m_\sigma^2+im_\sigma\Gamma_\sigma)^{-1}m_\sigma g_{\sigma\pi\pi}e^{b_\sigma t/2}\,, \quad (2.3)$$

where, for a spin-0 target

$$B^\mu=\frac{1}{m_\sigma}\left[p^\mu-\left(\frac{K\cdot p}{K\cdot P_i}\right)P_i{}^\mu\right]U\,. \quad (2.4)$$

The $\sigma\pi\pi$ coupling is chosen to be $m_\sigma g_{\sigma\pi\pi}$ for dimensional reasons. The couplings are related to the resonance widths in the usual way:

$$\Gamma_\rho=\frac{g_{\rho\pi\pi}{}^2}{48\pi}\frac{1}{m^2}(m^2-4m_\pi{}^2)^{3/2}\,, \quad (2.5a)$$

$$\Gamma_\sigma=\frac{3g_{\sigma\pi\pi}{}^2}{32\pi}(m^2-4m_\pi{}^2)^{1/2}\,. \quad (2.5b)$$

In the resonance denominators of (2.1) and (2.3), we use the usual parametrizations of the resonance widths[5]

$$\Gamma_\rho=\Gamma_{\rho0}\left(\frac{m_\rho}{m}\right)\left(\frac{m^2-4m_\pi{}^2}{m_\rho{}^2-4m_\pi{}^2}\right)^{3/2}\,, \quad (2.6a)$$

$$\Gamma_\sigma=\Gamma_{\sigma0}\left(\frac{m_\sigma}{m}\right)\left(\frac{m^2-4m_\pi{}^2}{m_\sigma{}^2-4m_\pi{}^2}\right)^{1/2}\,. \quad (2.6b)$$

The spin-averaged square of the ρ-production amplitude (2.1) is proportional to

$$q^2+\left(\frac{K\cdot q}{K\cdot p}\right)^2 p^2=q^2\sin^2\theta^*\,, \quad (2.7)$$

where $q=p^+-p^-$ and θ^* is the pion decay angle with respect to the incident photon direction in the rest frame of the ρ. In the data of Ref. 7, the pion decay angle is not specified and, therefore, in a comparison with the data, we average (2.7) over θ^*.

The spin-averaged square of the σ-production amplitude (2.3) is proportional to

$$(|t|+m^2)^2/4k^2-|t|=-|\mathbf{P}|^2\sin^2\theta\,, \quad (2.8)$$

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[8] S. Drell, Rev. Mod. Phys. **33**, 458 (1961); P. Söding, Phys. Letters **19**, 702 (1966).
[9] A. Krass, Phys. Rev. **159**, 1496 (1967).
[10] J. Pumplin, Phys. Rev. D **2**, 1859 (1970).
[11] See Ref. 13 of Ref. 7 and also Ref. 10.
[12] H. Alvensleben *et al.*, Phys. Rev. Letters **24**, 786 (1970).

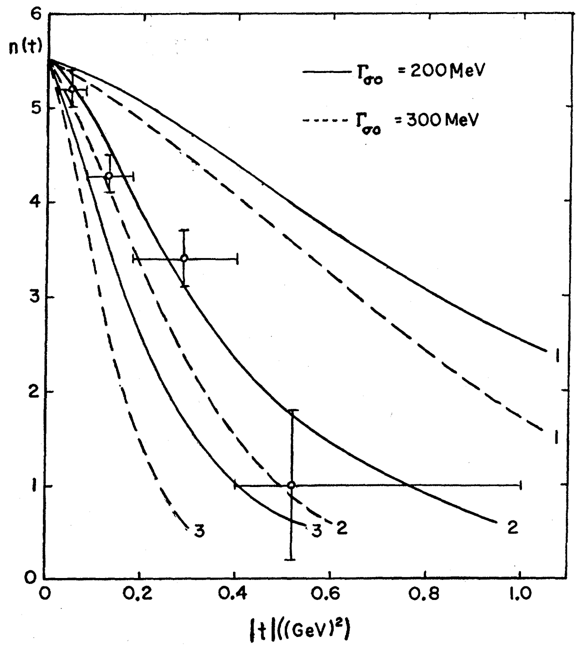

FIG. 2. The effective exponent in the MS factor as a function of t when one includes a σ resonance in the pion pair production amplitude. The incident photon energy is 2.8 GeV. The curves are labeled by the value of $|U/T_1|$ defined in (2.2) and (2.4). The parameters other than $\Gamma_{\sigma 0}$ are given in (2.11). The data are from Ref. 7.

where $t=(K-p)^2$ and k is the laboratory photon energy. On the right-hand side of (2.8), $\mathbf{P}$ is the laboratory momentum of the σ and θ is its production angle with respect to the photon beam. The σ is not produced in

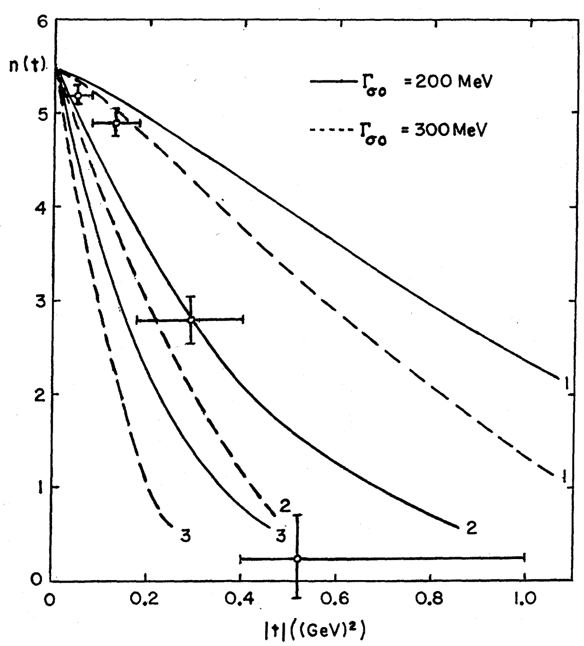

FIG. 3. Same as Fig. 2 but with an incident photon energy of 4.7 GeV.

in the forward direction. The ρ-σ interference term, proportional to the product of the square root of (2.7) and (2.8) and therefore to $\sin\theta$, is antisymmetric about the incident photon direction. The cross term will not be considered in what follows since it is assumed that the detection apparatus in the experiment of interest has integrated over this antisymmetric term and therefore eliminated it.

The cross section for the production of pion pairs is now the sum of the spin-averaged squares of the ρ and σ amplitudes. A MS factor $(m_\rho/m)^n$, with n t independent, is included in the ρ term but not in the σ term. The t dependence in the MS factor is then defined by assuming that the pion pairs were produced only via the ρ with a MS factor $(m_\rho/m)^{n(t)}$. The exponent $n(t)$ is defined by

$$\left(\frac{m_\rho}{m}\right)^{n(t)}=\left(\frac{m_\rho}{m}\right)^{n}+C^2e^{(b_\sigma-b_\rho)t}\frac{(m^2-m_\rho^2)+m_\rho^2\Gamma_\rho^2}{(m^2-m_\sigma^2)^2+m_\sigma^2\Gamma_\sigma^2}\times\frac{3}{2}\frac{1}{m^2}\left(|t|-\frac{(|t|+m^2)^2}{4k^2}\right),\quad(2.9)$$

where

$$C^2=\frac{2}{9}\frac{\Gamma_{\sigma 0}}{\Gamma_{\rho 0}}\frac{(m_\rho^2-4m_\pi^2)^{3/2}}{m_\rho^2(m_\sigma^2-4m_\pi^2)^{1/2}}\left|\frac{U}{T_1}\right|^2.\quad(2.10)$$

In the calculation, the following parameters are used[13]:

$$m_\rho=765\text{ MeV},\quad\Gamma_{\rho 0}=120\text{ MeV},\quad m_\sigma=740\text{ MeV},\quad m_\pi=140\text{ MeV},\quad b_\sigma=b_\rho,\quad n=5.5.\quad(2.11)$$

The results are shown in Fig. 2 for $k=2.8$ GeV and in Fig. 3 for $k=4.7$ GeV and compared with the data at these energies.[7] Good fits can be obtained to the data with $\Gamma_{\sigma 0}=200$ MeV and $|U/T_1|\approx 2$ or $\Gamma_{\sigma 0}=300$ MeV and $|U/T_1|\approx 1.5$.

Both the $|t|$ dependence and the energy dependence of $n(t)$ are reproduced by the ρ-plus-σ model, since, if one compares Figs. 2 and 3, a more rapid falloff is seen in the data as the photon energy increases and this more rapid falloff is contained in (2.9).

Using the above model in which both the ρ- and σ-production amplitudes contain exponential t dependence, the ratio of the total cross sections can be shown to be

$$\frac{\sigma_{\rm tot}(\gamma\to\sigma)}{\sigma_{\rm tot}(\gamma\to\rho)}=\frac{b_\rho}{2b_\sigma^2k^2}\left|\frac{U}{T_1}\right|^2.\quad(2.12)$$

Since above $\sim$3 GeV $\sigma_{\rm tot}(\gamma\to\rho)$ is constant, (2.12) predicts that the total cross section for σ production decreases with increasing energy, which is expected since the σ is not diffractively produced. Using the

[13] At $t=0$ the value $n=5.5$ seems to be much preferred over the value $n=4$ predicted by Ross and Stodolsky, Ref. 6.

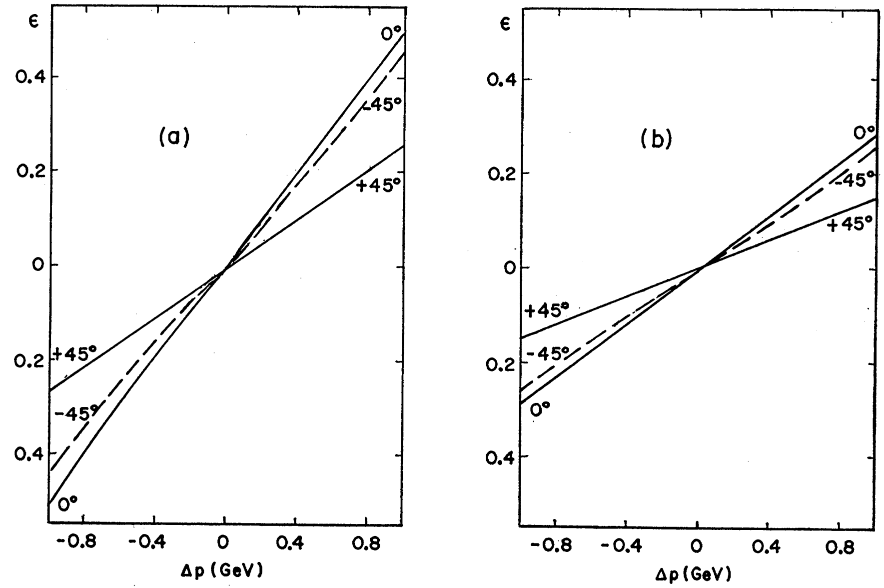

FIG. 4. The antisymmetric part of the pion pair production cross section when a σ resonance is included in the amplitude. The parameters are given in (2.11) and $|U/T_1|=2$, $\Gamma_{\sigma 0}=200$ MeV. The curves are labeled by the relative σ-ρ phase angle $\Delta_{\sigma\rho}$. (a) $k=2.8$ GeV; (b) $k=4.7$ GeV.

hydrogen target values[14]

$$\sigma_{\text{tot}}(\gamma p\rightarrow \rho p)\approx 16\ \mu\text{b}\,,\quad b_\rho=b_\sigma\approx 8\ \text{GeV}^{-2}$$

the total cross section for σ production should be $\sim 0.4\ \mu$b at $k=2.8$ GeV and $\sim 0.1\ \mu$b at $k=4.7$ GeV.[15]

Until now the nonforward terms T_2 and T_3 have been ignored in the ρ amplitude (2.2). If the optical theorem is applied to ρ-nucleus scattering, it can be shown that the magnitudes of T_2 and T_3 can be no greater than that of T_1. When these terms are included in the ρ amplitude with orders of magnitude equal to that of T_1, the change in the results of Figs. 2 and 3 is only as much as 20% for large values of $|t|$ and approaches zero as $|t|\rightarrow 0$.

III. ρ-σ INTERFERENCE TERM

By looking for an antisymmetric component in pion pair photoproduction, it can be determined whether or not σ production is present in the cross section. The Söding mechanism will not give an antisymmetric part since both the one-pion-exchange amplitude and the ρ-production amplitude (2.1) are antisymmetric under the interchange $\pi^+\leftrightarrow\pi^-$, and therefore all terms in the cross section, including the cross term, will be symmetric.

The ρ-σ interference term can be measured in the following way.[4] For a fixed difference in momenta between right and left spectrometer arms Δp, one measures the yield first with the π^+ passing through the right arm (N_+), and then the π^- passing through the right arm (N_-). The quantity

$$\epsilon=\frac{N_+(\Delta p)-N_-(\Delta p)}{N_+(\Delta p)+N_-(\Delta p)} \tag{3.1}$$

is then proportional to the ρ-σ interference term.

In a typical interference experiment of this type,[4] the pions are detected at equal symmetric angles with respect to the incident photon beam but with different energies. In this case,

$$\epsilon=\frac{2\,\text{Re}(W_\rho{}^*W_\sigma)}{|W_\rho|^2+|W_\sigma|^2}, \tag{3.2}$$

where

$$W_\rho=\left(\frac{m_\rho}{m}\right)^{2.75}e^{b_\rho t/2}\frac{m}{k}[k^2-(\Delta p)^2]^{1/2}\times\frac{1}{m^2-m_\rho{}^2+im_\rho\Gamma_\rho}, \tag{3.3}$$

$$W_\sigma=Ce^{b_\sigma t/2}\frac{m(\Delta p)}{[k^2-(\Delta p)^2]^{1/2}}\frac{e^{i\Delta_{\sigma\rho}}}{m^2-m_\sigma{}^2+im_\sigma\Gamma_\sigma}.$$

Using the values in (2.11) and $\Gamma_{\sigma 0}=200$ MeV, $|U/T_1|=2$, which gave good fits to $n(t)$, results for ϵ are shown in Fig. 4 for incident photon energies of 2.8 and 4.7 GeV. The curves correspond to various values of the relative σ-ρ phase angle $\Delta_{\sigma\rho}$. Asymmetries as large as 30–50% are possible at these energies, indicating that if there is a σ term in the pion pair

[14] E. Lohrmann, in *Proceedings of the Third International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, 1967* (Clearing House of Federal Scientific and Technical Information, Washington, D. C., 1968).

[15] The results in Figs. 2 and 3 could be reproduced in other models for ρ production. For instance, the σ could be produced via one-photon exchange. If one does not include the exponential t dependence in the amplitude, then the ratio of the σ to the ρ total photoproduction cross sections is proportional to $b_\rho M_T{}^2 \times\ln(4k^2/|t_{\min}|)$.

production cross section, it should be relatively easy to observe in such an experiment.

IV. SUMMARY

The scalar dipion resonance $\sigma(740)$ may very well be present in pion pair production experiments away from the forward direction. The t dependence of the exponent of the mass suppression factor can be accounted for by the addition of a σ-resonance term in the amplitude with reasonable values of the σ width and magnitude of its production amplitude. A measurement of the antisymmetric part of the pion pair production cross section would be a means of detecting the ρ-σ interference term. Antisymmetries of the order of 30% are expected using the previously obtained values of the σ width and magnitude of its production amplitude.

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Scaling Symmetry of Scattering Amplitudes

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We introduce and study a possible symmetry relation among the scattering amplitudes of two reactions whose corresponding masses are related by a common factor σ ($m_i'/m_i=\sigma$ for all i). We discuss two major physical examples of this symmetry involving electromagnetic and weak interactions. We consider the possibility of this symmetry in strong interactions at high energies and deduce from it interesting predictions on the asymptotic behavior of cross sections.

I. INTRODUCTION

If $R=(1+2\to 3+4)$ and $R'=(1'+2'\to 3'+4')$ are two reactions such that the corresponding particles have the same spins $S_i'=S_i$ and the *same* mass ratio $\sigma=m_i'/m_i$ for all i, then it is theoretically possible (see Sec. II) for their amplitudes to satisfy the following linear relation in all Lorentz frames:

$$H_h(R',\sigma K)=\eta(s,t)H_h(R,K)\,, \tag{1.1}$$

where $H_h(R,K)$ are general helicity amplitudes which are the generalizations to all frames of the usual c.m. helicity amplitudes. R and R' stand for the reactions involved, $K=(k_3k_4;\,k_1k_2)$ and $K'=(\sigma k_3,\sigma k_4;\,\sigma k_1,\sigma k_2)$ for their momenta, and $h=(\lambda_3\lambda_4;\,\lambda_1\lambda_2)$ for their helicities. η is a scalar function of the invariant variables $s=(k_1+k_2)^2$ and $t=(k_1-k_3)^2$.

We shall say that two reactions obey *scaling symmetry* if their amplitudes can be related as in (1.1). This symmetry is not widely realized in nature, but there are a few instances where it is actually realized to a good accuracy. For example, there is scaling symmetry between $R_{ee}=(e^+e^-\to e^+e^-)$ and $R_{\mu\mu}=(\mu^+\mu^-\to\mu^+\mu^-)$, and also between $R_{\nu e}=(\nu e^-\to\nu e^-)$ and $R_{\nu\mu}=(\nu\mu^-\to\nu\mu^-)$. We shall also discuss the hypothesis of a possible scaling symmetry at high energies between strong interactions such as $R_p=(p+p\to p+p)$ and $R_\Sigma=(\Sigma+\Sigma\to\Sigma+\Sigma)$.

The purpose of this article is the study of scaling symmetry. In Sec. II we discuss some of its properties, and compare it with the usual scaling *invariance*. In Sec. III we study the compatibility of scaling symmetry with unitarity. In Secs. IV–VI, we discuss in detail the above mentioned three examples involving electromagnetic, weak, and strong interactions. Finally, in Sec. VII we give a brief summary. We shall restrict our discussions to two-body reactions for simplicity; all arguments can be generalized easily to other reactions.

II. PROPERTIES OF SCALING SYMMETRY

We arrived at Eq. (1.1) while studying all possible linear symmetries among scattering amplitudes that are covariant under Lorentz transformations.[1] The relations hold for all Lorentz frames ΛK, where Λ is any proper Lorentz transformation, since the Wigner rotation $W^H(\Lambda,k)$ [which determine the transformation properties of $H_h(R,K)$] depend only on the velocities $\mathbf{v}=\mathbf{k}/k_0$ and hence they are independent of the mass of the particles. Therefore, $H(R',\sigma K)$ and $H(R,K)$ have the same transformation properties, so that if (1.1) holds in one frame K it must also hold in all other Lorentz frames ΛK. For this reason η must be a scalar.

Scaling symmetry with an arbitrary function $\eta(s,t)$ is trivial for the scattering of spinless particles, since for these reactions there is only one *scalar* scattering amplitude, and $\eta(s,t)$ can always be defined as the

[1] J. Daboul, Fortschr. Physik (to be published).