free field, these Fourier components (i.e., plane-wave solutions) would lead to an explicitly noncovariant representation in which the three-momentum is always real, but the energy complex. In the coordinate space, the equation of a free field with complex masses is, of course, manifestly covariant; however, its plane-wave solutions diverge exponentially in the asymptotic region. (One notes that, even for those solutions that diverge only along the time direction in a specific Lorentz frame, the same solutions viewed in other systems of reference would diverge in the asymptotic region along the spatial directions as well, thus violating the condition for the validity of the Fourier theorem.) The mathematical procedure of applying the usual quantization rules to these Fourier components must, therefore, be regarded as a purely formal one; its general validity is clearly questionable. Thus, both the naive and the CLOP prescriptions are, at present, merely recipes for evaluating the S matrix, connected only heuristically to the Lagrangian field theory.

Between these two prescriptions, the one that leads to a relativistically invariant unitary S matrix is clearly to be preferred.[8] The alternative raised by Nakanishi, *either* to sacrifice the Lorentz invariance *or* the Lagrangian field-theoretical formulation, appears to be an artificially created issue. His novel suggestion that there may be some merit in a "very slightly" noninvariant S matrix, obtained by rigidly adhering to the original naive prescription, is not, at any rate, a line of thought we would like to encourage.

[8] See also the remarks given near the end of Sec. V of Ref. 6.

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Comments on Eikonalization at High Energy*

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We previously showed how s-channel iteration of general kinds of exchanges in high-energy e-e scattering in quantum electrodynamics leads to eikonalization. In this note we clarify this result and briefly discuss its implications.

IN a previous publication,[1] we demonstrated that under certain conditions, s-channel iteration of general kinds of exchanges in high-energy e-e scattering in quantum electrodynamics (QED) leads to eikonalization.[2] In this note we would like to clarify this result and briefly discuss its implications. For a more complete discussion of the terms and methods we use, we refer the reader to the original article.

Consider a high-energy two-body → two-body scattering process which proceeds by the multiple exchange of *any* connected unit, as in Fig. 1. Then the conditions under which eikonalization takes place, for both e-e scattering in QED and for a φ^3 theory, are twofold: First, the external particles retain their respective large momenta; that is, the left-hand particle has a large plus component both before, during, and after the collision, and the right-hand particle similarly retains its large minus component. This condition implies that t/s remains small, which is a commonly stated eikonalization condition. Second, we require that there be no vertex corrections on the external lines.

Under these two conditions we may consider the exchange of an arbitrary connected unit, keeping both the

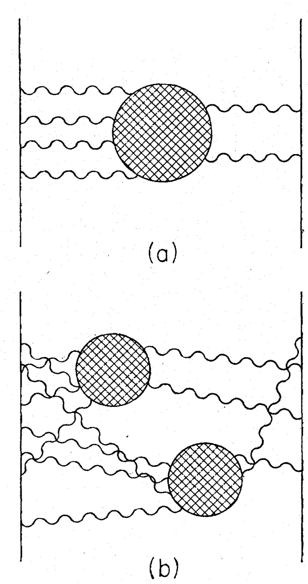

FIG. 1. Multiple exchange of connected units: (a) an example of single exchange of a connected unit; (b) one of the second-order iterations of diagram (a).

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[1] S.-J. Chang and Paul M. Fishbane, Phys. Rev. D 2, 1104 (1970).

[2] Similar results have been obtained by R. Sugar in φ^3 theory. We would like to thank Professor R. Blankenbecler (private communication).

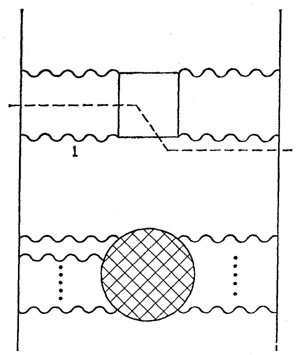

FIG. 2. Example of a unitarity cut corresponding to to Mandelstam-type diagrams. The dashed line represents the unitarity cut. Our high-energy approximations imply that such contributions to unitarity are small.

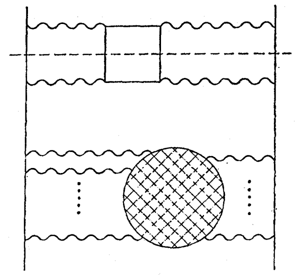

FIG. 3. A cut which contributes a finite amount to unitarity at high energies. Cuts of this type are sufficient to saturate s-channel unitarity.

leading and nonleading contributions in $\ln s$ of this unit.[3] Let the s and t dependence of this exchange be $F(s,\mathbf{k})$ [see, e.g., Eq. (A5) of Ref. 1] with Fourier transform

$$-i\chi(s,\mathbf{b})=\int\frac{d^2k}{(2\pi)^2}e^{i\mathbf{k}\cdot\mathbf{b}}F(s,\mathbf{k})\,.$$

Then the sum of all possible exchanges of our unit is of the form

$$\sim is\int d^2b\, e^{-i\mathbf{k}\cdot\mathbf{b}}(e^{-i\chi(s,\mathbf{b})}-1)\,.$$

Note that there is no *a priori* reason why $\chi(s,\mathbf{b})$ has to be real or imaginary. In general, we would expect $\chi(s,\mathbf{b})$ to be a complex function describing a complex s-dependent force. This eikonalization is like nonrelativistic eikonalization under iteration of an s-dependent potential, and in that sense tends to provide support to recent attempts to calculate Regge (cut) contributions by using an s-dependent potential.[4]

Physically we would expect that by including *all* connected pieces the resulting $\chi(s,\mathbf{b})$ should have a negative absorptive part for all $\mathbf{b}$. That this is true can be established rigorously by considering inelastic processes as well. A model calculation based on φ^3 theory also supports this point.[5] What we would like to emphasize here is that the effective s-dependent potential is generated by summing over *all* connected exchanges. The result is potentially very useful in understanding and computing the s-dependent potential from both a given theoretical model and a phenomenological analysis.

Our arguments suggest that under the conditions stated above, Mandelstam-type diagrams[6] are not important in the saturation of unitarity, as in Fig. 2, and thus that off-mass-shell effects on the external lines are unimportant. Indeed, the infinite-momentum rules suggest that this is so, since cutting the diagram as in Fig. 2 necessarily transmits a finite fraction of large momentum into the exchange line (e.g., line 1 of Fig. 2). In turn, this result would suggest that s-channel unitarity may be verified by applying Cutkosky rules to individual connected pieces independently (i.e., to cuts which would be possible if the only exchange were the unit being cut). A possible cut is given in Fig. 3. This suggestion is explicitly true; a detailed discussion may be found in Ref. 5.

Note added in proof. Recently there has been some discussion on the question of alternative infinite momentum pathways[7] in $\lambda\varphi^3$. We would like to make two comments. First, this difficulty does not arise in QED, as has already been pointed out. Second, even in a scalar theory, this difficulty does not arise if one insists that the incident particles retain their large momenta, i.e., the legs of the exchanged mesons do not carry any appreciable fraction of the large incident momentum (these mesons are "soft"). In an exact treatment of this process the incident infinite momentum will be shared by more than one intermediate particle. The scattering process can be regarded as the exchange of soft mesons between two sets of particles, each set carrying large but opposite infinite momentum. We regard the process in which only one particle carries infinite momentum as the first step in a new kind of perturbation theory.

The authors wish to thank Professor L. Durand and Professor T.-M. Yan for discussion and for suggesting that we clarify the general eikonalization result.

[3] Since our general proof is good only to $O(1/s)$, it does not make much sense to keep lower-order terms in s in the connected unit. We *can*, however, keep the leading terms in s and all (nonleading) orders in $\ln s$.

[4] See, for example, S. C. Frautschi and B. Margolis, Nuovo Cimento **57A**, 427 (1968), or F. Henyey, G. L. Kane, Jon Pumplin, and M. H. Ross, Phys. Rev. **182**, 1579 (1969). Typically in such approaches the potential is given by Pomeranchon contributions, lower-lying Regge poles, or both.

[5] S.-J. Chang and T.-M. Yan, Phys. Rev. Letters **25**, 1586 (1970).

[6] S. Mandelstam, Nuovo Cimento **30**, 1148 (1963).

[7] George Tiktopoulos and S. B. Treiman, Princeton University report (unpublished); B. Hasslacher, D. K. Sinclair, G. M. Cicuta, and R. L. Sugar, Phys. Rev. Letters **25**, 1591 (1970).