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Essential Breaking of Scale Invariance in Nonlinear Chiral Lagrangian Models*

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We show that, in the conventional chiral $SU(2)\otimes SU(2)$-symmetric nonlinear Lagrangian models of pseudoscalar mesons, scale invariance is essentially broken. Although a dilatation current $\mathfrak{D}^\mu(x)$ generating a scale transformation for the meson fields can be explicitly constructed, it is impossible to choose a canonical frame in which the axial-vector current transforms with scaling dimension 3 under this scale transformation. The conflict between scale invariance and the nonlinear realization of chiral symmetry is exhibited through the chiral nonvariance of the new energy-momentum tensor $\theta^{\mu\nu}$, which gives $\mathfrak{D}^\mu(x)=x_\nu\theta^{\mu\nu}(x)$ and $\partial_\mu\mathfrak{D}^\mu(x)=\theta_\mu{}^\mu(x)$. Furthermore, we also show that, in nonlinear models, there is no chiral-symmetric tensor which can approach this new energy-momentum tensor in the free-field limit.

SINCE the possible relationship between scale transformations and the dynamical structure of field theories became a subject of various investigations, it was suggested that the breaking of scale invariance and its interplay with chiral symmetry are important for the understanding of hadron dynamics.[1] In this paper, we investigate the role of scale invariance in the conventional nonlinear Lagrangian models of chiral symmetry.[2] As we shall see, in contrast with the linear models,[3] scale invariance and chiral $SU(2)\otimes SU(2)$ symmetry turn out to be two conflicting symmetries in these nonlinear models.

The general $SU(2)\otimes SU(2)$-symmetric nonlinear Lagrangian for a set of pseudoscalar mesons can be written as[4,5]

$$\mathfrak{L}=f^{-2}\operatorname{tr}(\partial_\mu U\partial^\mu U^\dagger),\tag{1}$$

where $U=U(if\tau_a\varphi_a)$ is a unitary, unimodular 2×2 matrix, and τ_a are the usual isospin matrices. From the unitary and unimodular condition, we can write

$$\frac{\partial}{\partial\varphi_a}U=id_{ab}(\varphi)\tau_bU,\tag{2}$$

where d_{ab} must satisfy the integrability condition, i.e.,

$$\frac{\partial}{\partial\varphi_a}d_{ab}-\frac{\partial}{\partial\varphi_b}d_{ac}=-2\epsilon_{dec}d_{ad}d_{bc}.\tag{3}$$

As shown in Ref. 4, the Lagrangian (1) is invariant under an $SU(2)$ transformation

$$\varphi_a\to\varphi_a'=\varphi_a-\epsilon_{abc}\delta\omega_b\varphi_c\tag{4a}$$

as well as under a chiral transformation

$$\varphi_a\to\varphi_a'=\varphi_a-\delta v_bF_{ab}(\varphi),\tag{4b}$$

where

$$F_{ab}(\varphi)\equiv d_{ab}^{-1}(\varphi)-\epsilon_{abc}\varphi_c.\tag{5}$$

The vector and axial-vector currents generating these transformations are given by

$$V_a{}^\mu=\epsilon_{abc}\pi_b{}^\mu\varphi_c,\tag{6}$$

$$A_a{}^\mu=F_{ab}\pi_b{}^\mu,\tag{7}$$

where

$$\pi_b{}^\mu=\frac{\partial}{\partial(\partial_\mu\varphi_b)}=4f^{-2}d_{bc}d_{dc}\partial^\mu\varphi_d.\tag{8}$$

From the canonical commutation relation

$$[\pi_a{}^0(\mathbf{x},t),\varphi_b(\mathbf{x}',t)]=-i\delta_{ab}\delta(\mathbf{x}-\mathbf{x}'),$$

we have

$$[V_a{}^0(\mathbf{x},t),\varphi_b(\mathbf{x}',t)]=i\epsilon_{abc}\varphi_c(\mathbf{x},t)\delta(\mathbf{x}-\mathbf{x}'),\tag{9}$$

$$[A_a{}^0(\mathbf{x},t),\varphi_b(\mathbf{x}',t)]=iF_{ab}(\varphi)\delta(\mathbf{x}-\mathbf{x}').\tag{10}$$

Different models of the nonlinear relization of the chiral symmetry[6] thus correspond to different forms of $d_{ab}(\varphi)$, and therefore $F_{ab}(\varphi)$, satisfying the integrability condition (3).

In all the models, it has been shown in Ref. 6 that

$$\lim_{f\to0}\operatorname{tr}(\partial_\mu U\partial^\mu U^+)=\tfrac{1}{2}f^2\partial_\mu\varphi_a\partial^\mu\varphi_a+O(f^4).\tag{11}$$

Therefore, in the limit $f\to0$, the Lagrangian (1) becomes the Lagrangian of a set of massless free fields which is exactly scale invariant. However, in the presence of chiral-symmetric interaction, since the parameter f has a physical dimension of $(\text{mass})^{-1}$, the scale invariance is expected to be broken. In the case

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[1] See, for example, G. Mack and A. Salam, Ann. Phys. (N. Y.) **53**, 174 (1969), and references therein; K. G. Wilson, Phys. Rev. **179**, 1499 (1969); M. Gell-Mann, Summer School of Theoretical Physics, University of Hawaii, 1969 (unpublished). P. Carruthers, Phys. Rev. D **2**, 2265 (1970).

[2] J. A. Cronin, Phys. Rev. **161**, 1483 (1967); L. S. Brown, *ibid.* **163**, 1802 (1967); P. Chang and F. Gürsey, *ibid.* **164**, 1752 (1967).

[3] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); G. Mack and A. Salam, Ref. 1.

[4] L. S. Brown, Ref. 2.

[5] We will use Greek indices as Lorentz indices, Latin index k as Euclidean index, and Latin indices from a to e as internal-symmetry indices. The metric tensor is $g^{00}=-g^{11}=-g^{22}=-g^{33}=1$. The repeated indices are to be summed over and the products of operators are to be properly symmetrized.

[6] P. Chang and F. Gürsey, Ref. 2; F. Gürsey, Acta Phys. Austriaca Suppl. **V**, 185 (1968).

of broken scale invariance, we even have to define what we mean by a scale transformation. This is usually done by requiring that there be an interpolating field transforming as[7]

$$\varphi_a(x) \to \varphi_a'(x') = S^l \varphi_a(sx), \tag{12}$$

where l is the scaling dimension of the field φ_a. The scaling dimension l may or may not be equal to the dimension 1 of the free meson field.[8] However, in the following discussion, it is not important to know the precise value of l for these pseudoscalar fields. For a constant l, the current generating the scale transformation (12) can be written down in the usual way as

$$\mathfrak{D}^\mu(x) = -l\pi_a{}^\mu \varphi_a + x_\nu T^{\mu\nu} \tag{13}$$

such that

$$[D(t), \varphi_a(\mathbf{x},t)] = i(l - x_\nu \partial^\nu)\varphi_a(\mathbf{x},t), \tag{14}$$

where $T^{\mu\nu}$ is the canonical energy-momentum tensor

$$T^{\mu\nu}(x) = \pi_a{}^\mu(x)\partial^\nu \varphi_a(x) - g^{\mu\nu}\mathfrak{L} \tag{15}$$

and $D(t)$ is the dilatation operator

$$D(t) = \int d^3x' \mathfrak{D}^0(\mathbf{x}',t). \tag{16}$$

Let us not investigate the scale transformation properties of the vector and axial-vector currents. If these currents have definite transformation properties, we should have

$$[D(t), V_a{}^\mu(\mathbf{x},t)] = i[l(V_a{}^\mu) - x_\nu \partial^\nu] V_a{}^\mu(\mathbf{x},t), \tag{17a}$$

$$[D(t), A_a{}^\mu(\mathbf{x},t)] = i[l(A_a{}^\mu) - x_\nu \partial^\nu] A_a{}^\mu(\mathbf{x},t), \tag{17b}$$

where $l(V)$ and $l(A)$ are, respectively, the scaling dimensions of the vector and axial-vector currents. For our purpose, it is sufficient and more convenient to consider all the space-time components of the left-hand side of Eq. (17a) but only the time component of the left-hand side of Eq. (17b). From the canonical commutation relation, it is straightforward to show that

$$[D(t), V_a{}^0(\mathbf{x},t)] = i(-3 - x_\nu \partial^\nu) V_a{}^0(\mathbf{x},t), \tag{18a}$$

$$[D(t), V_a{}^k(\mathbf{x},t)] = i(-1 - x_\nu \partial^\nu) V_a{}^k(\mathbf{x},t) + l\int d^3x' [V_a{}^k(\mathbf{x},t), \pi_b{}^0(\mathbf{x}',t)\varphi_b(\mathbf{x}',t)], \tag{18b}$$

$$[D(t), A_a{}^0(\mathbf{x},t)] = i(-3 - x_\nu \partial^\nu) A_a{}^0(\mathbf{x},t) + l\left[\frac{\partial F_{ab}(\varphi)}{\partial \varphi_d}\varphi_d - F_{ab}(\varphi)\right]\pi_b{}^0, \tag{18c}$$

where the commutator on the right-hand side of Eq. (18b) can be explicitly calculated by use of the equation of motion. For $l=0$, we can immediately see from Eqs. (18a) and (18b) that the time component of the vector current transforms with scaling dimension 3 but the spatial components transform with scaling dimension 1. When the spatial and time components transform differently under the scale transformation, one cannot have asymptotic scale invariance without violating the principle of relativity, and therefore the scale invariance must be essentially broken.[9]

For $l \neq 0$, we can then see the conflict between the scale invariance and the nonlinear realization of the chiral symmetry. It has been shown that[8] the time components of the vector and axial-vector currents should have scaling dimension 3 in order for the current algebra to be preserved under a scale transformation. On the other hand, as seen from Eq. (18c), the condition for $l_A = 3$ is

$$\frac{\partial F_{ab}}{\partial \varphi_d}\varphi_d - F_{ab} = 0, \tag{19}$$

which requires $F_{ab}(\varphi)$ to be a homogeneous function of φ_a. As shown in Ref. 4, the general form of F_{ab} can be written as

$$F_{ab}(\varphi) = \delta_{ab} A(\varphi^2) + \varphi_a \varphi_b B(\varphi^2). \tag{20}$$

The integrability condition (3) together with the definition (5) for F_{ab} implies that A and B must satisfy

$$AB - 1 - 2(A + \varphi^2 B)\frac{dA}{d\varphi^2} = 0. \tag{21}$$

The functions A and B can thus be determined by Eqs. (19) and (21). The result is

$$A(\varphi^2) = i(\varphi^2)^{1/2}, \tag{22a}$$

$$B(\varphi^2) = b(\varphi^2)^{-1/2}, \tag{22b}$$

where b is an arbitrary constant. In this case, the axial-vector current is given by Eqs. (7), (19), and (22) as

$$A_a{}^\mu(x) = [i\delta_{ab}(\varphi^2)^{1/2} + b\varphi_a \varphi_b (\varphi^2)^{-1/2}]\pi_b{}^\mu. \tag{23}$$

Due to the factor i in front of the δ_{ab} term, the axial-vector current becomes non-Hermitian. Therefore, from the point of view of chiral symmetry, the condition (19) for the axial-vector current to have the desired scaling dimension 3 possesses no acceptable solutions.

We have thus shown that, within the general formalism of chiral symmetric nonlinear Lagrangian models, it is impossible to find any canonical frame, specified by A and B, in which the current algebra is scale invariant. The conflict between the nonlinear realization of chiral symmetry and scale invariance can further be made clear by considering the new energy-momentum tensor $\theta^{\mu\nu}$.[10] From the isospin

[7] G. Mack and A. Salam, Ref. 1.

[8] K. Wilson, Ref. 1.

[9] M. A. B. Bég, J. Bernstein, D. J. Gross, R. Jackiw, and A. Sirlin, Phys. Rev. Letters **25**, 1231 (1970).

[10] C. G. Callan, S. Coleman, and R. Jackiw, Ann. Phys. (N. Y.) **59**, 42 (1970).

invariance, $d_{ab}(\varphi)$ can be written as

$$d_{ab}(\varphi)=\delta_{ab}\alpha(\varphi^2)+\varphi_a\varphi_b\beta(\varphi^2)+c_{abc}\varphi_c\gamma(\varphi^2)\,, \quad (24)$$

where α, β, and γ are related to A and B by the condition

$$d_{ab}(\varphi)d_{bc}^{-1}(\varphi)=\delta_{ac}. \quad (25)$$

From Eqs. (5), (20), (24), and (25), we have

$$A\alpha+\varphi^2\gamma=1\,, \quad (26a)$$

$$A\beta+B(\alpha+\varphi^2\beta)+\gamma=0\,, \quad (26b)$$

$$A\gamma+\alpha=0. \quad (26c)$$

From Eqs. (8) and (25), it can be shown that

$$\pi_a{}^\mu\varphi_a=4f^{-2}\varphi_a\partial^\mu\varphi_a[\alpha(\varphi^2)+\varphi^2\beta(\varphi^2)]^2. \quad (27)$$

The right-hand side of Eq. (27) can, therefore, be expressed as a total derivative,

$$\pi_a{}^\mu\varphi_a=\partial^\mu G(\varphi^2) \quad (28)$$

with $G(\varphi^2)$ given by

$$dG/d\varphi^2=2f^{-2}(\alpha+\varphi^2\beta)^2. \quad (29)$$

Then, via the prescription of Ref. 10, we can construct a new energy-momentum tensor

$$\theta^{\mu\nu}=T^{\mu\nu}+\tfrac{1}{3}l(g^{\mu\nu}\Box-\partial^\mu\partial^\nu)G(\varphi^2) \quad (30)$$

such that

$$\mathfrak{D}^\mu(x)=x_\nu\theta^{\mu\nu}(x)\,, \quad (31)$$

$$\partial_\mu\mathfrak{D}^\mu(x)=\theta_\mu{}^\mu(x). \quad (32)$$

The relationship between the scale invariance and chiral symmetry can, therefore, be understood by investigating the properties of this new energy-momentum tensor $\theta^{\mu\nu}$.

It is obvious that $\theta^{\mu\nu}$ is isospin invariant. Furthermore, for the completely chiral symmetric Lagrangian (1), the canonical energy-momentum tensor $T^{\mu\nu}$ is chiral symmetric. Therefore, we only have to investigate the chiral transformation property of the second term on the right-hand side of Eq. (32). Since the axial charge operator

$$Q_a{}^5=\int d^3x\, A_a{}^0(x)$$

is independent of time, the commutation with $Q_a{}^5$ and the space-time differentiation of $G(\varphi^2)$ is interchangeable. The chiral transformation property of $\theta^{\mu\nu}$ is thus completely determined by the commutator $[Q_a{}^5,G(\varphi^2)]$. From Eqs. (10), (29), and (30), we have

$$[Q_a{}^5,G(\varphi^2)]=F_{ab}\varphi_b\frac{dG}{d\varphi^2}$$
$$=2lf^{-2}\varphi_a(A+\varphi^2B)(\alpha+\varphi^2\beta)^2. \quad (33)$$

On the other hand, from Eqs. (25) and (26), we must have

$$(A+\varphi^2B)(\alpha+\varphi^2B)=1\,, \quad (34)$$

which implies that neither $A+\varphi^2B$ nor $\alpha+\varphi^2B$ can vanish if both $d_{ab}(\varphi)$ and $d_{ab}^{-1}(\varphi)$ exist. As a result $G(\varphi^2)$, and therefore $\theta^{\mu\nu}$, cannot be a chiral scalar if f is a finite c-number parameter. Therefore, in no canonical frame can we find a new energy tensor satisfying (31), (32), and chiral invariance at the same time. The chiral noninvariance of $\theta^{\mu\nu}$ reflects the noncommutativity of the dilatation and axial-vector charge operators (which can also be shown directly), and therefore the conflict between the symmetries generated by these two operators.

Furthermore, we also want to point out the following question associated with the chiral noninvariance of the new energy-momentum tensor. As seen from Eqs. (1) and (11), the nonlinear models have a massless free-field limit when f approaches zero. If the new energy-momentum tensor

$$\theta_{\rm free}{}^{\mu\nu}=\partial^\mu\varphi\partial^\nu\varphi-\tfrac{1}{2}g^{\mu\nu}\partial^\lambda\varphi\partial\lambda\varphi+\tfrac{1}{6}(g^{\mu\nu}\Box-\partial^\mu\partial^\nu)\varphi^2 \quad (35)$$

is indeed *the* energy-momentum tensor for a set of massless free scalar (pseudoscalar) fields, then can we construct a new tensor, satisfying Eqs. (31) and (32) or not, for the nonlinear models, such that it becomes $\theta_{\rm free}{}^{\mu\nu}$ in the limit $f\to 0$? As we shall see, the answer is again negative.

It was shown in Ref. 6 that in all the known models we have

$$\lim_{f\to 0}\pi_a{}^\mu=\partial^\mu\varphi_a. \quad (36)$$

In other words, we have

$$\lim_{f\to 0}G(\varphi^2)=\tfrac{1}{2}\varphi^2. \quad (37)$$

Therefore, taking $l=1$, the new energy-momentum tensor $\theta^{\mu\nu}$, given by Eq. (30), does have the limit (35). As seen from Eq. (4b), a chiral transformation cannot transform φ_a into some function containing the derivatives of φ_a. Hence the only possibility of making $\theta^{\mu\nu}$ chiral invariant and still having the limit (35) is to add some other function of φ^2 to $G(\varphi^2)$ such that the result is chiral invariant but the additional function approaches zero in the limit $f\to 0$. However, if we call the resulting new function $F(\varphi^2)$, it can be seen immediately that

$$[Q_a{}^5,F(\varphi^2)]=\varphi_a(A+\varphi^2B)\frac{dF}{d\varphi^2}, \quad (38)$$

which can not vanish unless $dF/d\varphi^2=0$. For $dF/d\varphi^2=0$, F must then be a constant c number and therefore

$$(g^{\mu\nu}\Box-\partial^\mu\partial^\nu)F(\varphi^2)=0. \quad (39)$$

However, the tensor thus constructed just becomes the canonical energy-momentum tensor which only approaches the canonical energy-momentum tensor of the massless free fields instead of $\theta_{\mathrm{free}}{}^{\mu\nu}$ in the limit $f \to 0$.

To conclude, we have shown that axial-vector current can not have scaling dimension 3 in the presence of the nonlinear realization of chiral symmetry for the conventional models. In other words, if we require the axial-vector current to have scaling dimension 3, then it becomes non-Hermitian. The conflict between scale invariance and chiral symmetry is further reflected by the fact that in no canonical frame can one find a chiral-invariant new energy-momentum tensor. Furthermore, we have also shown that it is impossible to construct a new chiral-invariant tensor for the conventional nonlinear models such that it will approach the new energy-momentum tensor in the free-field limit. The scale invariance is thus essentially broken in the conventional nonlinear Lagrangian models of chiral symmetry for a set of pseudoscalar fields, unless the models are drastically modified.

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Veneziano Model for the Process $V_1+\pi \to V_2+\pi$*†

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A Veneziano model is presented for the scattering of pions on isoscalar vector mesons of arbitrary mass, including photons. In addition to the leading term of the amplitude, we allow for satellite terms. The first ones are determined by the requirements of the absence of daughters and the correct widths for the first resonances in all channels.

I. INTRODUCTION

In this work we build a Veneziano-type amplitude[1] for the process $\pi+V_1 \to \pi+V_2$ (V_i is an isosinglet, $C=-1$, vector meson). The main feature distinguishing this work from previous papers[2] is that particular emphasis is given to the structure of the amplitude in the lowest resonance region.

Veneziano-type amplitudes enjoy the benefit of crossing symmetry and correct asymptotic behavior, displayed by the leading term. Their main defect is generally the lack of unitarity.[3] On the other hand, if one wants to describe low-energy scattering, the best-known way to do it is by means of dispersion relations. There we run into the problem of subtraction constants. In order to match these two approaches, the validity of the Veneziano amplitude should be ensured at lower energies. In doing so one has to resort to additional criteria[4] in order to determine the next-to-leading terms. The criteria used by us promise that the Veneziano-type amplitudes will be free of daughter trajectories and we fulfill this requirement, at least in the region of lowest resonances. The amplitude derived here will be matched with a dispersion-relation approach in a theoretical study of the decay $\omega \to 2\pi+\gamma$.[5]

The plan of this article runs as follows. In Sec. II we define invariant and parity-conserving helicity amplitudes. In Sec. III the asymptotic behavior of the amplitudes is given. In Sec. IV we compute the contributions of the ρ and f_0 exchanges. In Sec. V we build the Veneziano amplitudes, and in Sec. VI the main features of our amplitude are discussed, with particular emphasis on the reasoning that leads us to eliminate daughter trajectories.

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[1] G. Veneziano, Nuovo Cimento 57A, 190 (1968).

[2] A. Capella, B. Diu, J. M. Kaplan, and D. Schiff, Nuovo Cimento Letters 1, 655 (1969); P. Carruthers and E. Lasley, Phys. Rev. D 1, 1204 (1970); in contrast to these works our model does not contain the $\epsilon(0^+)$ in the s channel. For π-ρ scattering see, e.g., E. S. Abers and V. L. Teplitz, Phys. Rev. Letters 22, 909 (1969); Phys. Rev. D 1, 624 (1970).

[3] For attempts of unitarizing Veneziano amplitudes, see C. Lovelace, *Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions* (Argonne National Laboratory, Argonne, Ill., 1969), p. 562.

[4] A Veneziano-type amplitude contains *a priori* an infinite number of satellites whose coefficients are usually determined by additional requirements. In some cases, all satellite coefficients may vanish. This may happen for special values of ϵ, $\epsilon=\alpha(s)+\alpha(t)+\alpha(u)$, such as $\epsilon=2$ in the original work of Veneziano (Ref. 1) or $\epsilon=3$ in our amplitude given in (5.1) and (5.2), while in C. Lovelace's work [Phys. Letters 28B, 264 (1968)], the requirement of Adler's consistency conditions on π-π and π-K amplitudes eliminates the satellite terms.

[5] N. Levy and P. Singer, Phys. Rev. D 3 (to be published).