Linear symmetric quark theories of baryon magnetic moments

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A phenomenological and theoretical analysis of the baryon magnetic moments leads to a definition of the class of linear symmetric quark theories for the baryon moments. In this class of theories there are members that can reproduce the experimental results fairly well. However, it seems that despite SU(3) breaking the average of the light-quark magnetic moments $\frac{1}{3}(\mu_u+\mu_d+\mu_s)$ is unexpectedly small. A possible explanation is an anomalous moment for the quarks.

I. INTRODUCTION AND $SU(3)_f$ ANALYSIS OF THE BARYON MOMENTS

Recently the magnetic moments of the Ξ hyperons have been measured with better than 10% accuracy.[1] This allows a model-independent evaluation of the $SU(3)_f$-irreducible components of the magnetic-moment operator. The results of such an evaluation reveal not only the leading terms already known in the symmetry limit but also expose the leading symmetry-breaking corrections. This merits a fresh look at the status of the magnetic-moment calculation trying to extract new information on the structure of baryons and their interactions.

The magnetic moments of the ground-state $SU(3)_f$ octet of $\frac{1}{2}^+$ baryons and the Σ-Λ transition moment may be considered as the nonvanishing matrix elements of a matrix μ_B. With an appropriate phase convention the matrix μ_B is a real symmetric 8×8 matrix. The matrix of the experimentally measured moments can be expanded in a complete set of tensorial matrices provided by the $SU(3)_f$ classification scheme:

$$\mu_B=m^1U_0^{\underline{1}}+m^FU_0^{\underline{8}F}+m_1^FU_1^{\underline{8}F}+m^DU_0^{\underline{8}D}+m_1^DU_1^{\underline{8}D}+m_1^{\underline{10}+}U_1^{\underline{10}+}+\hat{m}^{\underline{27}}I_0^{\underline{27}}+\hat{m}_1^{\underline{27}}I_1^{\underline{27}}+\hat{m}_2^{\underline{27}}I_2^{\underline{27}} . \tag{1.1}$$

The tensorial matrices $U_u^{\underline{n}\gamma}$ are neutral $U_3=0$ members of the representation $\underline{n}$ of $SU(3)_f$ with U-spin u. The index γ distinguishes between the symmetric (D) and antisymmetric (F) coupling schemes for $\underline{n}=\underline{8}$, while $U_1^{\underline{10}+}$ is given by

$$U_1^{\underline{10}+}=\frac{1}{\sqrt{2}}(U_1^{\underline{10}}+U_1^{\overline{\underline{10}}}) . \tag{1.2}$$

The tensorial matrices $I_i^{\underline{27}}$ are $Y=I_3=0$ members of the representation $\underline{27}$ with I-spin i. All the basis matrices employed in the expansion (1.1) are orthogonal to each other in the trace scalar product and the trace of the square of any basis matrix is 8.

The choice of basis matrices with well-defined U-spin properties is motivated by the fact that in the $SU(3)_f$-symmetry limit the magnetic-moment operator is the U-spin-scalar member of an octet. However μ_{Σ^0}, the Σ^0 magnetic moment, is unmeasurable with present-day experimental techniques. We therefore need a theoretical substitute for the unmeasured value of μ_{Σ^0}. Such a substitute is the Marshak-Okubo-Sudarshan (MOS) sum rule[2] giving μ_{Σ^0} as the average of μ_{Σ^+} and μ_{Σ^-}:

$$\mu_{\Sigma^0}=\tfrac{1}{2}(\mu_{\Sigma^+}+\mu_{\Sigma^-}) . \tag{1.3}$$

The MOS sum rule expresses the absence of an isospin-2 term in the magnetic moment of the Σ isospin triplet. Since isospin 2 appears only in the $\underline{27}$ representation it is convenient to span the three-dimensional subspace of matrices in the $\underline{27}$ representation by matrices with well defined isospin properties. With such a basis the MOS sum rule simply gives

$$\hat{m}_2^{\underline{27}}=0 . \tag{1.4}$$

With the restriction (1.4) and the MOS sum rule we can now calculate the other eight expansion coefficients in (1.1) in terms of the eight measured entries in μ_B. Table I gives the measured elements of μ_B, the explicit form of the basis matrices, and the calculated value of the expansion coefficients.

The expansion coefficients fall into three distinct classes:

(1) The coefficients m^F and m^D are clearly determined. They are similar in magnitude to the experimental moments themselves.

(2) The coefficients m_1^F and m_1^D are clearly smaller but being more than 4 standard deviations away from zero are statistically inconsistent with zero.

(3) All other coefficients, namely, m^1, m^{10^+}, $\hat{m}^{27}$, $\hat{m}_1^{27}$, are less than 2 standard deviations away from zero. Thus the signal for these coefficients is presently masked by the statistical noise. As better experiments become available some of them may turn out to be statistically significant but numerically small compared with m_1^F or m_1^D while others continue to be statistically consistent with zero.

This brings out clearly the fact that most of the strength of the matrix element is concentrated in the octet subspaces. It is therefore possible to give an approximate description of the magnetic moments in terms of the four octet coefficients.

The coefficients for the approximate descriptions are determined by a least-squares fit. Namely, we minimize the expression

$$\chi^2 = \sum_i{}' \left[\frac{\mu_i^{\text{expt}} - \mu_i^{\text{fit}}}{\sigma_i} \right]^2 , \tag{1.5}$$

where the summation is over the $\Sigma\Lambda$ transition moment and the experimentally measured magnetic moments. The prime on the summation sign indicates the omission of the Σ^0 magnetic moment. The expression μ_i^{fit} involves the fitted expansion coefficients and can be easily constructed with the help of Table I. For $\sigma_i{}^2$ we put

$$\sigma_i{}^2 = (\Delta\mu_i)^2 + (x\mu_i^{\text{expt}})^2 , \tag{1.6}$$

where $\Delta\mu_i$ is the statistical error in μ_i while x is a number between zero and one. The number x gives a rough measure of the quality of the approximate description. It expresses the fact that we are giving an approximate description for numbers like μ_p or μ_n which are determined experimentally to a part per million, namely, several orders of magnitude better than the approximation we have in mind.

The approximate values for the nonvanishing expansion coefficients are

$$\begin{aligned} m^F &= 1.32 \pm 0.05, \quad m_1^F = -0.18 \pm 0.04 , \\ m^D &= 1.26 \pm 0.05, \quad m_1^D = -0.18 \pm 0.04 , \end{aligned} \tag{1.7}$$

with $x = 0.07$ and $\chi^2_{\min} = 4.18/4$ degrees of freedom. Out of this value of $\chi^2_{\min}$ 1.83 is contributed by μ_{Σ^+} and 1.68 by $\mu_{\Sigma\Lambda}$.

II. QUARK THEORIES FOR THE MAGNETIC MOMENTS

The description of the baryon magnetic moments in the $SU(3)_f$-symmetry limit was predicted 20 years ago by Coleman and Glashow[3] and by Okubo.[4] The basis for the prediction was the assumption that the magnetic-moment operator transforms under $SU(3)_f$ like the charge operator. This leads to the sum rules

$$m^1 = m_1^F = m_1^D = m_1^{10^+} = \hat{m}^{27} = \hat{m}_1^{27} = \hat{m}_2^{27} = 0 . \tag{2.1}$$

It should be noted that this rough description serves to define the phase of $\mu_{\Sigma\Lambda}$ since experiments determine only $|\mu_{\Sigma\Lambda}|$. The Coleman-Glashow theory is reasonable at a level of $x = 0.24$ or about 25% accuracy. This is the level at which most $SU(3)_f$ results are broken.

Static SU(6) theory[5] reproduces the $SU(3)_f$ results (2.1) for the magnetic moments but goes one step further and supplements them by predicting a definite numerical value for the D-to-F ratio r, namely,

$$r = \frac{m^D}{m^F} = \frac{\sqrt{5}}{2} \; (\cong 1.118) . \tag{2.2}$$

To go beyond this rough description we invoke the quark model in which baryons are composed of quarks.[6] In such a framework one ascribes magnetic moments to the quarks and calculates the magnetic moments of the baryons in terms of quark moments.

We introduce the irreducible combinations of quark moments,

$$\begin{aligned} \mu^1 &= \tfrac{1}{3}(\mu_u + \mu_d + \mu_s), \\ \mu^8 &= \frac{\sqrt{2}}{6}(2\mu_u - \mu_d - \mu_s) , \\ \mu_1^8 &= \frac{\sqrt{6}}{6}(\mu_d - \mu_s) . \end{aligned} \tag{2.3}$$

In a naive quark picture where all three quarks are degenerated in mass and have only a Dirac normal magnetic moment, μ^1 and μ_1^8 vanish. However there is no reason to impose such a vanishing in a more realistic quark picture.

The simplest version of the quark model has the three quarks in a baryon in a vanishing relative angular momentum state. In this simple version one gets

TABLE I. The matrix elements of the magnetic moment (in nuclear magnetons), the basis matrices used in the analysis, and the irreducible expansion coefficients.

	μ	$\Delta\mu$	$U^{\underline{1}}$	$U_0^{\underline{8}F}$	$U_1^{\underline{8}F}$	$U_0^{\underline{8}D}$	$U_1^{\underline{8}d}$	$U_1^{\underline{10}+}$	$I_0^{\underline{27}}$	$I_1^{\underline{27}}$	$I_2^{\underline{27}}$
p	2.793	$\sim 10^{-6}$	1	1	1	1	-1	1	3	1	0
n	-1.913	$\sim 10^{-6}$	1	0	2	-2	0	-1	3	-1	0
Λ	-0.6129	0.0045	1	0	0	-1	-1	0	-9	0	0
Σ^+	2.33	0.13	1	1	-1	1	1	-1	-1	0	1
Σ^0	—	—	1	0	0	1	1	0	-1	0	-2
Σ^-	-1.41	0.25	1	-1	1	1	1	1	-1	0	1
Ξ^0	-1.250	0.014	1	0	-2	-2	0	1	3	-1	0
Ξ^-	-0.75	0.06	1	-1	-1	1	-1	-1	3	1	0
Σ-Λ	-1.82	$^{+.18}_{-.25}$	0	0	0	$-\sqrt{3}$	$\sqrt{3}/3$	0	0	$\sqrt{3}$	0
Matrix-normalization factor			1	$\sqrt{2}$	$\sqrt{6}/3$	$\sqrt{10}/5$	$\sqrt{30}/5$	$2\sqrt{3}/3$	$\sqrt{15}/15$	$2\sqrt{5}/5$	$2\sqrt{3}/3$
Expansion coefficient			-0.04	1.29	-0.16	1.32	-0.29	0.07	0.025	-0.12	0
Statistical error			0.05	0.05	0.03	0.08	0.07	0.04	0.015	0.10	

$$\begin{aligned}
&m^{\underline{1}}=\mu^{\underline{1}} ,\\
&m^F=\mu^{\underline{8}},\ \ m_1^F=\mu_1^{\underline{8}} ,\\
&m^D=\frac{\sqrt{5}}{2}\mu^{\underline{8}},\ \ m_1^D=\frac{\sqrt{5}}{2}\mu_1^{\underline{8}} ,\\
&m_1^{\underline{10}+}=\hat{m}^{\underline{27}}=\hat{m}_1^{\underline{27}}=\hat{m}_2^{\underline{27}}=0 .
\end{aligned} \tag{2.4}$$

Thus the simple version of the quark model describes the magnetic moments in terms of five expansion cofficients but in addition it gives two SU(6)-type relations,

$$r=\frac{m^D}{m^F}=\frac{m_1^D}{m_1^F}=\frac{\sqrt{5}}{2} . \tag{2.5}$$

This leaves us with three independent coefficients which is to be expected in a model where all magnetic moments are simple linear homogeneous functions of the three quark moments.

Comparing Eq. (2.4) with the results in Table I, we see that out of the nonoctet coefficients only $m^{\underline{1}}$ does not vanish identically. Thus for (2.4) to be the theoretical explanation of the approximation (1.7) we need $\mu^{\underline{1}}$ to vanish. The vanishing of $\mu^{\underline{1}}$ is unexpected because once the quark moments deviate from the naive SU(3) Dirac values it is not clear why the magnitude of $\mu_1^{\underline{8}}$ should be significantly larger than $\mu^{\underline{1}}$. To check further the consistency of (1.7) with (2.5) we calculate the two ratios

$$\frac{m^D}{m^F}=0.95\pm0.05,\quad \frac{m_1^D}{m_1^F}=1.00\pm0.31 . \tag{2.6}$$

Thus although the second ratio is consistent with the predicted value of $\sqrt{5}/2$ the first one is just about three standard deviations away from it. Therefore the simple version of the quark model needs some improvement.

We can improve the simple version of the quark model by modifying the simple angular momentum structure of the wave function. Franklin[7] enumerated all possible corrections to the magnetic moments calculation due to spin and orbital angular momentum configurations mixing. Geffen and Wilson[8] worked out in detail a particular model in which the two like quarks in a hadron have probability $\cos^2\theta$ to be in a spin-triplet state and probability $\sin^2\theta$ to be in a spin-singlet state. The predictions of the Geffen-Wilson model for the moments are given in Table II. With these predictions one

TABLE II. The Geffen-Wilson model. The angle θ is the configuration-mixing angle. The value $\theta=0$ gives the predictions of the simple quark model. The table is taken from the University of Minnesota report of Ref. 8. The sign of $\mu_{\Sigma\Lambda}$ was changed to conform with the Coleman-Glashow conventions.

$$\begin{aligned}
&\mu_p=\tfrac{1}{3}(4\mu_u-\mu_d)\cos^2\theta+\mu_d\sin^2\theta\\
&\mu_n=\tfrac{1}{3}(4\mu_d-\mu_u)\cos^2\theta+\mu_u\sin^2\theta\\
&\mu_\Lambda=\mu_s\cos^2\theta+\tfrac{1}{3}(2\mu_u+2\mu_d-\mu_s)\sin^2\theta\\
&\mu_{\Sigma^+}=\tfrac{1}{3}(4\mu_u-\mu_s)\cos^2\theta+\mu_s\sin^2\theta\\
&\mu_{\Sigma^0}=\tfrac{1}{2}(\mu_{\Sigma^+}+\mu_{\Sigma^-})=\tfrac{1}{3}(2\mu_u+2\mu_d-\mu_s)\cos^2\theta+\mu_s\sin^2\theta\\
&\mu_{\Sigma^-}=\tfrac{1}{3}(4\mu_d-\mu_s)\cos^2\theta+\mu_s\sin^2\theta\\
&\mu_{\Xi^0}=\tfrac{1}{3}(4\mu_s-\mu_u)\cos^2\theta+\mu_u\sin^2\theta\\
&\mu_{\Xi^-}=\tfrac{1}{3}(4\mu_s-\mu_d)\cos^2\theta+\mu_d\sin^2\theta\\
&\mu_{\Sigma\Lambda}=-\frac{\sqrt{3}}{3}(\mu_u-\mu_d)(2\cos^2\theta-1)
\end{aligned}$$

gets

$$\begin{aligned}
&m^{1}=\mu^{1}\,,\\
&m^{F}=\mu^{8}\cos^2\theta,\quad m_1^{F}=\mu_1^{8}\cos^2\theta\,,\\
&m^{D}=\mu^{8}\frac{\sqrt{5}}{2}(2\cos^2\theta-1)\,,\\
&m_1^{D}=\mu_1^{8}\frac{\sqrt{5}}{2}(2\cos^2\theta-1)\,,\\
&m_1^{10^+}=\hat{m}^{27}=\hat{m}_1^{27}=\hat{m}_2^{27}=0\,.
\end{aligned}\tag{2.7}$$

Once again we have a description of the baryon magnetic moments in terms of five expansion coefficients and two relations among them, namely,

$$m^{D}m_1^{F}-m^{F}m_1^{D}=0\,,\tag{2.8a}$$

$$r=\frac{m^{D}}{m^{F}}=\frac{\sqrt{5}}{2}(1-\tan^2\theta)\,.\tag{2.8b}$$

Since Eqs. (2.7) give an equality between m^1 and μ^1 it is desirable to have μ^1 very small so that (2.7) serves as a theoretical explanation of the approximate description (1.7). Evaluating the left-hand side of (2.8a) we get

$$m^{D}m_1^{F}-m^{F}m_1^{D}=0.01\pm0.08\tag{2.9a}$$

while from (2.8b) we get

$$\tan^2\theta=0.15\pm0.05\,.\tag{2.9b}$$

This is a rather small mixing angle but we have no other independent determination of θ to compare with.

Thus if the quark moments are such that μ^1 vanishes, the Geffen-Wilson model with the mixing angle (2.9b) provides a reasonable description of the baryon magnetic moments at an accuracy level of about 7%.

The simple version of the quark model and the Geffen-Wilson model may be considered as two members of a class of theories which we now define. We say that a theory belongs to the class of linear symmetric theories if it fulfills the following two demands:

(1) The theory gives the baryon-magnetic-moment expansion coefficients as linear homogeneous functions of the quark-moment expansion coefficients.

(2) The coefficients in the linear homogeneous functions are $SU(3)_f$ symmetric.

With these definitions it follows that for any theory in this class we have the following relations:

$$\begin{aligned}
&m^{1}=\alpha\mu^{1}\,,\\
&m^{F}=\beta\mu^{8},\quad m_1^{F}=\beta\mu_1^{8}\,,\\
&m^{D}=\gamma\mu^{8},\quad m_1^{D}=\gamma\mu_1^{8}\,,\\
&m_1^{10^+}=\hat{m}^{27}=\hat{m}_1^{27}=\hat{m}_2^{27}=0\,.
\end{aligned}\tag{2.10}$$

Theories belonging to the class may differ from each other by the values of the SU(3)-symmetric constants α, β, and γ that they give. If simple quark ideas are right we may expect α, β, and γ to be close to $+1$ in any reasonable theory in the class. In (2.10) we have a description of the baryon magnetic moments in terms of five expansion coefficients with the following relations:

$$m^{D}m_1^{F}-m^{F}m_1^{D}=0\,,\tag{2.11a}$$

$$r=\frac{m^{D}}{m^{F}}=\frac{\gamma}{\beta}\,.\tag{2.11b}$$

A best fit determining the independent coefficients in (2.10) gives

$$m^{1}=-0.04\pm0.06,\quad m^{F}=1.31\pm0.06\,,$$
$$m_1^{F}=-0.20\pm0.04,\quad r=0.95\pm0.04$$

with $x=0.07$ and $\chi^2_{\min}=3.78/4$ degrees of freedom. Out of this value of $\chi^2_{\min}$ 1.80 is contributed by $\mu_{\Sigma\Lambda}$ and 1.17 by μ_{Σ^+}.

Once more, for (2.10) to be able to reproduce the approximate description (1.7) m^1 would have to vanish. But now it is not clear if the approximate vanishing of m^1 is produced by an approximate vanishing of μ^1 or an approximate vanishing of α. To gain some understanding of the situation let us estimate[9] μ^1 and μ_1^8. For quark moments which are normal Dirac moments[10] we get

$$\begin{aligned}
\mu^{1}&=\frac{e}{18}\left[\frac{2}{m_\mu}-\frac{1}{m_d}-\frac{1}{m_s}\right]\Big/\left[\frac{e}{2m_p}\right]\\
&=\frac{m_p}{9}\left[\frac{2}{m_u}-\frac{1}{m_d}-\frac{1}{m_s}\right],
\end{aligned}\tag{2.12a}$$

$$\mu_1^{8}=\frac{\sqrt{6}m_p}{18}\left[-\frac{1}{m_d}+\frac{1}{m_s}\right],\tag{2.12b}$$

where m_u, m_d, and m_s are the quark constituent masses while m_p is the proton mass. In view of the approximate isospin invariance in hadronic phenomena we assume $m_u\cong m_d$ giving

$$\mu^{1}=\frac{1}{9}\frac{m_p}{m_u}\left[1-\frac{m_u}{m_s}\right],\tag{2.13a}$$

$$\mu_1^{8}=-\frac{\sqrt{6}}{18}\frac{m_p}{m_u}\left[1-\frac{m_u}{m_s}\right].\tag{2.13b}$$

Since in any reasonble model $m_u/m_s<1$ we get from (2.13) a positive $\mu^{\underline{1}}$ and a negative $\mu^{\underline{8}}_{\overline{1}}$. Moreover (2.13) leads to

$$\mu^{\underline{1}}/\mu^{\underline{8}}_{\overline{1}}=-\frac{\sqrt{6}}{3}(\cong -0.816) \;. \tag{2.14}$$

Thus with normal Dirac moments for the quark $\mu^{\underline{1}}$ and $\mu^{\underline{8}}_{\overline{1}}$ have similar magnitudes leading to an approximate vanishing of α. We consider such a deviation from the simple quark-model value $\alpha=1$ large and undesirable and suggest that the way to understand the approximate vanishing of $m^{\underline{1}}$ is to blame it on the approximate vanishing of $\mu^{\underline{1}}$. In other words in the framework of linear symmetric models we should get

$$\mu^{\underline{1}}=0 \;. \tag{2.15}$$

This contradicts the estimate we derived for quarks with normal Dirac moments. This result was in essence known to Geffen and Wilson although they phrased it differently and restricted themselves to their version of a linear symmetric model. The explanation they suggest is that besides normal Dirac moments quarks have anomalous magnetic moments.

With $\mu^{\underline{1}}$ vanishing (2.10) corresponds to the approximate description (1.7). Actually according to (2.10) the four octet coefficients can be expressed in terms of three parameters namely m^F, $m^F_{\overline{1}}$, and r. A best fit gives

$$m^F=1.32\pm0.06,\quad m^F_{\overline{1}}=-0.19\pm0.03 \;, \qquad r=0.95\pm0.04 \tag{2.16}$$

with $x=0.07$ and $\chi^2_{\min}=4.18/5$ degrees of freedom. Out of this value of $\chi^2_{\min}$, 1.86 is contributed by μ_{Σ^+} and 1.70 by $\mu_{\Sigma\Lambda}$.

III. CONCLUSIONS

We have seen that there is a reasonable phenomenological description of the baryon magnetic moments in terms of four expansion coefficients (1.7) or in terms of three parameters (2.16). A closer look at the various fits we get reveals that μ_{Σ^+} and $\mu_{\Sigma\Lambda}$ may be important sources of discrepancy. It would be nice to have more accurate measurements of μ_{Σ^+}, μ_{Σ^-}, and $\mu_{\Sigma\Lambda}$.

On the theoretical side we have seen that any linear symmetric model which agrees with the fit (2.16) gives a reasonable explanation of the data at the 7% level. Moreover, it seems clear that the $SU(3)_f$-singlet quark moment $\mu^{\underline{1}}$ is small and approximately zero. The vanishing of $\mu^{\underline{1}}$ contradicts the idea that quarks have pure Dirac magnetic moments and leads to the suggestion that they have anomalous magnetic moments. Geffen and Wilson assumed that the anomalous part of the magnetic moment is independent of the charge of the quark but may depend on its mass. Therefore, in this isospin-invariant approximation the anomalous moment is an isoscalar. This gives

$$\mu_u=\frac{m_p}{3m_u}(2+\delta_{\rm an}) \;, \tag{3.1}$$

$$\mu_d=\frac{m_p}{3m_u}(-1+\delta_{\rm an}) \tag{3.2}$$

while the vanishing of $\mu^{\underline{1}}$ gives

$$\mu_s=\frac{m_p}{3m_u}(-1-2\delta_{\rm an}) \;. \tag{3.3}$$

Substituting in Eq. (2.7) for m^F and $m^F_{\overline{1}}$ we get

$$\frac{m^F_{\overline{1}}}{m^F}=\frac{\sqrt{3}\delta_{\rm an}}{2+\delta_{\rm an}} \tag{3.4}$$

giving, with (2.17),

$$\delta_{\rm an}=-0.15\pm0.02 \tag{3.5}$$

and

$$\mu_u/\mu_d=-1.61\pm0.05 \tag{3.6}$$

which agrees with the Geffen-Wilson estimate of -1.6. An anomalous moment for the quark may be a hint that quarks are not elementary but possess an internal structure.

To avoid this conclusion one has to avoid linear symmetric models, e.g., by a more complicated configuration-mixing scheme, where baryon moments receive an orbital contribution. The vanishing of $\mu^{\underline{1}}$ means that the magnetic moment operator is proportional to a generator of flavor SU(3) in the space of the light quarks u,d,s. We know of no reason why this should be the case. The question whether magnetic momentlike operators constructed with flavor-changing currents have a similar property merits some attention.

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