# Two- and three-body contributions to cosmological monopole annihilation

Duane A. Dicus and Don N. Page*
*Center for Particle Theory and Center for Theoretical Physics, University of Texas, Austin, Texas 78712*

Vigdor L. Teplitz
*Strategic Affairs Division, U.S. Arms Control and Disarmament Agency, Washington, D. C. 20451*
*and Physics Department, University of Maryland, College Park, Maryland*
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We show (1) that the two-body cross section for monopole-antimonopole annihilation is smaller when photon discreteness effects are included than previously believed; (2) that the three-body annihilation process should dominate the two-body process; but (3) that, even when monopole clumping and monopole dilution by entropy production is taken into account, these processes do not appear to provide a solution to the cosmological monopole problem in that either the monopoles do not annihilate or they annihilate so late that an unacceptable amount of entropy is produced.

## I. INTRODUCTION

Grand unified theories[1] of the strong and electroweak interactions offer the possibility of explaining the baryon excess of the Universe[2] but have the apparent problem of producing too many magnetic monopoles.[3,4] Mechanisms have been suggested for reducing the initial monopole production,[5–7] but they are uncertain and generally have undesirable side effects. Gravitational clumping has been suggested[4] and analyzed[8,9] as a possible means for enhancing the radiative recombination and subsequent annihilation of monopole-antimonopole pairs, but this mechanism is not sufficient.

In this paper we shall show that the cross section for two-body annihilation is even smaller than that previously used. We then investigate the consequence of a number of multibody interactions, particularly three-body recombinations, that have not been considered before for monopoles. Some of these mechanisms are much more effective than those previously analyzed and *a priori* might lead to the annihilation of most monopoles. However, we have not succeeded in finding a mechanism which can get rid of the monopoles early enough so that they do not produce more entropy than is allowed by present upper limits on the photon-to-baryon ratio.

For notational simplicity, we shall set $\hbar=c=G=k=4\pi\epsilon_0=1$ to obtain Planck units in which every quantity is dimensionless. To obtain the estimates which are sufficient for our purposes, and to emphasize the basic physics without getting bogged down in numerical details, we shall usually neglect factors that are close to unity, such as $4\pi$, the number of participating particle species, various logarithms, etc. Only when one finds a marginally effective mechanism for solving the monopole problem would it be worthwhile to go back and put in these numerical refinements. When it is necessary to put in numbers, we shall take the electric charge as $e=\alpha^{1/2}\sim 10^{-1}$, the magnetic charge of monopoles as $g=$integral multiple of $\frac{1}{2}e^{-1}\sim\alpha^{-1/2}\sim 10$, and the mass of the monopole[10] as $m\sim 10^{16}$ GeV $\sim 10^{-3}$ in Planck units.

The early Universe contains number densities $n_f$ of free monopoles, $n_b$ of bound monopoles, $n_M=$spatial average of $n_f+n_b$ (all monopoles), $n_p$ of charged light particles (plasma), $n_\gamma$ of photons, and $n_{\rm other}$ of other particles. During the times of interest, $n_p\sim n_\gamma\sim n_{\rm other}\sim T_p{}^3$, where $T_p$ is the plasma temperature. The plasma will be relativistic for $T_p\gtrsim m_{\rm electron}\sim 10^{-22}$, so the charged-particle rest mass will be ignored, but the monopoles will have temperature $T_M \ll m$ and so will be nonrelativistic. The entropy density is then

$$s\sim n_M+n_p+n_\gamma+n_{\rm other}\sim T_p{}^3\ ,$$

and the energy density is $\rho=\rho_M+\rho_p\sim mn_M+T_p{}^4$. When the Universe is nearly homogeneous and isotropic, the spatial curvature is negligible at early times, and the Einstein equations imply an expansion time scale $t\sim\rho^{-1/2}$ (roughly the age of the Universe then).

It is convenient to choose a comoving scale

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length $a$ such that a cell of this size would contain, on a spatial average, one unit of entropy $S$ (i.e., roughly one particle) at some initial time after monopole production. As the monopoles annihilate far out of equilibrium, $S \equiv sa^3$ will increase above its initial value of unity. Then one may distinguish between the decreasing number $\nu \equiv n_M a^3$ of monopoles per comoving cell volume and the more rapidly decreasing ratio

$$\epsilon \equiv \nu/S \equiv n_M a^3/sa^3 = n_M/s \sim n_M/n_\gamma \sim n_M T_p^{-3} \tag{1}$$

of monopoles to entropy. Since the annihilation of one monopole releases energy $m$ which is then thermalized at temperature $T_p$ to increase the plasma entropy by $m/T_p \gg 1$,

$$dS = -\frac{m}{T_p}d\nu = -\frac{m}{T_p}(\epsilon\, dS + S\, d\epsilon) \tag{2}$$

or

$$\ln S = -\int_{\epsilon_i}^{\epsilon} \frac{d\epsilon}{\epsilon + T_p/m}\ , \tag{3}$$

where $\epsilon_i = \nu_i$ is the initial monopole-to-entropy ratio at $S = S_i = 1$. Contributions to the integral are only significant during periods of monopole domination of the Universe's energy density. Since the effective initial time is after the production of the supermassive bosons that decay into a baryon excess[2] of no more than $10^{-2}$ of the entropy, one must have today $S_f \le 10^9$ to avoid diluting the baryon-to-entropy ratio below the presently observed lower bound $(10^{-11})$.[8] On the other hand, the present cosmological limits on the density of the Universe imply[4,8] that $\epsilon_f \le 10^{-24}$. Then if we assume that the plasma temperature is monotonically decreasing with time (since it is hard to see how the monopole annihilation rate could be fast enough to raise $T_p$ without thereby cutting itself back), we have

$$\begin{aligned}\ln 10^9 \ge \ln S_f &= \ln S - \int_{\epsilon}^{\epsilon_f} \frac{d\epsilon}{\epsilon + T_p/m} \\ &\ge \ln S + \ln\left[\frac{\epsilon + T_p/m}{\epsilon_f + T_p/m}\right],\end{aligned} \tag{4}$$

which implies

$$\nu = S\epsilon \le 10^{-15} + (10^9 - S)T_p/m \tag{5}$$

at the time the plasma temperature is $T_p$. Thus the monopole number per initial entropy must decrease at least to $10^{-15}$ by $T_p \sim 10^{-24} m \sim 10$ eV. There are corresponding looser bounds on $\nu$ at higher temperatures. There may also be a problem with thermalizing the annihilation radiation if it occurs too late. See Appendix A for a very brief discussion of thermalization.

Zeldovich and Khlopov[3] estimated the annihilation of monopoles diffusing through the plasma when their mean-free path is short compared with the size $r_0$ of bound pairs with an ionization energy $\sim \alpha^{-1} r_0^{-1}$ equal to $T_p$; $r_0 \sim (\alpha T_p)^{-1}$. Preskill[4] used an improved estimate of the monopole-plasma scattering time

$$\tau_s \sim m T_p^{-2} \tag{6}$$

to deduce by the same method that the monopole number is reduced to

$$\nu \sim \epsilon \sim m\alpha^3 \sim 10^{-9} \tag{7}$$

at a temperature $T_p \sim m\alpha^2 \sim 10^{12}$ GeV, which is well within the limit (5). (Preskill actually obtained $10^{-10}$ by including a factor $\xi^{-1/2}$ for $\xi \sim 100$ relativistic particle species.) However, at lower temperatures the scattering by the plasma becomes ineffective in binding monopoles into pairs with $r \lesssim r_0$ so negligible additional annihilation can occur by this mechanism. Zeldovich and Khlopov[3] perform a classical calculation, which Preskill[4] and later references[8,9] also cite to give the radiative recombination cross section for a monopole-antimonopole pair to go into a bound orbit by the emission of bremsstrahlung radiation. They find that for an incident velocity $v \sim (T/m)^{1/2}$,

$$\sigma_c \sim m^{-2}\alpha^{-2} v^{-14/5} \sim m^{-3/5}\alpha^{-2} T_M^{-7/5}\ , \tag{8}$$

which is too small to reduce $\nu$ significantly.

At temperatures below $m\alpha^{5/2} \sim 10^{11}$ GeV, radiative recombination is even less effective than this estimate because the classical calculation becomes invalid. The incident motion of the monopole-antimonopole pair may be treated classically since they can be in wave packets which spread negligibly during the magnetic Coulomb collisions for the relevant large values of the angular momentum. However, it is also important to consider discreteness effects for the bremsstrahlung photons. The classical calculation predicts capture for incident angular momenta $l \le l_c \sim \alpha^{-1} v^{-2/5}$, but for $l \ge l_Q \sim \alpha^{-3/2}$ (which is less than $l_c$ for $v \lesssim \alpha^{5/4}$) the expected energy emitted is less than the dominant emission frequency $\omega_{\text{peak}} \sim m\alpha^{-2} l^{-3}$ corresponding to the time scale at closest approach. In a quantum analysis of the electromagnetic field radiated by the monopoles during their classical motion, the number of photons emitted with $\omega \gtrsim mv^2$ has a Poisson probability distribution with an expectation number $N_l \sim (l_Q/l)^2\theta(l_m - l)$, with the cutoff at $l_m \sim \alpha^{-2/3}v^{-2/3}$ where the dominant photon frequency falls below the incident kinetic energy. For $v \lesssim \alpha^{5/4}$, most captures occur when at least one photon is emitted with $\omega \gtrsim mv^2$, leading to a quantum capture cross section

$$\begin{aligned}\sigma_Q &\approx 4\pi m^{-2}v^{-2}\sum_l (2l+1)[1-\exp(-N_l)]\\ &\sim m^{-2}v^{-2}l_Q{}^2\ln(l_m/l_Q)\\ &\sim m^{-2}\alpha^{-3}v^{-2}\ln(\alpha^{5/4}/v)\ .\end{aligned} \tag{9}$$

For $\alpha^{5/4} \lesssim v \ll 1$, $l_m < l_c$ so that capture generally occurs by the emission of many photons, each with $\omega \lesssim mv^2$ but whose total energy exceeds the incident kinetic energy. This gives the classical cross section (8).

Appendix B gives a more detailed analysis of the classical and quantum cross sections, including the numerical factors omitted in the body of this paper. For monopoles of equal mass $m$ and equal but opposite magnetic charges $\pm\frac{1}{2}n\alpha^{-1/2}$, the cross section to leading order in $\alpha^{-5/4}v$ or its reciprocal is

$$\begin{aligned}\sigma \sim\; & 3^{-3/2}2.5\pi m^{-2}n^6\alpha^{-3}v^{-2}\\ & \times\ln(1+2^{-1/5}3^{3/2}5^{-1}\pi^{2/5}n^{-2}\alpha v^{-4/5})\ .\end{aligned} \tag{10}$$

In any case, radiative recombination produces only a negligible amount of monopole annihilation after $\epsilon$ is reduced to $\sim 10^{-10}$ by diffusion-induced annihilation, because this two-body process has a time scale

$$\tau_2 \sim \langle n_M \sigma v\rangle^{-1} \sim \epsilon^{-1}T_p{}^{-3}m^{3/2}\alpha^3 T_M{}^{1/2}\ , \tag{11}$$

which is far longer than the expansion time scale $t$, just as it is for electron-positron annihilation in the late Universe.[11,12]

## II. THREE-BODY RECOMBINATION BEFORE DECOUPLING

Three-body collisional (nonradiative) recombination ($M+\overline{M}+M \rightarrow M\overline{M}+M$) very early becomes more important than radiative recombination for the monopoles, as it also does for electrons and positrons in the late Universe.[11,12] Applying detailed balance and an electromagnetic duality transformation to the atomic collisional ionization cross section[13] gives a recombination rate coefficient into the $n$th bound level of

$$C_{i,n} \sim n^4 m^{-3}T_M{}^{-1}(T_M + I_n)^{-1}\ , \tag{12}$$

where the binding energy of the monopole-antimonopole pair is

$$I_n \sim m\alpha^{-2}n^{-2}\ . \tag{13}$$

If we consider only binding into states with $I_n > T_M$ ($n < n_T \sim m^{1/2}\alpha^{-1}T_M{}^{-1/2}$) which are not likely to be broken up again, the three-body recombination rate per monopole is

$$\begin{aligned}\tau_3{}^{-1} &\sim n_f{}^2\sum_{n=1}^{n_T} C_{i,n}\\ &\sim n_f{}^2 m^{-1/2}\alpha^{-5}T_M{}^{-9/2}\ .\end{aligned} \tag{14}$$

Using $n_M \sim n_f \sim \epsilon T_p{}^3$ the time scale $\tau_3$ is shorter than $\tau_2$ for $T_p \sim T_M \lesssim m\epsilon\alpha^{-2} \sim 10^{-8} \sim 10^{11}$ GeV but is longer than the expansion time $t$ until much later when the monopole temperature $T_M$ can drop below the plasma temperature $T_p$.

So long as the monopole-plasma scattering time $\tau_s \sim mT_p{}^{-2}$ is short compared with the expansion time $t$, the monopoles are kept in thermal equilibrium with the plasma. This has the advantage of producing a drag force[8] on the monopoles which causes bound pairs with $I_n > T_p$ (and hence orbital velocities higher than thermal velocities) to spiral together and annihilate in a time $\tau_s < t$, but it has the disadvantage of keeping $T_M \sim T_p$ so that $\tau_3$ is long. The ratio $\tau_s/t \sim m\rho_p{}^{-1/2}\rho^{1/2}$ is always small when the Universe is radiation-(plasma-) dominated and becomes of order unity only when the monopole mass density dominates that of the plasma by $m^{-2} \sim 10^6$. This occurs at the decoupling temperature $T_d \sim m^3\epsilon$. Only after decoupling can $T_M$ drop faster than $T_p$.

During radiation domination ($T_p \gtrsim T_1 \sim m\epsilon \sim 10^7$ GeV), $T_M \sim T_p$, $n_f \sim n_M$, and $\tau_3/t \sim m^{1/2}\alpha^5\epsilon^{-2}T^{1/2} \gtrsim 1$, so not very much three-body binding occurs. Once the Universe becomes monopole dominated at $T_p \sim T_1$, $\tau_3/t \sim m\alpha^5\epsilon^{-3/2}$, which for $\epsilon \sim 10^{-9}$ is within an order of magnitude or so of unity. Hence there may be a small amount of three-body binding followed by plasma-induced spiraling and annihilation at this stage. Although the number of monopoles is not reduced much, a small reduction in number can reduce $\epsilon$ significantly by increasing the entropy per comoving volume, $S \equiv sa^3 \propto T_p{}^3a^3$. Differentiating this expression and using (1) and (2) gives

$$\frac{d\ln T_p}{d\ln\epsilon}=\left[1+\frac{T_p}{m\epsilon}\right]^{-1}\left[-\frac{T_p}{m}\frac{d\ln a}{d\ln\nu}-\frac{1}{3}\right]$$

$$\approx\frac{T_p}{m\epsilon}\frac{\tau_3}{t}-\frac{1}{3}$$

$$\sim\alpha^5\epsilon^{-5/2}T_p-\tfrac{1}{3}\ , \tag{15}$$

which has the approximate solution $\epsilon\sim\alpha^2 T_p{}^{2/5}$ for $T_p<m\epsilon$. This is valid until the temperature drops to the decoupling temperature

$$T_d\sim m^3\epsilon_d\sim m^3\alpha^2 T_d{}^{2/5}\sim m^5\alpha^{10/3}\sim 10^{-22}\sim 1\text{ MeV}\ , \tag{16}$$

at which point the rate of monopoles to entropy is

$$\epsilon_d\sim m^2\alpha^{10/3}\sim 10^{-13}\ , \tag{17}$$

a factor of $m\alpha^{1/3}\sim 10^{-4}$ of what it was at $T>T_1$. Very little of this decrease directly reflects a change in the monopole number $\nu$, since $\ln\nu$ decreases by only a small amount $\sim m^{1/5}\alpha^{-1/5}\sim 1$. Most of the decrease reflects an increase in the plasma entropy $S$ by a factor $\sim 10^4$. This by itself lies within the bound $S_f\leq 10^9$, but now the limit (5) is being violated; the temperature is now so low that if the remaining $\nu\sim m\alpha^3\sim 10^{-9}$ monopoles annihilate they will produce too much entropy (unless the temperature can be raised again without cutting off the annihilation).

## III. RECOMBINATION AND ANNIHILATION AFTER DECOUPLING

Despite the entropy problem that apparently arises if the monopoles do annihilate, let us examine what may happen to them after decoupling. Three-body recombination becomes more effective as $T_M$ drops below $T_p$, but now the plasma drag no longer causes the monopoles to spiral together and annihilate on a time scale short compared with the expansion time. Thus even if $\tau_3$ drops below $t$ so that most remaining monopoles become bound, they will exist in bound states a long time before they annihilate; this worsens the entropy problem. Later we shall examine the question of gravitational clumping, which may increase the annihilation rates somewhat,[4,8,9] but first simply ask how fast bound monopole pairs can annihilate when $\tau_s>t$ and $T_M<T_p$. For pairs with absolute energy $I_n>T_p$, the plasma drag force[8] on the monopoles causes them to spiral together with

$$\frac{d\ln I_n}{dt}=\frac{1}{\tau_s}\sim\frac{T_p{}^2}{m}\ . \tag{18}$$

But if $\tau_s>t$, only a small amount of spiraling in occurs before $T_p$ becomes too low to be effective in this way.

When the plasma and monopoles reach densities so low they do not have a significant effect on the bound pair, it will spiral in from high principal quantum number $n$ and angular momentum $l$ in a time[14]

$$t_{\text{dipole}}\approx\frac{2n^3l^5}{mg^{10}(n+l)^2}$$

$$\sim m^{-1}\alpha^5 n^6\sim m^2\alpha^{-1}I_n{}^{-3} \tag{19}$$

by the spontaneous emission of magnetic dipole radiation. This can be a very long time. For example, in the case $I_n\sim T_d\sim m^5\alpha^{10/3}$, we have $t_{\text{dipole}}\sim m^{-13}\alpha^{-11}\sim 10^{62}\sim 10^{11}$ yr, so such monopole pairs would still be around today, probably greatly in excess of the cosmological upper limits.

One must consider, however, the effects of the plasma and other monopoles. For a long time the orbital frequency of the bound monopole pair, $\omega\sim m\alpha^{-2}n^{-3}\sim m^{-1/2}\alpha I_n{}^{3/2}$, is much lower than $T_p$. One might think there would be a large number of photons per mode, leading to absorption and stimulated emission of radiation. But such electromagnetic modes do not exist below the relativistic plasma frequency

$$\omega_p\sim\alpha^{1/2}T_p\ , \tag{20}$$

so no photons can be emitted or absorbed for $\omega<\omega_p$. One can calculate the energy dissipation of the orbiting monopoles' evanescent fields due to ohmic losses in the plasma, which has a mean-free path $\lambda_p\sim\alpha^{-2}T_p{}^{-1}$ and a conductivity $\sim\alpha^{-1}T_p$. This turns out to be essentially the same as the dissipation (18) due to the plasma drag force, which is calculated[8] by assuming the charged particles act independently on the monopoles. (The individual particle effects must go over to the collective particle effects at an impact parameter roughly that of the mean-free path, which explains the approximate equality between the two since the ohmic losses are greatest near this distance.) There is also some energy radiated into phonons, but generally much less power goes into sound than into heat. Thus none of these plasma effects lead to much monopole annihilation if $\tau_s>t$.

Now consider magnetic Coulomb collisions between monopoles. Free monopoles with temperature $T_M$ undergo large-angle scattering with a cross section $\sigma \sim \alpha^{-2} T_M^{-2}$, which leads to a free-free thermalization time

$$\tau_{\mathrm{ff}} \sim m^{1/2} \alpha^2 n_f^{-1} T_M^{3/2} . \quad (21)$$

This is less than the expansion time for

$$\epsilon \gtrsim m^2 \alpha^4 (n_M/n_f)^2 (T_M/T_p)^3 ,$$

so until or unless $\epsilon$ becomes smaller than $\epsilon_d$, the free monopoles are kept nearly in thermal equilibrium with themselves, though not with the plasma when $\tau_s > t$. Bound monopole pairs with $I_n < T_M$ will also be kept in thermal equilibrium at the Saha abundances, but there will be a deficit of pairs with $I_n > T_M$ as the three-body recombination rates are not fast enough to maintain the thermal abundances. Collisions with free monopoles will cause the abundances to diffuse inward to lower quantum levels $n$, but this will be limited by the collision rates. For bound pairs with $I_n > T_M$ and $l \sim n - l$, the magnetic analog of the Stark effect pulls in an incident-free monopole with energy $E \sim T_M$ and angular momentum $L \lesssim n$ to a close collision[11] which should lead to a nonperturbative redistribution of energy (e.g., with the ejected monopole carrying off energy $\sim I_n$ and the remaining pair being left roughly twice as strongly bound as before). The cross section for this $M\overline{M} + M \to M\overline{M} + M$ is $\sigma \sim n^2 m^{-1} E^{-1}$, which gives a bound-free collision time per bound pair of

$$\tau_{\mathrm{bf}} \sim m^{1/2} \alpha^2 n_f^{-1} I_n T_M^{1/2} . \quad (22)$$

These collisions will cause the pairs to become more tightly bound until

$$\tau_{\mathrm{bf}}/t \sim m \alpha^2 \epsilon^{-1/2} n_M n_f^{-1} I_n T_M^{1/2} T_p^{-3/2}$$

becomes of order unity, thus leaving most bound pairs with

$$n \sim m n_M^{1/4} n_f^{-1/2} T_M^{1/4} . \quad (23)$$

If most monopoles become bound ($n_b > n_f$), it may also become important to consider bound-bound collisions between pairs of bound monopoles. Since the force between pairs falls off more rapidly with distance than the angular momentum barrier (unlike the case of a free monopole attracted to a bound pair with a $1/r^2$ potential), the cross section will be given roughly by the size of the larger pair,

$$\sigma \sim r_n^2 \sim m^{-2} \alpha^2 n^4 \sim \alpha^{-2} I_n^{-2} ,$$

leading to a bound-bound collision time scale

$$\tau_{\mathrm{bb}} \sim m^{1/2} \alpha^2 n_b^{-1} I_n^2 T_M^{-1/2} . \quad (24)$$

However, this will be longer than $\tau_{\mathrm{bf}}$ unless $n_f/n_b \lesssim T_M/I_n$, which is unlikely to occur while these collisions are important, since they themselves tend to replenish the free monopoles by $M\overline{M} + M\overline{M} \to M\overline{M} + M + \overline{M}$, which occurs as well as $M\overline{M} + M\overline{M} \to M\overline{M} + M\overline{M}$.

To calculate the three-body collisional recombination rate by (14), we need to know the time evolution of the number density $n_f$ of free monopoles and their temperature $T_M$. After decoupling from the plasma, the bound monopoles take a much longer time to annihilate than they did to form, so during the recombination stage we may ignore the decrease in $\nu$ due to annihilation. Then $y \equiv n_f/n_M \sim n_f m t^2$ decreases as $d \ln y/dt = -\tau_3^{-1}$ with the rate given by (14). If the monopoles were completely decoupled from the plasma, we would have $T_M \propto a^{-2} \propto t^{-4/3}$ from the expansion of the Universe, assuming for now it is isotropic and homogeneous (no gravitational clumping), and ignoring the heat released by the binding of the monopoles into pairs. But the coupling with the plasma only decreases gradually, so there will be heating by the plasma and $T_M$ will not drop quite so fast. There are far more plasma particles than monopoles, so this coupling will have a negligible cooling effect upon the plasma itself. The slow annihilation of the monopole pairs will actually cause the plasma entropy to increase somewhat, but as a first approximation we may ignore this and assume the plasma cools purely by the expansion of the Universe:

$$T_p \sim \epsilon^{-1/3} n_M^{1/3} \approx \epsilon_d^{-1/3} n_M^{1/3} \sim m^{-1} \alpha^{-10/9} t^{-2/3} . \quad (25)$$

Combining the effects of the expansion and the plasma coupling on the monopoles, using (6), we get

$$d \ln T_M/dt \sim 2d \ln T_p/dt + m^{-1} T_p^2 (T_p T_M^{-1} - 1) , \quad (26)$$

which for $T_p \ll T_d$ has the solution

$$T_M \sim T_d (T_p/T_d)^{3/2} \sim m^{-2} \epsilon_d^{-1} t^{-1} \sim m^{-4} \alpha^{-10/3} t^{-1} . \quad (27)$$

Then the three-body recombination proceeds by

$$-d \ln y/dt \sim \tau_3^{-1} \sim y^2 n_M^2 m^{-1/2} \alpha^{-5} T_M^{-9/2} \sim y^2 m^{31/2} \alpha^{10} t^{1/2} , \quad (28)$$

with the solution

$$y \equiv n_f/n_M \sim (1 + m^{31/2} \alpha^{10} t^{3/2})^{-1/2} . \quad (29)$$

Extrapolating to the present time $t_0 \sim 10^{10}$ yr $\sim 10^{61}$ gives $y_0 \sim 10^{-13}$, so nearly all monopoles would be bound today by this process, though disruptive collisions between bound pairs are likely to make the fraction of free monopoles somewhat higher. The majority would be bound ($y < \frac{1}{2}$) by

$$t_b \sim m^{-31/3}\alpha^{-20/3} \sim 10^{44} \sim 1 \text{ sec.} \tag{30}$$

Collisions with the free monopoles would, according to (23), leave most pairs bound somewhere near the quantum level

$$n \sim m^{-7/3}\alpha^{-5/2} \sim 10^{12} , \tag{31}$$

which decays to small $n$ and annihilates only after the dipole emission time

$$t_{\text{dipole}} \sim m^{-1}\alpha^5 n^6 \sim m^{-15}\alpha^{-10} \sim 10^{66} \sim 10^{15} \text{ yr.} \tag{32}$$

Thus, most of the monopole pairs in a homogeneously expanding universe would not have annihilated yet, in violent disagreement with the limits implied by the density of universe.[4,8]

## IV. GRAVITATIONAL CLUMPING

Preskill suggested[4] that the gravitationally induced growth of density inhomogeneities (clumping) may increase the annihilation of monopoles. Goldman, Kolb, and Toussaint[8] (hereafter GKT) and Fry[9] found that the enhancement of the (invalid) classical two-body radiative recombination rate is not sufficient to produce enough monopole annihilation. As discussed above, correcting the error for the radiative recombination rate merely makes the problem worse. However, we need to examine to what degree the more effective three-body nonradiative recombination rate is enhanced by gravitational clumping. We find that this does not work either, for hydrodynamically stable clumps can form only after decoupling,[8] which is too late for the monopoles to annihilate without producing too much entropy. However, it is of interest to consider what would happen to monopole clumps that form after decoupling.

Density inhomogeneities can grow gravitationally only on scales larger than the Jeans length,[15] and only after this length comes into the horizon, which requires that the Universe be matter dominated.[9] As GKT discuss,[8] this first occurs long before decoupling, so there can be clumps of coupled monopoles and radiation. But the pressure is then dominated by the relativistic plasma, so such clumps cannot attain hydrostatic equilibrium unless they expand to $R > \epsilon^{-1}M$, before which decoupling occurs. Hence, the gravitationally bound clumps which stay coupled will either recollapse all the way into black holes of unacceptably large mass,[8] or else they will fragment into pieces eventually small enough for the plasma to leak out. In any case there will not be a significantly enhanced (say more than a factor of 2) annihilation of monopoles before decoupling.

After decoupling, the monopoles can collapse independently of the plasma, in which case the Jeans mass becomes[16]

$$M_J \sim m^{-2} n_M{}^{-1/2} T_M{}^{3/2} \lesssim M_{\text{Jd}} \sim m^{-2}\epsilon_d{}^{-1/2} \sim m^{-3}\alpha^{-5/3} \sim 10^{12} \sim 10^7 \text{ g} . \tag{33}$$

$M_{\text{Jd}}$, the Jeans mass at decoupling, corresponds to

$$N_{\text{Jd}} = m^{-1}M_{\text{Jd}} \sim m^{-4}\alpha^{-5/3} \sim 10^{15} \tag{34}$$

monopoles. Clumps will first form with this number of monopoles and greater, and then later as $M_J$ decreases smaller clumps may form or possibly fragment out of the larger clumps. If an isolated clump starts at radius $R_i$ as a small density perturbation $(\delta\rho/\rho)_i$, $\delta\rho/\rho$ will grow $\propto t^{2/3} \propto R$ until it becomes of order unity at $R_m \sim R_i(\delta\rho/\rho)_i{}^{-1}$. Then the clump will stop expanding and undergo violent relaxation[17] as it condenses to a virialized state at $R \sim \frac{1}{2}R_m$, $T_M \sim mM/R$. The virialized state will have a gravitational potential energy $V \sim -M^2/R$, kinetic energy $K = -\frac{1}{2}v$, and hence negative total energy $E = K + V = -K = \frac{1}{2}V$ (not counting the rest mass energy or the internal kinetic and Coulomb energies of monopole-antimonopole pairs, though these latter energies also obey the virial relations).

Once a quasiequilibrium clump forms, it will evolve primarily by gravitational and magnetic Coulomb interactions, with radiative processes being much less important. Since in the nonrelativistic limit both kinds of interaction energies go as $1/r$, the evolution of the clump is invariant under the scaling transformation $\vec{x} \to \lambda\vec{x}$, $t \to \lambda^{3/2}t$, ignoring radiation and quantum effects. Thus the ratios of different time scales of interest may depend upon the number $N$ of monopoles in the clump but not upon the clump radius $R$. Competing processes will be the formation of magnetically bound monopole-antimonopole pairs (which releases energy and thus tends to make the clump expand) and the evaporation of monopoles from the clump (which carries away energy and thus tends to make the clump contract). If the clump has $N_f$ free monopoles and $N_b = N - N_f$ bound monopoles in $\frac{1}{2}N_b$ pairs,

$$E = -K = -\tfrac{3}{2}(N_f + \tfrac{1}{2}N_b)T_M \sim -m^2 N(N+N_f)R^{-1} . \quad (35)$$

The time scale for three-body recombination (per free monopole) is

$$\tau_3 \sim m^{19/2}\alpha^5 N^{9/2} N_f^{-2} R^{3/2} . \quad (36)$$

The evaporation time scale has two limiting forms, depending upon whether the free-free monopole scattering time (21),

$$\tau_{\rm ff} \sim m^{7/2}\alpha^2 N^{3/2} N_f^{-1} R^{3/2} , \quad (37)$$

is greater or less than the gravitational crossing time

$$\tau_{\rm grav} \sim m^{-1/2} N^{-1/2} R^{3/2} . \quad (38)$$

If $\tau_{\rm ff} > \tau_{\rm grav}$, which occurs for

$$N \gtrsim m^{-4}\alpha^{-2} N_f/N \sim 10^{16} N_f/N ,$$

the free monopoles oscillate through the clump many times before scattering significantly. When a monopole does experience large-angle scattering, there is a probability of order unity that it will gain sufficient energy to escape the gravitational field of the clump, thus giving an evaporation time scale $\sim \tau_{\rm ff}$. If on the other hand $N \lesssim m^{-4}\alpha^{-2} N_f/N$ so $\tau_{\rm ff} < \tau_{\rm grav}$, the free monopoles scatter many times in passing through the clump and hence evaporate from the suface at the diffusion time $\sim \tau_{\rm ff}^{-1}\tau_{\rm grav}^2$. In either case we get

$$\tau_{\rm evap} \sim \tau_{\rm ff} + \tau_{\rm ff}^{-1}\tau_{\rm grav}^2 \quad (39)$$

for free monopoles. Including the evaporation induced by the collisions with or between bound monopole pairs will decrease the evaporation time, especially if $N_f \ll N$, but the actual rates will depend upon the orbital sizes of the bound pairs. In any case

$$\tau_{\rm evap}/\tau_3 \lesssim m^{-6}\alpha^{-3} N^{-3} N_f + m^{-14}\alpha^{-7} N^{-7} N_f^3 \quad (40)$$

is less than unity for $N \gtrsim N_0 \equiv m^{-7/2}\alpha^{-7/4} \sim 10^{14}$, so for a clump with a larger number of monopoles (e.g., with $N_{\rm Jd} \sim m^{-4}\alpha^{-5/3} \sim 10^{15}$), the collisions will cause the free monopoles to evaporate from the clump faster than they form bound pairs.

Thus, gravitational clumping cannot be effective in leading to the magnetic binding and subsequent annihilation of a very large fraction of monopoles, unless somehow nearly all monopoles condense into small clumps.

For clumps with $N \lesssim N_0$, if we ignore collisions with and between bound monopole pairs in order to obtain a minimum for $N_f$ (since such collisions can only tend to replenish $N_f$), we obtain $dN_b/dt \lesssim N_f \tau_3^{-1}$ by three-body recombination, and $-dN/dt \gtrsim N_f \tau_{\rm evap}^{-1}$ by evaporation of free monopoles. (Evaporation of pairs or evaporation of the free monopoles ejected during three-body recombination increases the evaporation rate.) Combining these with $N = N_f + N_b$ and (40) gives

$$dN_f/dN \lesssim 1 + \tau_{\rm evap}/\tau_3 \lesssim 1 + m^{-6}\alpha^{-3} N^{-3} N_f + m^{-14}\alpha^{-7} N^{-7} N_f^3 . \quad (41)$$

If one starts with a clump of mostly free monopoles with initial monopole number $N_i > N_0 \sim 10^{14}$, then (41) implies that the monopoles do not form pairs significantly until $N \lesssim N_0$, so most monopoles escape. If $N_i < N_0$,

$$\Delta N \gtrsim (m^2\alpha N_i)^{7/3} = N_i (N_i/N_0)^{4/3}$$

escape while the rest go into bound pairs, except for a possible imbalance of monopoles and antimonopoles,[18] $\delta N = |N_M - N_{\overline{M}}|$. In order for the excess particles to be bound to the clump despite their mutual magnetic repulsion, $\delta N \lesssim m^2\alpha N$, which is less than $\Delta N$ for $N_i \gtrsim m^{-2}\alpha^{-2} \sim 10^8$ or $\Delta N \gtrsim 1$. Thus this imbalance, which will eventually escape as $N$ decreases, is relatively unimportant.

To reduce $\nu$ from $\sim 10^{-9}$ to $\lesssim 10^{-15}$ as required so that they do not today have a mass density more than 10 or so times the baryon mass density, we need $\Delta N/N_i \lesssim 10^{-6}$, which requires all but $\sim 10^{-6}$ of the monopoles to condense into clumps with $N_i \lesssim N_0(\Delta N/N_i)^{3/4} \lesssim 10^{10}$. Comparing this with the Jeans number with $T_M$ given by (27),

$$N_J \sim m^{-3} n_M^{-1/2} T_M^{3/2} \sim m^{-5/2} t T_M^{3/2} \sim m^{-17/2}\alpha^{-5} t^{-1/2} , \quad (42)$$

one sees that such clumps can form only after

$$t \sim m^{-10}\alpha^{-13/2}(N_i/N_0)^{-2} \gtrsim (1\ {\rm sec})(\Delta N/N_i)^{-3/2} \gtrsim 10\ {\rm yr} . \quad (43)$$

At this time the monopole energy density exceeds the plasma density by a factor of

$$\rho_M/\rho_p \sim m^4\alpha^{40/9}t^{2/3} \sim m^{-8/3}\alpha^{1/9}(N_i/N_0)^{-4/3} \gtrsim 10^{14} , \quad (44)$$

so far too much entropy would be produced by the annihilation of the monopoles.

## V. CONCLUSIONS

Three-body nonradiative recombination of monopoles into bound pairs is far more effective than the radiative recombination heretofore considered. However, it is not effective enough to lead to sufficient monopole annihilation before the annihilation would produce far too much entropy. Gravitational clumping may also help but occurs too late to avoid too much entropy generation.

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## APPENDIX A: THERMALIZATION OF ANNIHILATION PRODUCTS

Assuming that $M\overline{M}$ annihilate into $A$ light particles ($A \sim 100-1000$), say $\gamma$'s for convenience, the rate for their scattering off the thermal background, $\gamma\gamma \rightarrow e^+e^-$, is

$$\tau^{-1} = \langle n\sigma v \rangle \approx m^{-1}\alpha^2 A T_p^{\,2} \ln(mT_p/Am_e^{\,2}) \sim 40m^{-1}\alpha^2 A T_p^{\,2} . \quad \text{(A1)}$$

Scattering continues until the rate becomes less than the reciprocal of the expansion time. This happens at

$$T_p \sim 10^{-3}m^3\alpha^{-4}A^{-2}\epsilon \sim 1 \text{ MeV} \quad \text{(A2)}$$

for $\epsilon = 10^{-13}$.

If the annihilation photons are not thermalized shortly after being produced, then their energy red shifts as $E_\gamma = (m/A)T/T_a$, where $T_a$ is the temperature when the monopoles annihilated, and $\tau^{-1}$ in (A1) should be multiplied by $T_a/T$. Thus scattering will begin again when

$$T \lesssim 10^3 m^{-3}\alpha^4 A^2 \epsilon^{-1} T_a^{\,2} \sim (T_a/1 \text{ MeV})T_a .$$

The above assumes the Universe is dominated by the monopoles. If the Universe somehow becomes radiation dominated then thermalization through the scattering of the annihilation photons off the background and the subsequent scattering of the produced electrons off the thermal background proceeds much more easily. In either case, however, for $T_p < 10^{-3}$ MeV photon production processes (bremsstrahlung and double Compton emission) become inefficient so that energy fed into the photon background distorts the background, raising $\gamma$ energies by $\delta T$ without increasing the $\gamma$ number density by $\delta n_\gamma/n_\gamma = 3\delta T/T$.

## APPENDIX B: RADIATIVE CAPTURE CROSS SECTION

Two monopoles of reduced mass $\mu = m_1m_2/(m_1+m_2)$ and equal but opposite magnetic charges $g$ will be captured radiatively if they emit more energy $E_r$ than their initial kinetic energy $E_k = \frac{1}{2}\mu v^2$, where $v \ll c = 1$ is their initial relative velocity. If the initial angular momentum $l$ is large compared with $\hbar = 1$, the monopoles may be considered to be in wave packets whose spread is small during the important part of the magnetic Coulomb collision. It may be easily checked that almost all collisions that lead to capture have nearly parabolic motion in the vicinity of the closest approach, the region where most of the radiation occurs. (Monopoles which are captured enter this region in hyperbolic orbits with eccentricity just slightly greater than unity and leave it in elliptic orbits with eccentricity just less than one.) Thus one may approximate the monopole motion while they are radiating by classical parabolic orbits, for which wave-packet spreading and radiation back-reaction are only small perturbations. (Of course, whether or not the monopoles remain unbound depends on these small perturbations, but they do not have much effect on the motion until the monopole separation becomes sufficiently large that the small difference between the kinetic and potential energies can assert itself.)

When the monopole motion is a given classical trajectory, the electromagnetic field radiated is a coherent state whose expectation value for the field strengths is given by the classical Maxwell equations.[19] This gives a Poisson distribution of photon numbers, with the expectation number in a small frequency interval being $N(\omega)d\omega = dE_r/\omega$. Classical Maxwell theory can be used to derive the expected energy $dE_r$ radiated in this frequency interval. Applying a duality transformation to the results of Landau and Lifshitz[20] [e.g., to their Eq. (70.18) in the limit $\epsilon^2-1=g^{-2}vl \ll 1$, or, more directly, to Eq. (70.10)—but note that their unnumbered equation following this has a factor of $(1-\epsilon^2)^2$ missing] yields

$$N(\omega)d\omega = 8\pi^{-1}g^6 l^{-2}[K_{1/3}{}^2(x)+K_{2/3}{}^2(x)]x\,dx \;, \tag{B1}$$

$$x=\tfrac{1}{3}g^{-4}\mu^{-1}l^3\omega=\tfrac{1}{6}g^{-4}v^2 l^3\omega/E_k \;. \tag{B2}$$

The total expected energy radiated is

$$\begin{aligned}\langle E_r\rangle &= \int_0^\infty N(\omega)\omega\, d\omega \\ &= 2\pi g^{10}\mu l^{-5} \\ &= 4\pi g^{10} v^{-2} l^{-5} E_k \;,\end{aligned} \tag{B3}$$

which is greater than the incident kinetic energy for

$$l < l_c = (4\pi)^{1/5} g^2 v^{-2/5} \;. \tag{B4}$$

Thus if the monopoles always radiated exactly the expectation value, they would bind for all incident angular momenta less than $l_c$, giving a classical cross section

$$\sigma_c = \pi\mu^{-2}v^{-2}l_c{}^2 = 2^{4/5}\pi^{7/5}g^4\mu^{-2}v^{-14/5} \;. \tag{B5}$$

But integrating (B1) shows that the total expected number of photons emitted,

$$\begin{aligned}\langle N\rangle &= \int_0^\infty N(\omega)d\omega \\ &= 2^3 3^{-1/2} g^6 l^{-2} \\ &= 2^{11/5}3^{-1/2}g^2 v^{4/5}(l/l_c)^{-2} \;,\end{aligned} \tag{B6}$$

is much less than one for $l \sim l_c$ if $v \ll g^{-5/2}$. Because photons are quantized, they will be emitted only probabilistically with this mean number per collision, so most collisions with $l \sim l_c$ will not result in radiation or binding. (An irrelevant infrared divergence in the number of very-low-energy photons does not show up in this analysis because of the approximation of a parabolic orbit.)

To calculate the actual cross section when the discreteness of photons is important, one must calculate the probability that the total energy emitted is greater than the incident kinetic energy of the monopoles. Since the probability of emission in each frequency range is an independent Poisson distribution, this probability for a given $v$ and $l$ is

$$P_l(E_r \ge E_k) = \exp(-\langle N\rangle)\sum_{n=1}^{\infty}\frac{1}{n!}\int_0^\infty \prod_{i=1}^{n} N(\omega_i)d\omega_i\,\theta\left[\sum_{j=1}^{n}\omega_i - E_k\right] , \tag{B7}$$

where the sum and integral is over all the ways that $E_r=\sum_i \omega_i \ge E_k$ can be emitted into $n$ photons. By writing the step function (for $\omega_i, E_k > 0$) as

$$\theta\left[\sum_{i=1}^{n}\omega_i - E_k\right] = 1 - \int_{-\infty}^{+\infty} d\tau \frac{\sin E_k\tau}{\pi\tau}\exp\left[\sum_{i=1}^{n} i\omega_i\tau\right] , \tag{B8}$$

a few steps of simple algebra lead to

$$P_l(E_r \ge E_k) = \int_{-\infty}^{+\infty} d\tau\frac{\sin E_k\tau}{\pi\tau}\left[1-\exp\left[\int_0^\infty (e^{i\omega\tau}-1)N(\omega)d\omega\right]\right] . \tag{B9}$$

This expression is well defined even when $\langle N\rangle$ has an infrared divergence. The cross section is then

$$\sigma = \frac{\pi}{\mu^2 v^2}\sum_{l=0}^{\infty}(2l+1)P_l(E_r \ge E_k) \;. \tag{B10}$$

It appears difficult to evaluate (B9) exactly with the spectrum (B1). However, one can easily get the cross section in the two limiting cases in which the photon discreteness is either unimportant or very important. When $g^{-5/2} \ll v \ll 1$, $\langle N\rangle$ is large for $l \lesssim l_c$, so the classical calculation is valid and leads to the classical cross section (B5). (Note that for elementary charged particles, $g^2$ is replaced by $e^2=\alpha \ll 1$, so the nonrelativistic classical calculation never applies for them.) On the other hand, when $v \ll g^{-5/2} \sim \alpha^{5/4} \sim 10^{-2}$, $\langle N\rangle \ll 1$ for

$l \gg l_Q \sim g^3$. The dominant part of the spectrum occurs for $x \sim 1$ or $\omega \sim g^4 \mu l^{-3} \sim g^4 v^{-2} l^{-3} E_k$, which is larger than $E_k$ for $l \lesssim l_m \sim g^{4/3} v^{-2/3} \gg l_Q$. Hence, when emission leads to capture, it almost always occurs because at least one photon emitted has $\omega \geq E_k$. That is, for $l \lesssim l_m$ most photons emitted each have $\omega \geq E_k$, and for $l \gtrsim l_m$, multiphoton emission is so rare they seldom add up to $E_r \geq E_k$ if the highest energy individual photon does not have $\omega \geq E_k$. Thus the probability of capture is always essentially the probability that at least one photon is emitted with $\omega \geq E_k$. This is $1-\exp(-N_l)$, where

$$N_l = \int_{E_k}^{\infty} N(\omega) d\omega = \frac{4g^2}{3\pi} v^2 l K_{1/3} \left[ \frac{v^2 l^3}{6g^4} \right] K_{2/3} \left[ \frac{v^2 l^3}{6g^4} \right] \tag{B11}$$

is the expected number of photons emitted with $\omega \geq E_k$. For $l \ll l_m$, $N_l \approx \langle N \rangle$, the total expectation number given by (B6), but for $l \gtrsim l_m$ it drops off exponentially with $l^3$. Then

$$\begin{aligned} \sigma_Q &\approx \pi \mu^{-2} v^{-2} \sum_l (2l+1)[1-\exp(-N_l)] \\ &\sim \pi \mu^{-2} v^{-2} \left[ l_Q{}^2 + \int_{l_Q}^{l_m} \frac{16 g^6 dl}{3^{1/2} l} \right] \\ &\sim 2^5 3^{-3/2} \pi \mu^{-2} g^6 v^{-2} \ln(g^{-5/2}/v) . \end{aligned} \tag{B12}$$

This is for $l_Q \sim g^3 \gg 1$; if $g^3 \ll 1$ (as for charged particles), $l_Q$ is replaced by unity and

$$\sigma_Q(g \ll 1) \approx 2^5 3^{-3/2} \pi \mu^{-2} g^6 v^{-2} \ln(g^2/v) . \tag{B13}$$

Setting $\mu = \frac{1}{2} m_e$ and $g = e$ in (B13) gives the low-velocity radiative capture cross section for positronium,[11] for which the numerical factors essentially go back to a very early calculation for hydrogen by Kramers.[21]

We may now combine (B5) and (B12) by an interpolation formula valid to leading order in $g^{5/2} v$ or its reciprocal for both velocity regimes, assuming $v \ll 1 \ll g^2$:

$$\begin{aligned} \sigma \sim\; & 3^{-3/2} 40 \pi \mu^{-2} g^6 v^{-2} \\ & \times \ln(1 + 2^{-11/5} 3^{3/2} 5^{-1} \pi^{2/5} g^{-2} v^{-4/5}) . \end{aligned} \tag{B14}$$

If the monopoles do not have equal but opposite magnetic charges $g_1 = -g_2$, set $g^2 = -g_1 g_2$ and replace $\mu^{-2}$ by $g^{-2}(g_1 m_1{}^{-1} - g_2 m_2{}^{-1})^2$ in the final cross-section formulas. (The analysis was given only for $g_1 = -g_2$ for simplicity.) In this appendix (only), all leading-order numerical factors have been carefully kept, using units in which $\hbar = c = 1$.

*Permanent address: Department of Physics, Pennsylvania State University, University Park, Pennsylvania 16802.

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