PHYSICAL REVIEW D VOLUME 25, NUMBER 9 1 MAY 1982

SO(10) and the invisible axion

G. Lazarides
The Rockefeller University, New York, New York 10021
(Received 20 November 1981)

We study the Dine, Fischler, and Srednicki mechanism for solving the strong *CP* problem in the context of the SO(10) unified gauge theory. We find that this mechanism can be implemented in a minimally extended SO(10) model, where the SO(10) symmetry breaks down to SU(3)×SU(2)×U(1) via SU(5). The resulting theory is consistent with the standard hot-big-bang cosmology and with present phenomenology. We also show that, in other minimally extended SO(10) models, where SO(10) breaks down to SU(3)×SU(2)×U(1) either directly or via SU(4)×SU(2)×SU(2), the Dine, Fischler, and Srednicki mechanism cannot be implemented.

One of the most attractive solutions of the strong *CP* problem is certainly the Peccei-Quinn (PQ) symmetry.[1] In its original version, however, this symmetry breaks at the electroweak mass scale and predicts[2] the existence of a light pseudoscalar particle, called the axion, which is not seen.[3] Recently, Dine, Fischler, and Srednicki[4] (DFS) have suggested a variant of the PQ scheme in which the global anomalous U(1) symmetry breaks at a superheavy mass scale and the resulting axion is superlight with its coupling to ordinary matter highly suppressed. In this scheme, therefore, the axion becomes *invisible.* The DFS mechanism has been implemented by Wise, Georgi, and Glashow[5] (WGG) in a minimally extended SU(5) unified gauge model. The Higgs sector of this model contains a *complex* $\underline{24}$ and two $\underline{5}$'s, all having nonzero charges with respect to the DFS symmetry. This symmetry, therefore, breaks at the superheavy mass scale $M_X \sim 10^{15}$ GeV and the axion mass is $\sim f_\pi m_\pi / M_X \sim 10^{-8}$ eV; the axion's pseudoscalar couplings are $\sim f_\pi / M_X \sim 10^{-16}$ and its scalar couplings $\sim \theta f_\pi / M_X \sim 10^{-31}$. Here f_π is the pion decay constant, m_π the pion mass, and θ the usual *CP*-violating QCD parameter.

The minimal WGG model is not rich enough to account for the observed fermion mass spectrum. In particular, it leads to the wrong asymptotic mass relations $m_e = m_d$ and $m_s = m_\mu$. It has recently been noticed[6,7] that this model is also inadequate to explain the observed baryon asymmetry of the Universe. All these difficulties can be cured by including more Higgs multiplets that couple directly to fermions.

We would like to point out another potential difficulty of this model which probably cannot be cured so easily. The *complex* scalar 24-plet Φ has a nonzero charge with respect to the DFS symmetry. Because of this, the Higgs potential $V(\Phi)$ does not contain terms cubic in Φ. Consequently, for natural values of the Higgs couplings, the transition from the SU(5) to the SU(3)×SU(2)×U(1), phase proceeds in the early Universe without any essential supercooling.[8–10] We, therefore, expect that during this transition an unacceptably large density of superheavy magnetic monopoles is produced.[15] This leads to a potential conflict between the WGG SU(5) gauge model and the standard hot-big-bang cosmology.

Motivated by this potential difficulty of the WGG SU(5) model, we study, in this paper, the implementation of the DFS mechanism in the context of the SO(10) unified gauge theory.[16] We consider the following three breaking patterns of SO(10) which can be achieved by employing a *minimal* Higgs system:

(a) $\mathrm{SO}(10) \to \mathrm{SU}(5) \to \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1) \to \mathrm{SU}(3)\times\mathrm{U}(1)$,

(b) $\mathrm{SO}(10) \to \mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{SU}(2) \to \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1) \to \mathrm{SU}(3)\times\mathrm{U}(1)$,

(c) $\mathrm{SO}(10) \to \mathrm{SU}(3)\times SU(2)\times\mathrm{U}(1) \to \mathrm{SU}(3)\times\mathrm{U}(1)$.

We argue that the DFS mechanism can be implemented in the model (a) and the emerging minimally extended model can be consistent with conventional hot-big-bang cosmology. In particular, we show that in this model, the transition from the SU(5) to the SU(3)$\times$SU(2)$\times$U(1) phase can exhibit a moderate supercooling by few orders of magnitude. The magnetic monopole production is then naturally suppressed. We also show that a catastrophic domain wall production[17,18] during this transition is avoided. The DFS symmetry breaks at the scale of the SO(10) breaking and the mass of the axion is $\lesssim 10^{-10}$ eV whereas its pseudoscalar couplings are $\lesssim 10^{-18}$. It is well known that the SO(10) model (a) possesses certain interesting features not shared by the WGG SU(5) model. Namely, it possesses a spontaneously broken local $B-L$ symmetry, admits a definition of parity (which we should break explictly to avoid domain wall production[17]), predicts[19] measurable neutrino masses ($\lesssim 40$ eV) and unites all fermions of one family in a single irreducible representation. These features of this SO(10) model together with our result and the fact that it reproduces the usual SU(5) predictions for $\sin^2\theta_W$ and the proton lifetime make it probably more attractive than SU(5).

In contrast to the case of model (a), the DFS mechanism cannot be implemented in the context of model (b). The model (c), on the other hand, allows for a DFS mechanism but at the price of an unsuppressed magnetic monopole production. (These results are true if we do not allow extensions of the minimal models other than complexifications of their real Higgs fields. This restriction prevents the proliferation of Higgs parameters.) So we see that, although the three SO(10)-breaking patterns considered are all consistent with present phenomenology, the requirement of a DFS symmetry together with cosmological considerations select out one of them.

It is interesting to note that previous cosmological considerations[18,20] in the context of the SO(10) theory without a DFS symmetry seem to exclude the SO(10)-symmetry-breaking pattern (a). Reference 20, in particular, seems even to favor model (b). We see that the requirement of a DFS symmetry reverses these results.

Let us now discuss in some detail the implementation of the DFS mechanism in the various SO(10)-symmetry-breaking schemes.

Model (a). We will consider the following conventional minimal breaking pattern of SO(10):

$$\mathrm{SO}(10)\xrightarrow[\underline{16}]{M_s\gtrsim 10^{17}\,\mathrm{GeV}}\mathrm{SU}(5)\xrightarrow[\underline{45}]{M_X\sim 10^{15}\,\mathrm{GeV}}\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\xrightarrow[\underline{10}]{M_W\sim 10^{2}\,\mathrm{GeV}}\mathrm{SU}(3)\times\mathrm{U}(1)\,. \tag{1}$$

To achieve this breaking pattern we employ a 16-plet, a 45-plet, and a 10-plet of Higgs fields denoted by χ, Φ, and H, respectively. The fermions belong to three $\underline{16}$'s ψ_α ($\alpha=1,2,3$). The right-handed neutrinos acquire superheavy masses through a two-loop diagram.[19] The induced superlight masses of the left-handed neutrinos are compatible with the cosmological bound, provided $M_s \gtrsim 10^{17}$ GeV.

We will now try to see whether the DFS mechanism can be implemented in this model. The scalar $\underline{10}$ H is necessarily *complex* (for real H we get equal mass matrices for the charge $\frac{2}{3}$ and $-\frac{1}{3}$ quarks) and the most general Yukawa Lagrangian is

$$L_Y=h_{\alpha\beta}\bar{\psi}^c_\alpha\psi_\beta H+h'_{\alpha\beta}\bar{\psi}^c_\alpha\psi_\beta H^*+\mathrm{H.c.} \tag{2}$$

There is only one way (apart from a trivial replacement of H by H^*) to impose a global anomalous U(1) symmetry on L_Y and still have nontrivial Yukawa interactions:

$$\begin{aligned}&\psi_\alpha\to e^{i\theta}\psi_\alpha\ (\alpha=1,2,3)\,,\\&H\to e^{-2i\theta}H\ \text{ for } e^{i\theta}\in\mathrm{U}(1)\,.\end{aligned} \tag{3}$$

This symmetry implies that $h'_{\alpha\beta}=0$ in Eq. (2). Nonvanishing right-handed neutrino masses require[19] the presence of the cubic Higgs coupling $e(\chi^c)^\dagger\Gamma^i\chi H_i$+H.c. (or this coupling with H replaced by H^*), where Γ^i ($i=1,2,\ldots,10$) are generalized Dirac matrices. Consequently, χ must transform under U(1) as

$$\chi\to e^{i\theta}\chi\,,\quad e^{i\theta}\in\mathrm{U}(1)\,. \tag{4}$$

So far we have no clue of how the Higgs $\underline{45}$ Φ transforms under the global U(1). Let us suppose for the moment that Φ is a U(1) singlet. The U(1) is then broken at the first stage of symmetry breaking in Eq. (1) by the SU(5) singlet expectation value of χ. However, a local $\mathrm{U}(1)_0$ [SU(5) $\times\mathrm{U}(1)_0\subset$SO(10] is also broken by $\langle\chi\rangle$ at this stage and a linear combination of U(1) and $\mathrm{U}(1)_0$

[call it U(1)′] survives as a global anomalous symmetry at the level of SU(5). At the next stage of symmetry breaking $\langle\Phi\rangle\neq 0$ but U(1)′ remains unbroken. This is so because Φ was assumed to be a U(1) singlet and its expectation value, lying in the SU(5) $\underline{1}$ and $\underline{24}$ directions, is a $U(1)_0$ singlet. We thus end up with an unbroken global anomalous U(1)′ symmetry at the level of SU(3)×SU(2)×U(1). This global U(1)′ then breaks at the electroweak mass scale and we obtain the ordinary PQ scheme with a *visible* axion. This undesirable conclusion can be avoided by replacing the real Φ by a *complex* one which transforms nontrivially under U(1):

$$\Phi\to e^{iq\theta}\Phi \quad \text{for } e^{i\theta}\in U(1) \quad (q\neq 0)\ . \tag{5}$$

The most general SO(10)- and U(1)-invariant renormalizable Higgs potential, for $q\neq 2$ and -4 (see below), is then given by

$$V_0(\chi,\Phi,H)=V_1(\chi)+V_2(\Phi)+V_3(H)+V_4(\chi,\Phi)+V_5(\chi,H)+V_6(\Phi,H)\ , \tag{6}$$

where

$$\begin{aligned}
V_1(\chi)&=-\mu_\chi{}^2\chi^\dagger\chi+\tfrac{1}{2}\lambda(\chi^\dagger\chi)^2+\tfrac{1}{2}\kappa[(\chi^c)^\dagger\Gamma^i\chi][(\chi^c)^\dagger\Gamma^i\chi]^*\ ,\\
V_2(\Phi)&=\tfrac{1}{4}\mu_\Phi{}^2\mathrm{Tr}(\Phi^\dagger\Phi)+\tfrac{1}{16}a_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2+\tfrac{1}{16}a_2\mathrm{Tr}(\Phi^\dagger\Phi^\dagger)\mathrm{Tr}(\Phi\Phi)\\
&\quad+\tfrac{1}{4}b_1\mathrm{Tr}(\Phi^\dagger\Phi\Phi^\dagger\Phi)+\tfrac{1}{4}b_2\mathrm{Tr}(\Phi^\dagger\Phi^\dagger\Phi\Phi)\ ,\\
V_3(H)&=-\tfrac{1}{2}\mu_H{}^2(H_i^*H_i)+\tfrac{1}{4}c(H_i^*H_i)^2+\tfrac{1}{4}d(H_i^*H_i^*)(H_jH_j)\ ,\\
V_4(\chi,\Phi)&=-\tfrac{1}{4}\alpha(\chi^\dagger\chi)\mathrm{Tr}(\Phi^\dagger\Phi)+\gamma\chi^\dagger\frac{\sigma^{ij}}{2}\frac{\sigma^{kl}}{2}\chi\Phi_{ij}^*\Phi_{kl}\ ,\\
V_5(\chi,H)&=e(\chi^c)^\dagger\Gamma^i\chi H_i+f(\chi^\dagger\chi)(H_i^*H_i)+g(\chi^\dagger\frac{\sigma^{ij}}{2}\chi)H_i^*H_j+\text{H.c.},\\
V_6(\Phi,H)&=k\,\mathrm{Tr}(\Phi^\dagger\Phi)H_i^*H_i+l_1(H^\dagger\Phi^\dagger\Phi H)+l_2(H^\dagger\Phi\Phi^\dagger H)\ .
\end{aligned} \tag{7}$$

Here $\sigma^{ij}=(1/2i)[\Gamma^i,\Gamma^j]$, the quartic couplings are $\sim g^2$, $e\sim gM_s$, and $\mu_\chi,\mu_\Phi,\mu_H\sim M_s$. This potential happens to respect not only the anomalous global U(1) symmetry defined in Eqs. (3), (4), and (5) but also the following *nonanomalous* global U(1)″ symmetry:

$$\psi_\alpha\to\psi_\alpha\ ,\ \chi\to\chi\ ,\ \Phi\to e^{i\theta''}\Phi\ ,\ H\to H \quad \text{for } e^{i\theta''}\in U(1)''\ . \tag{8}$$

It is then easy to see that we still end up with the undesirable ordinary PQ scheme and a *visible* axion. There are only two ways to break U(1)″ explicitly and obtain a DFS scheme with an *invisible* axion. We can add to $V_0(\chi,\Phi,H)$ either the couplings

$$V_6'(\Phi,H)=\delta\,\mathrm{Tr}(\Phi\Phi)(HH)+\epsilon(H\Phi\Phi H)+\text{H.c.} \tag{9}$$

(or these couplings with H replaced by H^*) or the couplings

$$V_7(\chi,\Phi,H)=J(\chi^c)^\dagger\Gamma^i\chi\Phi_{ij}H_j^*+\text{H.c.} \tag{10}$$

The new Higgs potential so obtained will be denoted by $V(\chi,\Phi,H)$. Then, the U(1) symmetry in Eqs. (3), (4), and (5) with $q=2$, for case (9), and $q=-4$, for case (10), becomes a genuine DFS symmetry which breaks at a superheavy scale. The choice (9) is the direct generalization of the WGG choice in SU(5), whereas the coupling (10) has no analog in the simplest SU(5) model. In summary, we see that the DFS mechanism can be implemented in the SO(10) model of Eq. (1). Essentially, this can be done in only two ways.

The potential $V(\chi,\Phi,H)$ is not invariant under any genuinely discrete symmetries and therefore does not lead to a cosmologically catastrophic domain wall[18] production in the early Universe.

We now turn to the discussion of phase transitions and the primordial magnetic monopole problem in the model of Eq. (1). The most general expectation value of the *complex* $\underline{45}$ Φ which is consistent with the symmetry-breaking pattern in Eq. (1) (we put $M_W=0$) is

$$\langle\Phi\rangle=e^{i\theta}(\tfrac{1}{5}s+e^{i\eta}\phi)i\sigma^2\ , \tag{11}$$

where σ^2 is the usual Pauli matrix, $s>0$ is an SU(5) singlet, and ϕ a *real* SU(5) 24-plet in its diagonal form $\phi=y$ diag $(1,1,1,-\frac{3}{2}-\frac{3}{2})$, $y>0$. Notice that there is in general a relative phase η between s and ϕ. At a critical temperature $T_{c1}\sim M_s/g$, χ starts developing an expectation value $\chi_0\sim M_s/g$ and SO(10) breaks down to SU(5). In addition to χ, s can in general also develop an expectation value at this stage. The s-dependent terms of V (we set $H=0$) are

$$V_{\rm eff}(s)=\left[-\tfrac{1}{10}\mu_\Phi{}^2+(\gamma+\frac{\alpha}{10})\chi_0{}^2\right]s^2+\left[\frac{a}{100}+\frac{b}{250}\right]s^4\,, \quad (12)$$

where $a=a_1+a_2$ and $b=b_1+b_2$. $V_{\rm eff}(s)$ does not contain a linear term since the Higgs-boson coupling $-(\beta/2)\chi^\dagger\sigma^{ij}\chi\Phi_{ij}$ is absent by virtue of the continuous DFS symmetry. So s is not forced to develop an expectation value before the SU(5) breaking. If $\langle s\rangle$ remains zero, the subsequent breaking of SU(5) is going to be driven by an effective potential $V_{\rm eff}\,(s,\phi)$ which contains no cubic terms. We then expect no supercooling[8] and an unacceptably large superheavy-magnetic-monopole density.[15] In this case, therefore, the SO(10) model under consideration suffers from the same disease as the WGG model. To avoid this difficulty we restrict ourselves to the range of parameters where the mass term in Eq. (12) is negative. The field s then develops an expectation value $s_0\sim M_s/g$ already at the level of unbroken SU(5). Consequently, the effective potential for the subsequent breaking of SU(5) becomes

$$V_{\rm eff}(\phi,\eta)=-\tfrac{1}{2}\mu_\phi{}^2{\rm Tr}\phi^2+\tfrac{1}{3}c{\rm Tr}\phi^3+\tfrac{1}{4}a\,({\rm Tr}\phi^2)^2+\tfrac{1}{2}b\,{\rm Tr}\phi^4\,, \quad (13)$$

where

$$\mu_\phi^2=\mu_\Phi^2-\tfrac{1}{5}as_0{}^2+\tfrac{2}{5}a_2s_0{}^2\sin^2\eta-\tfrac{4}{25}bs_0{}^2\cos^2\eta-\tfrac{2}{5}bs_0{}^2-\alpha\chi_0{}^2\,,$$
$$c=\tfrac{6}{5}bs_0\cos\eta\,. \quad (14)$$

The parameters of this potential must satisfy[21] the inequalities $b>0$ and $15a+7b>0$. The phase η is determined by minimizing $V_{\rm eff}(\phi,\eta)$. There are two possible solutions:

$$\text{(i)}\ \sin\eta=0\,,$$
$$\text{(ii)}\ \cos\eta=-\frac{5b\,{\rm Tr}\phi^3}{(5a_2+2b)s_0{\rm Tr}\phi^2}\,. \quad (15)$$

In the case where the solution (i) corresponds to the absolute minimum of $V_{\rm eff}(\phi,\eta)$, $c\sim gM_s$ and the potential of Eq. (13) either breaks SU(5) down to SU(4)$\times$U(1) or gives $M_s\sim M_X$ with unacceptably large neutrino masses.[18] Fortunately, for $5a_2+2b\gtrsim 0$, the solution (i) becomes locally unstable and the solution (ii) is favored. In this case $\cos\eta\sim M_X/M_s$ and therefore $c\sim gM_X$. The potential in Eq. (13) has a cubic term of normal strength and the above problems are avoided. Moreover, because of this cubic term, the transition from SU(5) to SU(3)$\times$SU(2)$\times$U(1) can proceed with a moderate supercooling of few orders of magnitude.[21] The production density of primordial superheavy magnetic monopoles can then be suppressed below the cosmolgical bound.

We see that the requirement of a DFS symmetry leads to the complexification of the SO(10) $\underline{45}$ Φ and the problems encountered in Ref. 18 can then be circumvented. In particular, a disastrous domain-wall production is avoided and the primordial-monopole problem can be solved. The latter property is not shared by the WGG SU(5) model.

At a temperature $T_{c1}\sim M_s/g$ the SO(10) symmetry breaks down to SU(5). The global anomalous U(1) symmetry defined by Eqs. (3), (4), and (5) is also broken at this stage. It is easy to see that, since both χ and s acquire their expectation values at this transition, there is no global U(1) which is left unbroken at the SU(5) level. Thus, the DFS symmetry breaks at the mass scale $M_s\gtrsim 10^{17}$ GeV and the axion mass is $\lesssim 10^{-10}$ eV whereas its pseudoscalar couplings are $\lesssim 10^{-18}$.

The axion field can be easily constructed. To this end we denote by F_1 and F_2 the SU(5)-singlet components of χ and Φ, respectively. The neutral and colorless component of the SU(5) $\underline{24}$ in Φ is denoted by F_3 whereas the neutral and colorless components of the SU(5) $\underline{5}$'s in H, $H^\dagger$, and $\chi^\dagger$ are denoted by K_1, K_2, and K_3, respectively. The expectation values of these fields are

$$\langle F_i\rangle=v_ie^{i\alpha_i}\,,\quad \langle K_i\rangle=w_ie^{i\beta_i}\quad (i=1,2,3) \quad (16)$$

with $v_i\sim M_s/g$ $(i=1,2)$, $v_3\sim M_X/g$, and $w_i\sim M_W/g$ $(i=1,2,3)$. Defining shifted fields

$$F_i=(v_i+\rho_i)e^{i(\alpha_i+\xi_i/v_i)}\ ,$$
$$K_i=(w_i+\sigma_i)e^{i(\beta_i+\eta_i/w_i)} \tag{17}$$
$$(i=1,2,3)\ ,$$

the axion field can be written as

$$a\simeq\xi_2+\frac{v_3}{v_2}\xi_3-\frac{4w_2^2}{qw^2v_2}(w_1\eta_1+w_3\eta_3)$$
$$+\frac{4(w_1^2+w_3^2)}{qw^2v_2}w_2\eta_2\ , \tag{18}$$

where $w^2=w_1^2+w_2^2+w_3^2$ and $q=2$ or $q=-4$ in the case of Eq. (9) or (10), respectively. The axion field has been defined so to be orthogonal to the massless scalar ghosts that have been eaten by the $U(1)_0$ and the Z^0 gauge bosons.

It should be noted that the SO(10) model under consideration is incomplete. The fact that there exists only one Yukawa coupling in this model implies that we get no generation mixing and consequently no *CP* violation. Moreover, the wrong asymptotic mass relations $m_e=m_d$ and $m_\mu=m_s$ are not avoided. These problems, however, can be easily circumvented by the introduction of more Higgs multiplets (probably a $\underline{120}$) that couple to fermions.[22] These modifications are not expected to alter our results in any essential way if Higgs $\underline{126}$'s are excluded.

Model (b). We will now study the following symmetry-breaking pattern of SO(10),

$$\mathrm{SO}(10)\xrightarrow[\underline{54}]{M_X\sim10^{15}\ \mathrm{GeV}}\mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{SU}(2)\xrightarrow[\underline{126}]{M_R\sim10^{13}\ \mathrm{GeV}}\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$$
$$\xrightarrow[\underline{10}]{M_W\sim10^{2}\ \mathrm{GeV}}\mathrm{SU}(3)\times\mathrm{U}(1)\ . \tag{19}$$

To achieve this breaking pattern we employ a 54-plet, a 126-plet, and a 10-plet of Higgs fields and we denote them by Φ, χ, and H, respectively. The fermions belong to three $\underline{16}$'s ψ_α ($\alpha=1,2,3$). The right-handed neutrinos acquire superheavy masses at the tree level[23] and the induced left-handed neutrino masses are $\sim$few eV.

There is only one way (apart from a trivial replacement of H by H^*) to impose a global anomalous U(1) symmetry on the Yukawa Lagrangian L_Y of this model without jeopardizing its successful features[24]:

$$\psi_\alpha\to e^{i\theta}\psi_\alpha\ ,\quad H\to e^{-2i\theta}H\ ,$$
$$\chi\to e^{2i\theta}\chi\ ,\quad \text{for } e^{i\theta}\in\mathrm{U}(1). \tag{20}$$

L_Y then takes the form

$$L_Y=h_{\alpha\beta}\bar\psi^c_\alpha\psi_\beta H+g_{\alpha\beta}\bar\psi^c_\alpha\psi_\beta\chi^\dagger+\mathrm{H.c.} \tag{21}$$

The 54-plet Φ must be *complex* and transform nontrivially under U(1). This is seen as follows. If Φ is a U(1) singlet then this symmetry remains unbroken at the level of SU(4)$\times$SU(2)$\times$SU(2). It breaks together with the local $U(1)_{B-L}$ symmetry only when the $\underline{126}$ χ acquires an expectation value. However, a linear combination of the U(1) and the $U(1)_{B-L}$ survives as a global anomalous PQ symmetry at the level of SU(3)$\times$SU(2)$\times$U(1). We then obtain the ordinary PQ scheme with a *visible* axion.

The fact that the complexified 54-plet Φ transforms nontrivially under U(1) restricts its potential to

$$V(\Phi)=-\tfrac{1}{2}\mu_\Phi{}^2\mathrm{Tr}(\Phi^\dagger\Phi)+\tfrac{1}{4}a_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2+\tfrac{1}{4}a_2\mathrm{Tr}(\Phi^\dagger\Phi^\dagger)\mathrm{Tr}(\Phi\Phi)$$
$$+\tfrac{1}{2}b_1\mathrm{Tr}(\Phi^\dagger\Phi\Phi^\dagger\Phi)+\tfrac{1}{2}b_2\mathrm{Tr}(\Phi^\dagger\Phi^\dagger\Phi\Phi)\ . \tag{22}$$

The most general form of $\langle\Phi\rangle$ which breaks SO(10) down to SO($10-m$)$\times$SO(m) ($m=1,2,3,4,5$) is

$$\langle\Phi\rangle=e^{i\theta}y[10m(10-m)]^{-1/2}\mathrm{diag}(\underbrace{10-m,\ldots,10-m}_{m};\underbrace{-m,\ldots,-m}_{10-m})\ ,\quad \theta,y\in R\ . \tag{23}$$

Substituting $\langle\Phi\rangle$ into the potential $V(\Phi)$ we obtain[20]

$$V(\langle\Phi\rangle)=V(y,m)=-\tfrac{1}{2}\mu_\Phi{}^2y^2+[\tfrac{1}{4}(a_1+a_2)+\tfrac{1}{2}(b_1+b_2)s_4(m)]y^4\ , \tag{24}$$

where $s_4(m)=[100m(10-m)]^{-1}[(10-m)^3+m^3]$. It is then obvious that the SO(6)×SO(4) [=SU(4)×SU(2)×SU(2)] phase is never an absolute minimum of the potential $V(\Phi)$, since, for $b_1+b_2>0$, $V(y,m=5)<V(y,m=4)$ and for $b_1+b_2<0$, $V(y,m=1)<V(y,m=4)$. We, thus, see that the requirement of a DFS mechanism is not consistent with the SO(10)-breaking pattern in Eq. (19).[25]

It is interesting to note that whereas the model under consideration is perfectly consistent with present phenomenology,[24,26] it is ruled out by the requirement of a DFS mechanism (together with some cosmological considerations)[27].

Model (c). The direct breaking of SO(10) down SU(3)×SU(2)×U(1) at a scale $M_X \sim 10^{15}$ GeV may be achieved by employing a 45-plet Φ and a 126-plet X of Higgs fields. A DFS U(1) symmetry, which is broken at the mass scale M_X, would require [by an argument similar to the one discussed in the case of model (a)] a *complex* Φ which transforms nontrivially under U(1). Thus, the Higgs potential $V(\Phi,X)$ contains no cubic terms and, for natural values of the couplings, the transition from the SO(10) to the SU(3)×SU(2)×U(1) phase proceeds with no supercooling. The potential conflict with the standard hot-big-bang cosmology is not avoided.[27]

To summarize, it is seen that the requirement of a DFS symmetry and an *invisible* axion together with some cosmological considerations select out one of the various simple breaking schemes of SO(10) discussed in this paper. In particular, we find that the DFS mechanism can be implemented in the SO(10) model where the SO(10) symmetry breaks down to SU(3)×U(1) via SU(5). The emerging minimally extended model can avoid any potential conflict with the standard hot-big-bang cosmology. This property is not shared by the WGG model.

This work was supported in part under the Department of Energy Contract No. DE-AC02-81ER40033 B000.

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[1]R. D. Peccei and H. Quinn, Phys. Rev. Lett. 38, 1440 (1977); Phys. Rev. D 16, 1791 (1977).

[2]S. Weinberg, Phys. Rev. Lett. 40, 223 (1978); F. Wilczek, *ibid.* 40, 279 (1978).

[3]T. Donnelly *et al.,* Phys. Rev. D 18, 1607 (1978); A. Zehnder, ETH report, 1981 (unpublished).

[4]M. Dine, W. Fischler, and A. Srednicki, Phys. Lett. 104B, 199 (1981).

[5]M. Wise, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. 47, 402 (1981).

[6]J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and S. Rudaz, Phys. Lett. 106B, 298 (1981).

[7]R. Barbieri, R. N. Mohapatra, and D. V. Nanopoulos, CERN Report No. TH. 3105, 1981 (unpublished).

[8]A. D. Linde, Phys. Lett. 99B, 391 (1981); M. Daniel and C. E. Vayonakis, Nucl. Phys. B180, 301 (1981); M. Daniel, Phys. Lett. 98B, 371 (1981).

[9]This is so, despite the fact that the transition can be strongly first order, i.e., the expectation value of Φ can exhibit a jump of the order of its zero-temperature value.

[10]Supercooling may be achieved by employing the Coleman-Weinberg potential for the SU(5) breaking (see Ref. 11). In this case, however, the universe is likely to remain in the SU(5) phase forever (see Ref. 12). This is so, in particular, because, due to massive particle creation in a rapidly expanding universe, the temperature remains always $\gtrsim 10^{11}$ GeV (see Ref. 13). Consequently, the nonperturbative and gravitational effects (see Ref. 14) that could drive the SU(5) breaking are probably irrelevant.

[11]A. Billoire and K. Tamvakis, Nucl. Phys. B200, 329 (1982); P. G. Cook and K. T. Mahanthappa, Phys. Rev. D 23, 1321 (1981); M. Sher, *ibid.* 24, 1699 (1981).

[12]K. Sato, Nordita Report No. 80-29, 1980 (unpublished).

[13]M. Horibe, Osaka Univ. Report No. OU-HET 41, 1981 (unpublished).

[14]K. Tamvakis and C. E. Vayonakis, Report No. TH. 3108, 1981 (unpublished); L. F. Abbott, Nucl. Phys. B185, 233 (1981).

[15]J. P. Preskill, Phys. Rev. Lett. 43, 1365 (1979); Ya. B. Zeldovich and M. Yu. Khlopov, Phys. Lett. 79B, 239 (1978).

[16]H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) 93, 193 (1975); H. Georgi, *Particles and Fields,* edited by C. E. Carlson (AIP, New York, 1975).

[17]Ya. B. Zeldovich, I. Yu. Kobzarev, and L. B. Okun', Zh. Eksp. Teor. Fiz. 67, 3 (1974) [Sov. Phys.—JETP 40, 1 (1975)]; T. W. B. Kibble, J. Phys. A 9, 1387 (1976); Phys. Rep. C67, 183 (1980).

[18]G. Lazarides, Q. Shafi, and T. F. Walsh, Nucl. Phys. B195, 157 (1982).

[19]E. Witten, Phys. Lett. 91B, 81 (1980).

[20]G. Lazarides, M. Magg, and Q. Shafi, Phys. Lett. 97B, 87 (1980).

[21]A. Kennedy, G. Lazarides, and Q. Shafi, Phys. Lett. 99B, 38 (1981).

[22]H. Georgi and D. V. Nanopoulos, Nucl. Phys. B159, 16 (1979); G. Branco and A. Masiero, Phys. Lett.

97B, 95 (1980).

[23]M. Gell-Mann, P. Ramond, and R. Slansky (unpublished).

[24]G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. B181, 287 (1981).

[25]This conclusion is not avoided even if we replace the 54 Φ by a more complicated Higgs field (say 210). The requirement of a DFS symmetry does not allow the presence of cubic self-couplings of this field and the transition from SO(10) to SU(4)$\times$SU(2)$\times$SU(2) proceeds with no supercooling. The production density of magnetic monopoles is then unsuppressed (see Ref. 20).

[26]It should be mentioned that this model in its minimal implementation does not account for the observed magnitude of the *CP* violation. This can be easily cured by the introduction of a scalar 120.

[27]Compatibility of this model with a DFS mechanism may be achieved by keeping the Higgs 54 (45) real and adding an extra Higgs 16. The DFS charge of this field should be half of the DFS charge of the Higgs $\overline{126}$.