Dalitz-Yennie-type equivalent-photon spectrum for $\gamma\gamma$ collisions in electron storage rings

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It is shown that the Dalitz-Yennie version of the equivalent-photon approximation, originally established for electroproduction in the target rest frame, becomes inappropriate, to some extent, for application to $\gamma\gamma$ collision experiments in electron storage rings, when the outgoing electrons are tagged at finite angles. For this type of experiment, a new Dalitz-Yennie-type formula is given, the use of which is, however, restricted in principle to single-tagging measurements. In the configurations thus considered, quantitative differences between the original and the new Dalitz-Yennie-type formula may become significant; a numerical comparison shows indeed that the new formula definitely provides a better approximation, with respect to an "effective-photon spectrum" derived from the exact calculation for various processes, than the former one.

The question is often raised, mainly by experimentalists, which one is the "best" equivalent-photon spectrum (EPS) to be used for predictions or analysis bearing on the so-called two-photon processes or $\gamma\gamma$ collisions in electron storage rings. There are indeed several different forms of that spectrum appearing in the literature. Numerical differences between them remain, in general, relatively small when they are used for the computation of total cross sections of processes $ee\to eeX$,[1] but may become more significant when one applies them to configurations where—as in experiments to be performed with the new high-energy storage rings PETRA or PEP—the outgoing electrons are tagged at finite, though small, angles.

In particular, for such finite-angle tagging experiments, two EPS fomulas are currently proposed: the standard formula[2]

$$N(\omega,\theta)=\frac{2\alpha}{\pi}\frac{1}{\omega}\left(1-\frac{\omega}{E_0}+\frac{\omega^2}{2E_0^{\,2}}\right)\frac{1}{\theta} \tag{1}$$

and the Dalitz-Yennie formula[3]

$$N(\omega,\theta)=\frac{2\alpha}{\pi}\frac{1}{\omega}\left[\left(1-\frac{\omega}{E_0}+\frac{\omega^2}{2E_0^{\,2}}\right)\frac{1}{\theta}-\left(1-\frac{\omega}{E_0}+\frac{\omega^2}{4E_0^{\,2}}\right)\frac{E_0(E_0-\omega)\theta}{\omega^2+E_0(E_0-\omega)\theta^2}\right], \tag{2}$$

where E_0 is the electron's initial energy (beam energy), ω its energy loss, and θ its scattering angle. A distinctive feature of formula (2), with respect to (1), is that the spectrum vanishes when ω goes to zero.

In this paper we shall show that the Dalitz-Yennie formula, which was established quite correctly for electroproduction from nuclear targets, becomes less appropriate when applied to $\gamma\gamma$ collisions in electron storage rings; and that, on the other hand, it is possible, as far as single-tagging measurements are concerned, to use another EPS formula, formally somewhat similar to the Dalitz-Yennie formula in the sense that it retains its distinctive feature mentioned above, but involving numerical implications which may be quite different from those of the latter.

Let us first consider the case of electroproduction from a target at rest (Fig. 1), where both formulas (1) and (2) can be rather simply derived from the expression of the transverse-photon spectrum, the photon's transverse polarization being defined—according to the helicity formalism—by

$$\epsilon_\pm^\mu q_\mu=0,\quad \epsilon_\pm^\mu p_{T\mu}=0\,. \tag{3}$$

The transverse-photon spectrum is written in an almost unambiguous way as follows:

$$N(\omega,Q^2)=\frac{\alpha}{4\pi}\frac{\omega}{E_0^{\,2}}\frac{1}{Q^4}L_{++}\beta_{\gamma T}, \tag{4}$$

where some arbitrariness is involved only in the choice of $\beta_{\gamma T}$, the relative velocity of the virtual photon and the target; the simplest choice, though

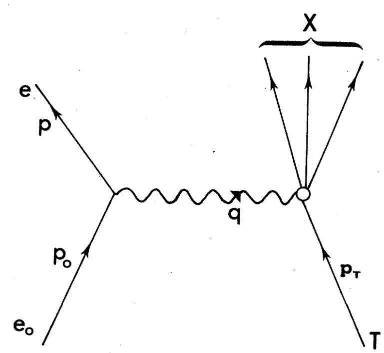

FIG. 1. Feynman diagram for electroproduction processes.

not the only one,[4] being $\beta_{\gamma T}=1$. Q is the absolute value of the virtual photon's mass; one has $Q^2 = -q^2 = E_0(E_0-\omega)\theta^2$. Finally L_{++}, the factor containing the dynamics of the electron-photon vertex, is defined (neglecting the electron mass) as

$$L_{++}=\mathrm{Tr}[(p_0\cdot\gamma)(\epsilon_+\cdot\gamma)(p\cdot\gamma)(\epsilon_+^*\cdot\gamma)]. \qquad (5)$$

Calling s the overall c.m., energy squared, M_T the target mass, and M_X the invariant mass of the particle system X produced, we get

$$L_{++}=\frac{4Q^2}{\Lambda(M_X{}^2, M_T{}^2, -Q^2)}[(s-M_T{}^2)(s-M_X{}^2-Q^2) - M_T{}^2Q^2]+2Q^2, \qquad (6)$$

where the function Λ is defined as

$$\Lambda(x,y,z)=x^2+y^2+z^2-2yz-2zx-2xy.$$

Using the set of kinematic relations

$$\Lambda=4M_T{}^2(\omega^2+Q^2), \quad s-M_T{}^2=2E_0M_T, \quad s-M_X{}^2-Q^2=2(E_0-\omega)M_T \qquad (7)$$

we get

$$L_{++}=\frac{4Q^2}{\omega^2+Q^2}\left(E_0{}^2-E_0\omega+\frac{\omega^2}{2}+\frac{Q^2}{4}\right). \qquad (8)$$

Substituting (7) into (3) we obtain

$$N(\omega,Q^2)=\frac{\alpha}{\pi}\frac{1}{\omega}\left[\left(1-\frac{\omega}{E_0}+\frac{\omega^2}{2E_0{}^2}\right)\frac{1}{Q^2} - \left(1-\frac{\omega}{E_0}+\frac{\omega^2}{4E_0{}^2}\right)\frac{1}{\omega^2+Q^2}\right], \qquad (9)$$

where the second term inside the bracket may be neglected or not, according to whether we consider Q^2/ω^2 a negligible quantity or not. Depending on that choice, we get either formula (1) or formula (2), once we change our variable from Q^2 to θ.

Let us now go over to the case of $\gamma\gamma$ collisions in electron storage rings (Fig. 2), where the laboratory frame is to be identified with the overall c.m. frame. Here the transverse polarization of both

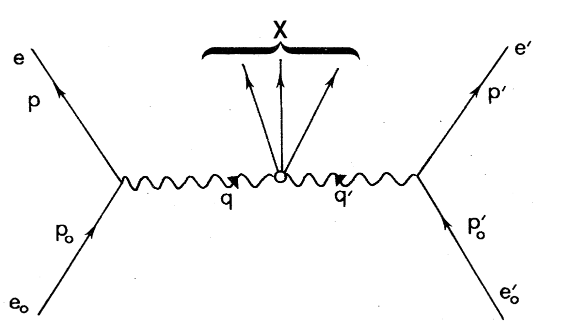

FIG. 2. Feynman diagram for $\gamma\gamma$ collisions in electron storage rings.

photons is defined in the helicity formalism by

$$\epsilon_\pm'^\mu=-\epsilon_\mp^\mu, \quad \epsilon^\mu q_\mu=0, \quad \epsilon^\mu q'_\mu=0. \qquad (10)$$

The double transverse-photon spectrum is written at the outset—again almost unambiguously—as follows:

$$\mathfrak{N}(\omega,Q^2,\phi;\omega',Q'^2,\phi') = \left(\frac{\alpha}{8\pi^2}\frac{\omega}{E_0{}^2}\frac{1}{Q^4}L_{++}\right)\left(\frac{\alpha}{8\pi^2}\frac{\omega'}{E_0{}^2}\frac{1}{Q'^4}R_{++}\right)\frac{\beta_{\gamma\gamma}}{2}, \qquad (11)$$

where parameters with and without (′) refer to the left-hand (LH) and right-hand (RH) vertex, respectively; θ (θ') are the azimuthal angles of the outgoing electrons; L_{++} (R_{++}) is the dynamic factor for the LH (RH) electron-photon vertex; for $\beta_{\gamma\gamma}$ the simplest choice, though again not the only one,[4] is $\beta_{\gamma\gamma}=2$.

The expression of L_{++} is derived in a trivial way from (6), substituting $-Q'^2$ for $M_T{}^2$, and $\tilde{s}$ (defined as the square of the total energy of the LH incoming electron and the RH virtual photon in their c.m. frame) for s. R_{++} is then derived from L_{++} by LH-RH symmetry.

It is fairly obvious that, in general, the double equivalent-photon spectrum (11) cannot be decorrelated into two independent spectra (i.e., the left-hand EPS will depend on right-hand variables and vice versa), except if we let both Q^2 and Q'^2 go to zero [and are then led to the standard formula (1)] or if we choose a procedure which *a priori* is hardly justified: Namely, neglecting Q'^2 in the expression of L_{++} and Q^2 in the expression of R_{++}.

It happens, however, that in the case of single-tagging measurements, treating the untagged electron as "tagged by absence" at almost 0° (more precisely, between 0° and the minimal tagging angle $\theta_{\min}$), the left-hand and right-hand transverse-photon spectra are disentangled naturally, as has been shown in a recent paper.[5] In that case, if we assume the LH electron to be the tagged one, we get

$$L_{++}\simeq(L_{++})_{Q'^2=0}=\frac{4Q^2\tilde{s}(\tilde{s}-M_X{}^2-Q^2)}{(M_X{}^2+Q^2)^2}+2Q^2. \qquad (12)$$

Here we have the following relations between the invariants used and laboratory-frame variables:

$$M_X{}^2+Q^2\simeq4\omega'\left(\omega+\frac{Q^2}{4E_0}\right), \quad \tilde{s}=4\omega'E_0. \qquad (13)$$

The expression of L_{++} thus becomes

$$L_{++}=\frac{4Q^2}{(\omega+Q^2/4E_0)^2}\left[E_0{}^2-E_0\left(\omega+\frac{Q^2}{4E_0}\right)+\frac{1}{2}\left(\omega+\frac{Q^2}{4E_0}\right)^2\right]. \qquad (14)$$

Substituting formula (14) into (11), the left-hand

EPS is obtained as follows (after integration over ϕ):

$$N(\omega, Q^2) = \frac{\alpha}{\pi}\frac{1}{\omega}\left[\left(1 - \frac{\omega}{E_0} + \frac{\omega^2}{2E_0{}^2}\right)\frac{1}{Q^2} - \left(1 - \frac{\omega}{E_0}\right)\frac{1}{4\omega E_0 + Q^2} - \frac{4\omega E_0}{(4\omega E_0 + Q^2)^2}\right], \tag{15}$$

where the second and third term inside the bracket may be neglected or not, according to whether one considers $Q^2/(4\omega E_0)$ a negligible quantity or not. Depending on that choice, we get either formula (1) or the following formula, once we change our variable from Q^2 to θ:

$$N(\omega, \theta) = \frac{2\alpha}{\pi}\left[\left(1 - \frac{\omega}{E_0} + \frac{\omega^2}{2E_0{}^2}\right)\frac{1}{\theta} - \left(1 - \frac{\omega}{E_0}\right)\frac{(E_0 - \omega)\theta}{4\omega + (E_0 - \omega)\theta^2} - \frac{4\omega(E_0 - \omega)\theta}{[4\omega + (E_0 - \omega)\theta^2]^2}\right]. \tag{16}$$

Like the Dalitz-Yennie spectrum, the EPS expressed by the new formula (16) vanishes when ω goes to zero. In that sense, it may be called a "Dalitz-Yennie-type" equivalent-photon spectrum.

Whereas we are led to formula (16) by the helicity treatment, some authors—as we said above—are currently using, for $\gamma\gamma$ processes, the original Dalitz-Yennie spectrum (2). This was first done by Brodsky, Kinoshita, and Terazawa,[6] who justified this approximation procedure by choosing a definition of "transverse polarization" quite different from that used in the helicity formalism. That definition, based on using the radiation or Coulomb gauge in the laboratory frame, appears to be the following:

$$\epsilon_\pm^\mu q_\mu = 0, \quad \epsilon_\pm^\mu P_\mu = 0, \quad \epsilon_\pm'^\nu q'_\nu = 0, \quad \epsilon_\pm'^\nu P_\nu = 0, \tag{17}$$

where P_μ $(= p_{0\mu} + p'_{0\mu})$ is the four-vector of the overall center of mass. Obviously this is a quite unnatural choice of polarization vectors, since it makes the transverse (and also longitudinal) amplitudes of the virtual $\gamma\gamma$ process $(\gamma\gamma \to X)$ depend on external variables $(q \cdot P, q' \cdot P)$ in addition to the internal ones $(Q^2, Q'^2, M_X{}^2)$. It is true that this dependence vanishes when one goes to the mass-shell limit of the photons, as one does in the equivalent-photon approximation. However, there is a certain price to pay for such an unnatural choice: Namely, the ratio of longitudinal to transverse amplitudes is here given[7] by

$$\frac{|\epsilon_0^\mu j_{\mu\nu}|}{|\epsilon_\pm^\mu j_{\mu\nu}|} = \frac{Q\,|j_{3\nu}|}{\omega\,|j_{\pm,\nu}|}, \tag{18}$$

where the three axis is taken as the beam-collision axis in the laboratory frame, and where one defines

$$j_{\pm,\nu} = \mp (1/\sqrt{2})(j_{1\nu} \pm i j_{2\nu}).$$

Therefore, this choice of polarization vectors implies the condition $Q^2 \ll \omega^2$ for neglecting longitudinal with respect to transverse contributions. For the validity of the equivalent-photon approximation, this condition comes in addition to $Q^2 \ll M_X{}^2$ which must be satisfied in any case. Such an additional condition of validity obviously makes the Dalitz-Yennie formula less convenient for $\gamma\gamma$ processes in the case of finite-angle measurements, where Q^2 becomes relatively important.

If no attention is paid to that additional condition (as in some tagging efficiency calculations performed last year in the framework of the LEP project; see the comment by J. Field[8]), one may be led to quite dramatic consequences. That is what we are going to show in the following numerical test.

We consider three reactions

$$ee \to ee\mu^+\mu^-, \quad ee \to ee\pi^+\pi^-, \quad ee \to ee\eta$$

proceeding via the $\gamma\gamma$ process. We assume the practical case of a single-tagging measurement at finite angle. The tagged electron is supposed to be measured at $\theta = 25$ or 50 mrad; as for the untagged one, we assume that it may be considered as "tagged by absence" between $\theta' = 0°$ and $\theta' = \theta_{\min} = 10$ mrad. For the beam energy, we take $E_0 = 10$ GeV.

We compare both Dalitz-Yennie-type equivalent-photon spectra (16) and (2)—and, by the way, also the standard EPS given by formula (1)—with an "effective-photon spectrum" defined as follows (e.g., in the $\mu^+\mu^-$ case):

$$N^{\text{eff}}(\omega, \theta) = \frac{\displaystyle\int \frac{d\sigma(e_0 e'_0 \to ee'\mu^+\mu^-)}{d\omega\, d\theta\, d\omega' d\theta' d\phi_{ee'}} d\omega' d\theta' d\phi_{ee'}}{\displaystyle\int \frac{d\sigma(\gamma e'_0 \to ee'\mu^+\mu^-)}{d\omega' d\theta'} d\omega' d\theta'}. \tag{19}$$

In that formula, both the numerator and the denominator are computed exactly, except that we make $\theta' = 0$ wherever permitted (due to the smallness of $\theta_{\min}$, that approximation should introduce only a negligible error). As for the limits of integration, $\phi_{ee'}$ (the relative azimuthal angle between the outgoing electrons) is taken between 0 and 2π; whereas (both in the numerator and in the denominator) θ' is taken between 0 and $\theta_{\min}$, and ω' is taken between $\omega'_{\min}$ and E_0, with $\omega'_{\min}$ given by

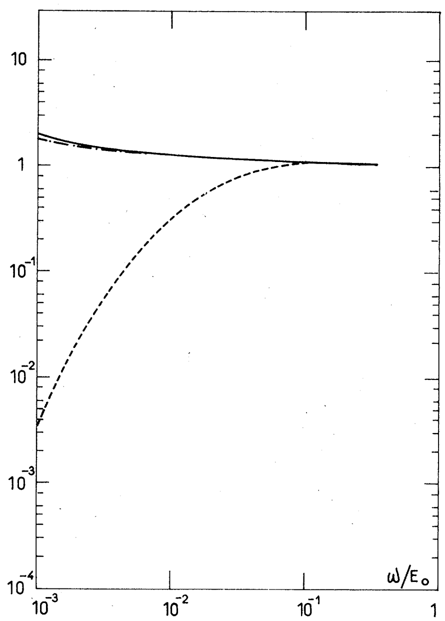

FIG. 3. Comparison of the standard EPS (1), the original Dalitz-Yennie spectrum (2), and the new Dalitz-Yennie-type spectrum (16), all three being normalized to the effective-photon spectrum (19) for the process $e_0e_0' \to ee'\mu^+\mu^-$ or $e_0e_0' \to ee'\pi^+\pi^-$ (using the Born-term model), assuming a single-tagging measurement under the following experimental conditions: Beam energy $E_0 = 10$ GeV; minimal angle of the tagging system $\theta_{\min} = 10$ mrad; scattering angle of the tagged electron $\theta = 25$ mrad. For simplicity, no limitation of the central detector's acceptance is assumed. The solid curve gives spectrum (1), normalized to (19); the dot-dashed curve, spectrum (16), normalized to (19); and the dashed curve, spectrum (2), normalized to (19).

the expression

$$\omega'_{\min} = \frac{4m_\mu^2 + E_0(E_0-\omega)\theta^2}{4\omega + (E_0-\omega)\theta^2} \tag{20}$$

which is obtained by taking account of the correlation between M_X^2, ω, ω', and θ.

The results of our comparison appear in Figs. 3–6, where we show all three spectra (1), (16), and (2), normalized to the effective-photon spectrum (19) (obviously, with that normalization, the best curve is that one which remains the closest to the value 1 at any ω value). One here notices that (i) the curves obtained for the case of muon-pair and pion-pair production, respectively, are so close to each other that they become indistinguishable, accounting for the logarithmic scale used; (ii) formula (16) shows indeed an improvement with respect to the standard EPS (1), though not a decisive one in the range considered; (iii)

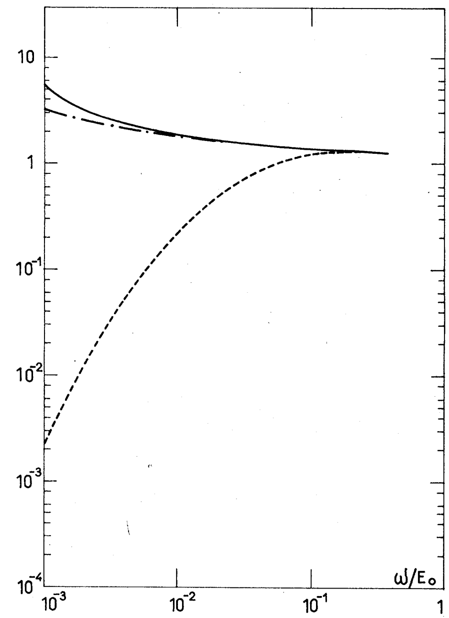

FIG. 4. Same as Fig. 3, except $\theta = 50$ mrad.

all three formulas tend to become bad when going to smaller and smaller values of ω/E_0 [except for formula (16) in the case of η production; but such a surprisingly good agreement, as shows up here,

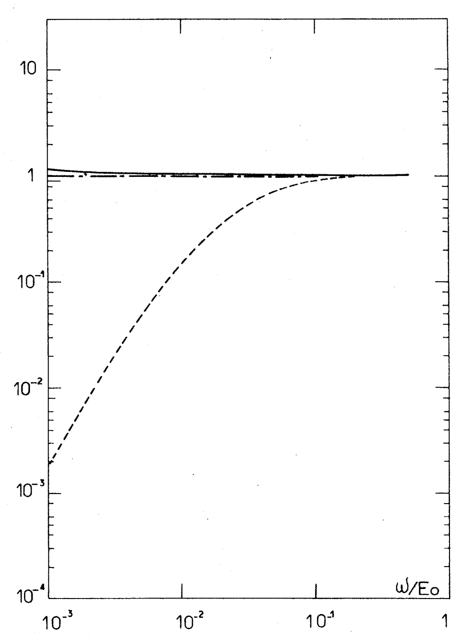

FIG. 5. Same as Fig. 3, except that the process considered here is $e_0e_0' \to ee'\eta$. [In the exact calculation, a vector-dominance-model type form factor, i.e., $F^2(Q^2) = (1+Q^2/m_\rho^2)^{-2}$, was used.]

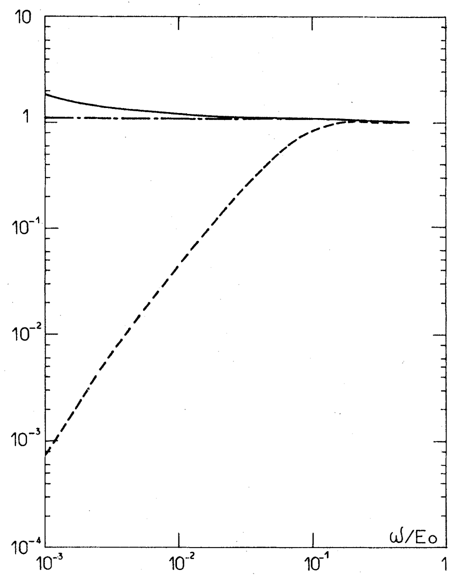

FIG. 6. Same as Fig. 5, except $\theta = 50$ mrad.

should be rather accidental]; and (iv) formula (2) is considerably worse than the two others.

While formula (16) thus appears to provide, for $\gamma\gamma$ experiments with finite-angle tagging systems, an acceptable substitute for the Dalitz-Yennie spectrum (2) and a slight improvement with respect to the standard EPS (1)—perhaps this is true even in the case of double-tagging measurements, as recently shown by J. Field[8]—experimentalists must be cautioned that there is no ideal equivalent-photon formula which they may use in any configuration. As we strongly emphasized in a recent paper,[9] it is essential to satisfy the basic condition $Q^2 \ll M_X^2$; otherwise, there is no substitute for an exact calculation.

The authors wish to thank Dr. John Field for having called their attention to the problem of discrepancies between various EPS formulas, in connection with tagging-efficiency calculations for LEP.

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[1]See, for instance, R. Bhattacharya, J. Smith, and G. Grammer, Jr., Phys. Rev. D 15, 3267 (1977).

[2]See, in particular, P. Kessler, Acta Phys. Austriaca 41, 141 (1975). One notices that integration over θ gives the familiar ln factor, whereas the nonlogarithmic term vanishes at finite scattering angles.

[3]R. H. Dalitz and D. R. Yennie, Phys. Rev. 105, 1598 (1957); see also Ref. 6.

[4]This factor $\beta_{\gamma T}$ is to be included in the spectrum in order to make up for the velocity factor entering, by definition, the denominator of the transverse "virtual photoproduction cross section." There is some arbitrariness in the choice of the value of that factor, only its limit for $Q \to 0$ being well defined, namely, $\beta_{\gamma T} \to 1$. Since in any variant of the equivalent-photon approximation the transverse virtual photoproduction is taken equal to its on-shell limit, i.e., to the cross section for a real photon (apart from a possible dynamic form factor), the arbitrariness is transferred from that cross section to the equivalent-photon spectrum. If one follows a prescription given by L. N. Hand [Phys. Rev. 129, 1834 (1963)] one would take, instead of $\beta_{\gamma T} = 1$, the value

$$\beta_{\gamma T} = (M_X^2 - M_T^2)/(2\omega M_T) = 1 - Q^2/(2\omega M_T).$$

In $\gamma\gamma$ collision processes, there is again some arbitrariness in choosing the value of $\beta_{\gamma\gamma}$, only its limit for $Q \to 0$ being well defined, i.e., $\beta_{\gamma\gamma} \to 2$. Following a prescription similar to that of L. N. Hand, one would be led to $\beta_{\gamma\gamma} = M_X^2/(2\omega\omega')$, which is again somewhat different from the value $\beta_{\gamma\gamma} = 2$ chosen here.

[5]C. Carimalo, P. Kessler, and J. Parisi, Phys. Rev. D 21, 669 (1980); see formulas (2.19)–(2.21) there. Actually, the possibility of tagging by absence was already studied some years ago by one of us [J. Parisi, thesis, University of Paris, 1974 (unpublished)]. Its obvious condition is that tagging counters (here acting as veto counters) with practically 100% efficiency should cover a continuous angular range between $\theta_{\min}$ and some large angle where the counting rate is expected to become exceedingly small.

[6]S. J. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. D 4, 1532 (1971).

[7]Using the same procedure as in Sec. III of Ref. 9.

[8]J. Field, DESY Report No. 79-78 (unpublished).

[9]C. Carimalo, P. Kessler, and J. Parisi, Phys. Rev. D 20, 1057 (1979).