Implications of PETRA QED tests on neutral-current structure

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Recent experiments performed at PETRA to test quantum electrodynamics are shown to provide nontrivial constraints on the structure of weak neutral currents.

Recently, stringent limits have been set at PETRA on the QED-violation parameters $\Lambda_\pm$, as defined below, in electron-positron annihilations into muon pairs,

$$e^+ + e^- \to \mu^+ + \mu^- . \tag{1}$$

Because we now have every reason to expect a weak-neutral-current contribution to (1) at some level, the significance of such QED tests should be assessed by considering *simultaneously* the electromagnetic *and* weak interactions. The purpose of this note is to show that the "QED experiments" already performed at PETRA provide nontrivial information on the parameters of neutral currents.

It has been traditional to characterize a possible violation of QED in the colliding-beam reaction (1) by parametrizing the data as follows[1]:

$$\sigma = \sigma_{\rm QED}[1 \mp s/(s - \Lambda_\pm^2)]^2 , \tag{2}$$

where s, as usual, stands for the center-of-mass energy squared. Recently, the Mark J Collaboration[2] at PETRA has published limits on $\Lambda_\pm$ based on data at $s \simeq 169$ to 999 GeV^2:

$$\begin{aligned} &\Lambda_- > 97 \text{ GeV} , \\ &\Lambda_+ > 71 \text{ GeV} , \end{aligned} \tag{3}$$

at the 95% confidence level. Similar results have been obtained by the PLUTO and JADE Collaborations.[3] More recently, it has been reported that new Mark J limits on $\Lambda_\pm$ are[4]

$$\begin{aligned} &\Lambda_- > 160 \text{ GeV} , \\ &\Lambda_+ > 120 \text{ GeV} . \end{aligned} \tag{4}$$

Because these limits for $\Lambda_\pm$ are comparable to, or even greater than, the conjectured mass of a Z boson, which is believed to mediate weak-neutral-current interactions with dimensionless coupling strength of order e, it is of obvious importance to examine simultaneously possible weak-interaction effects.

Suppose the effective Lagrangian for the neutral-current interactions of electrons and muons can be written in the following four-fermion form[5]:

$$\begin{aligned} \mathfrak{L} = -(G/\sqrt{2})[&h_{VV}(\bar e\gamma_\lambda e + \bar\mu\gamma_\lambda\mu)(\bar e\gamma_\lambda e + \bar\mu\gamma_\lambda\mu) \\ &+ 2h_{VA}(\bar e\gamma_\lambda e + \bar\mu\gamma_\lambda\mu)(\bar e\gamma_\lambda\gamma_5 e + \bar\mu\gamma_\lambda\gamma_5\mu) \\ &+ h_{AA}(\bar e\gamma_\lambda\gamma_5 e + \bar\mu\gamma_\lambda\gamma_5\mu)(\bar e\gamma_\lambda\gamma_5 e + \bar\mu\gamma_\lambda\gamma_5\mu)] , \end{aligned} \tag{5}$$

where μe universality has been assumed. If the neutral-current interactions are due to the exchange of one or more Z bosons, h_{VV} and h_{AA} must be positive semidefinite. For a single-Z-boson model we have the factorization constraint

$$h_{VA}{}^2 = h_{VV} h_{AA} . \tag{6}$$

For orientation purposes we first concentrate on first-order effects in G, i.e., weak-electromagnetic interference. To this order only the vector-type interaction (the h_{VV} term) contributes to the cross section for (1), and the fractional deviation from the QED prediction is readily derived to be

$$\Delta\sigma/\sigma_{\rm QED} = -(G/\sqrt{2}\pi\alpha) h_{VV} s . \tag{7}$$

Comparing (7) with the expansion of (2) in powers of $s/\Lambda_\pm{}^2$, we obtain

$$h_{VV} = 4\pi\alpha/\sqrt{2}G\Lambda_-{}^2 , \tag{8}$$

where we have considered only the $h_{VV} > 0$ case.[6] A model-independent comparison therefore yields an upper limit on h_{VV} for a lower limit on Λ_-:

$$\begin{aligned} &\Lambda_- > 97 \text{ GeV} \Rightarrow h_{VV} < 0.59 , \\ &\Lambda_- > 160 \text{ GeV} \Rightarrow h_{VV} < 0.22 . \end{aligned} \tag{9}$$

It is noteworthy that the most recent Mark J limit on Λ_- already rules out, at the 95% confidence level, the SU(2)-symmetric $V-A$ model[7] and the once-popular "hybrid model,"[8] both of which predict h_{VV} to be $\frac{1}{4}$.

Within the context of the standard SU(2)$\otimes$U(1) model,[9] which predicts

$$h_{VV} = \tfrac{1}{4}(1 - 4\sin^2\theta_W)^2 , \tag{10}$$

the experimental limits on Λ_- can be used to provide constraints on $\sin^2\theta_W$, *independently of any other experiments*. The Mark J limits imply

$$\begin{aligned} &\Lambda_- > 97\ \mathrm{GeV} \Rightarrow \sin^2\theta_W < 0.63\,, \\ &\Lambda_- > 160\ \mathrm{GeV} \Rightarrow 0.02 < \sin^2\theta_W < 0.48\,, \end{aligned} \tag{11}$$

which are already seen to be nontrivial results.

So far we have considered only the effects due to weak-electromagnetic interference (of order G). Let us now include corrections from the pure weak term (of order G^2) which contains both vector and axial-vector couplings. We also take into account the finite mass of a Z boson within the context of single-Z-boson models where the coupling constants are normalized to agree with the four-fermion Lagrangian (5) in the low-s (i.e., $s \ll m_Z^2$) limit.[10] Equation (8) is now modified as follows:

$$\begin{aligned} \Delta\sigma/\sigma_{\mathrm{QED}} = -4(s/4\pi\alpha)[G(s,m_Z^2)/\sqrt{2}] \\ \times \{h_{VV} - (s/4\pi\alpha)[G(s,m_Z^2)/\sqrt{2}] \\ \times (h_{VV} + h_{AA})^2\}\,, \end{aligned} \tag{12}$$

where

$$G(s,m_Z) \equiv G/(1 - s/m_Z^2) \xrightarrow[s \ll m_Z^2]{} G\,. \tag{13}$$

We see from (12) that there exist three independent parameters h_{VV}, h_{AA}, and m_Z. A limit for Λ_-, for particular values of s and m_Z, defines an allowed region in an h_{VV}-h_{AA} plane, as illustrated in Fig. 1 for $s = 1000\ \mathrm{GeV}^2$ and $\Lambda_- = 160$ GeV. We have done this with $m_Z = \infty$ (four-fermion limit), $m_Z = 90$ GeV (mass predicted by the standard model for $\sin^2\theta_W = 0.23$), and two small values of m_Z (50 and 40 GeV). The allowed region is to the left of the curves. We observe that for expected values of h_{AA} ($\simeq\frac{1}{4}$) a smaller value for m_Z constrains h_{VV} more rigidly. Hopefully a measurement of forward-backward asymmetry in (1) will soon provide a rough value for h_{AA}.

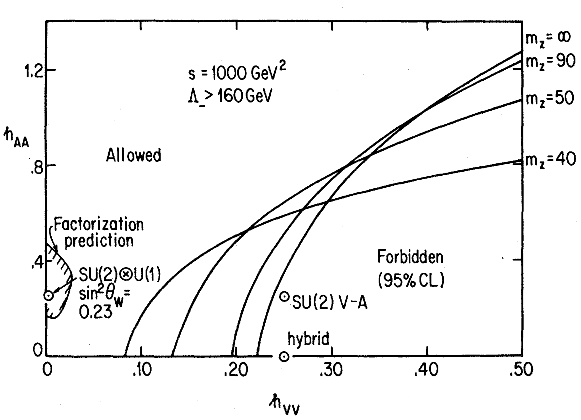

FIG. 1. Allowed regions in h_{VV}-h_{AA} plane for $\Lambda_- = 160$ GeV and $s = 1000\ \mathrm{GeV}^2$, with $m_Z = \infty$, 90, 50, 40, GeV. The predictions of various models are also indicated.

In single-Z-boson models with μe universality, h_{VV} and h_{AA} are related to other neutral-current parameters by the factorization relations[11]

$$h_{VV} = g_V^2/c_\nu^2\,, \quad h_{AA} = g_A^2/c_\nu^2\,, \tag{14}$$

where g_V and g_A are the vector and axial-vector coupling constants in neutrino-electron scattering, and c_ν stands for the elastic neutrino-neutrino scattering constant. In a wide class of gauge models c_ν^2 is predicted to be unity; c_ν^2 can also be determined experimentally by comparing inelastic neutrino-nucleon scattering, inelastic electron-deuteron scattering (the "SLAC asymmetry") and neutrino-electron scattering. The factorization predictions are exhibited in Fig. 1 using the most recent compilation of neutrino-electron scattering data.[12] Also shown in Fig. 1 are the predictions of the SU(2)-symmetric $V-A$ model, the hybrid model, and the standard model with $\sin^2\theta_W$ set to 0.23.

Curves analogous to those in Fig. 1 may also be drawn from limits on Λ_+, but they turn out to lie too high above those in Fig. 1 to be of practical interest. There is, however, an interesting constraint supplied by the Mark J limit on Λ_+. Within the context of single-Z boson models constrained to give the same low-s predictions as the standard model, with $\sin^2\theta_W = 0.23$, we obtain a lower limit on the Z-boson mass as follows[13]:

$$\Lambda_+ > 120\ \mathrm{GeV}\ \text{at}\ s = 1000\ \mathrm{GeV}^2 \Rightarrow m_Z > 36.3\ \mathrm{GeV}\,. \tag{15}$$

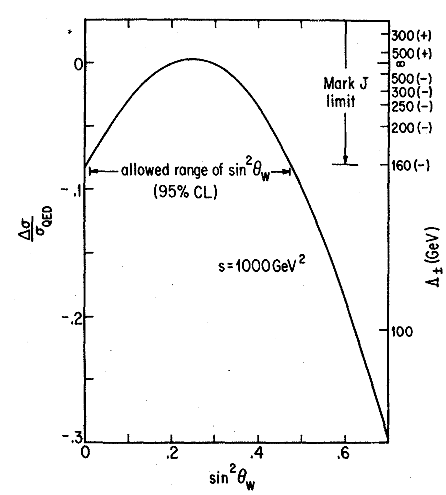

FIG. 2. $\Delta\sigma/\sigma_{\mathrm{QED}}$ vs $\sin^2\theta_W$ in the standard SU(2)⊗U(1) model at $s = 1000\ \mathrm{GeV}^2$, together with appropriate values of $\Lambda_\pm$. The range of $\sin^2\theta_W$ allowed by the Mark J limit on Λ_- is indicated.

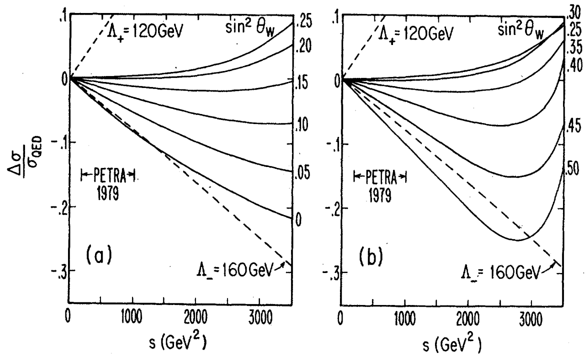

FIG. 3. $\Delta\sigma/\sigma_{QED}$ vs s in the standard SU(2) ⊗ U(1) model for (a) $\sin^2\theta_W = 0$–0.25, and (b) $\sin^2\theta_W = 0.25$–0.50. Also shown is $\Delta\sigma/\sigma_{QED}$ vs s using (2) for $\Lambda_+ = 120$ GeV and $\Lambda_- = 160$ GeV.

Let us now turn to the predictions of the standard SU(2)⊗ U(1) model. The number of parameters now decreases from three to one; the weak-mixing angle θ_W is the sole parameter in terms of which we can express h_{VV} [see (10)], h_{AA} ($=\frac{1}{4}$ independent of θ_W), and m_Z (= 74.6 GeV/ $\sin 2\theta_W$). In Fig. 2 we plot the predicted value of $\Delta\sigma/\sigma_{QED}$ vs $\sin^2\theta_W$ at $s = 1000$ GeV^2. The scale on the right gives the appropriate values of $\Lambda_\pm$. The allowed range of $\sin^2\theta_W$ for $\Lambda_- > 160$ GeV is very close to that obtained earlier using only first-order considerations with $m_Z = \infty$ [see (11)], which is expected because $s \ll m_Z^2$ even for $\sin^2\theta_W \simeq 0.5$. We also note that if $\sin^2\theta_W$ is really around 0.23, finite deviations from the QED cross section are virtually impossible to observe at present energies.

In Figs. 3(a) and 3(b) the energy dependence of $\Delta\sigma/\sigma_{QED}$ predicted by the standard model is illustrated for $\sin^2\theta_W$ between 0 and 0.5. Shown also for comparison is $\Delta\sigma/\sigma_{QED}$, given by (2) for $\Lambda_+ = 120$ GeV and $\Lambda_- = 160$ GeV.

To conclude, the level of sensitivity reached by the PETRA QED tests is such that weak-interaction effects can no longer be ignored. These tests should therefore be regarded more properly as tests of *electroweak* theories (quantum flavor dynamics). In particular, the Mark J results recently reported already rule out, at better than the 95% confidence level, the SU(2)-symmetric $V-A$ model and the once popular hybrid model and have begun to constrain the single parameter of the standard SU(2)⊗ U(1) model, $\sin^2\theta_W$, independently of any other measurement. Furthermore, within the framework of single-Z-boson models that give the same low-q^2 predictions as the standard model with $\sin^2\theta_W = 0.23$, the Z-boson mass must be greater than 36.3 GeV.

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