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Ghost-free axial gauge in supergravity

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A formulation of a ghost-free axial gauge is constructed in supergravity. In this paper we concentrate mainly on the spin-3/2 fermion field, since the discussion on the gravitational field is the same in our supergravity theory as that given in a previous paper by the present author on Einstein gravity. In our axial gauge we investigate a general structure of a free propagator, an exact propagator with eight form factors, two physical polarization vectors, and the physical polarization sum for a spin-3/2 fermion.

I. INTRODUCTION

Supergravity has been expected since its first construction[1] to have improvements over quantized Einstein gravity with regard to renormalizability.[2] Indeed, the cancellation of the ultraviolet divergences of S-matrix elements has been found in this supergravity at one- and two-loop levels. The structure of quantum supergravity is, however, very complicated due to the presence of three kinds of Faddeev-Popov (FP) ghosts. These ghosts come from the requirement of invariance of the supergravity Lagrangian under the Becchi-Rouet-Stora (BRS) transformation[3] which is the quantum counterpart of the general coordinate, local Lorentz, and local supersymmetry transformations.

Recently a formulation on an axial gauge condition was constructed in Einstein gravity.[4] In that condition it was shown that gravitational fields decouple from the FP ghosts. In this derivation an essential point is that a propagator of the FP ghost is proportional to $(k\cdot n)^{-m}$, where k_μ is a momentum, n_μ is a constant vector ($n^2\neq 0$), and m is a positive integer, in the gauge condition given by

$$n_\mu h^{\mu\nu} = \text{an arbitrary function}\,,$$

where $h^{\mu\nu}$ is a gravitational field. In this paper, we construct a formulation of such a gauge condition in supergravity, adopting the Lagrangian with the minimal set of auxiliary fields which are necessary to close the supergravity algebra.[5,6]

This paper is organized as follows. In Sec. II, we shall construct a ghost-free axial gauge in quantum supergravity following the paper of the present author on the quantum Einstein gravity.[4] We also discuss in this section the decoupling of the FP ghosts from gravitons and spin-$\frac{3}{2}$ fermions. For a gravitational field, the formulas of free and exact propagators, polarization vectors, and the physical polarization tensor are also the same in our supergravity theory as those previously given by us in Einstein gravity.[4] This is due to the fact that the gauge condition and the Ward-Takahashi (WT) identity of a two-point vertex function for a gravitational field are chosen the same as those in Einstein gravity. Section III is devoted to deriving three kinds of the Ward-Takahashi identities which correspond to three kinds of invariances of the supergravity Lagrangian under the general coordinate, local Lorentz, and local supersymmetry transformations. It is easily proved by using the Ward-Takahashi identity that the radiative corrections to a two-point vertex function for a spin-$\frac{3}{2}$ fermion are transverse. Then, it is shown that there are eight independent transverse tensors. In calculation of the exact propagator for a spin-$\frac{3}{2}$ fermion these tensors should be added to the free kinetic terms. Therefore, we must introduce eight unknown form factors which are in principle calculable by perturbation. Corresponding to these tensors $(K_i)_{\mu\nu}$, it is shown that there are also eight tensors $(L_i)_{\mu\nu}$ which vanish when contracted with a constant vector n_μ. The exact propagator for a spin-$\frac{3}{2}$ fermion is proved to be the sum of two terms: The first term consists of eight tensors of the latter type $(L_i)_{\mu\nu}$ with coefficients depending on eight form factors and the second term has a dependence on the gauge parameter. Finally we shall calculate the explicit forms of two physical polarization vectors and the physical polarization tensor for a spin-$\frac{3}{2}$ fermion. In Sec. IV, we shall summarize and discuss our results.

II. GHOST-FREE AXIAL GAUGE

The Lagrangian for supergravity with a minimal set of auxiliary fields[5] is given by

$$L_{\rm SG}=\frac{1}{\kappa^2}eR(e,\omega(e,\psi))-\frac{i}{2}\bar\psi_\mu R_\mu(\omega(e,\psi)) -\tfrac{1}{3}eM^2-\tfrac{1}{3}eN^2+\tfrac{1}{3}eg_{\mu\nu}L^\mu L^\nu\,, \qquad (2.1)$$

where M, N, and L^μ are auxiliary fields necessary to close the supersymmetry algebra[5] and[7]

$$\omega_{\mu ab}(e,\psi)=\omega_{\mu ab}(e)+\tfrac{1}{4}i\kappa^2(\bar\psi_\mu\gamma_a\psi_b+\bar\psi_a\gamma_\mu\psi_b-\bar\psi_\mu\gamma_b\psi_a)\,,$$
$$D_\mu=\partial_\mu+\tfrac{1}{2}\omega_{\mu ab}\sigma^{ab}\,,\quad \tfrac{1}{2}R_{\mu\nu}{}^{ab}\sigma_{ab}=[D_\mu,D_\nu]\,,\tag{2.2}$$
$$R=e_a{}^\mu e_b{}^\nu R_{\mu\nu}{}^{ab}\,,\quad R^\mu=-i\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D_\rho\psi_\sigma\,.$$

The gauge-fixing term $L_{\rm GF}$ which enables gravitons and spin-$\frac{3}{2}$ fermions to decouple from the FP ghosts is easily found:

$$\begin{aligned}L_{\rm GF}=&-\frac{1}{4\kappa}(\delta_a^\mu e^{a\nu}+\delta_a^\nu e^{a\mu}-2\eta^{\mu\nu})(n_\mu b_\nu+n_\nu b_\mu)\\&+\frac{\alpha}{2}\eta^{\mu\nu}b_\mu b_\nu-\tfrac{1}{2}\eta^{\mu\nu}n_\mu\partial_\nu(\eta_{a\lambda}e_b{}^\lambda-\eta_{b\lambda}e_a{}^\lambda)b^{ab}\\&+\frac{\beta}{2}b_{ab}b^{ab}-\eta^{\mu\nu}\bar b n_\mu\psi_\nu+\frac{\gamma}{2\kappa}\bar b b\,,\end{aligned}\tag{2.3}$$

where α, β, and γ are constants and b_μ, b_{ab}, and b are Lagrange multiplier fields. The FP ghost term associated with the gauge-fixing term is given by

$$\begin{aligned}L_{\rm FP}=&-\frac{i}{4\kappa}(n_\mu c_{*\nu}+n_\nu c_{*\mu})\,[\delta_a^\mu\delta(e^{a\nu})+\delta_a^\nu\delta(e^{a\mu})]\\&-\frac{i}{2}c_*^{ab}\eta^{\mu\nu}n_\mu\partial_\nu[\eta_{a\lambda}\delta(e_b{}^\lambda)-\eta_{b\lambda}\delta(e_a{}^\lambda)]\\&+\eta^{\mu\nu}\bar c_* n_\mu\delta(\psi_\nu)\,,\end{aligned}\tag{2.4}$$

where $\delta(F)$ means the BRS transformation of the field F. The BRS transformation which makes invariant the sum of $L_{\rm GF}$ and $L_{\rm FP}$ as well as $L_{\rm SG}$ itself is given by[5]

$$\begin{aligned}\delta(e^{a\mu})=&\,c^\lambda e^{a\mu}{}_{,\lambda}-c^\mu{}_{,\lambda}e^{a\lambda}+c^a{}_b e^{b\mu}-\kappa g^{\mu\nu}\bar\psi_\nu\gamma^a c\,,\\
\delta(\psi_\mu)=&\,c^\lambda\psi_{\mu,\lambda}+c^\lambda{}_{,\mu}\psi_\lambda+\tfrac{1}{2}c_{ab}\sigma^{ab}\psi_\mu+2i\kappa^{-1}D_\mu c\\&+\gamma_5(g_{\mu\nu}-\tfrac{1}{3}e_{a\mu}e_{b\nu}\gamma^a\gamma^b)L^\nu c\\&-\tfrac{1}{3}ie_{a\mu}\gamma^a(M-i\gamma_5N)c\,,\\
\delta(M)=&\,c^\lambda M_{,\lambda}+\tfrac{1}{2}e^{-1}\bar R^\mu e_{a\mu}\gamma^a c+\tfrac{1}{2}i\kappa\bar\psi_\lambda L^\lambda\gamma_5 c\\&+\tfrac{1}{2}\kappa\bar\psi_\lambda e_a{}^\lambda\gamma^a(M+i\gamma_5N)c\,,\\
\delta(N)=&\,c^\lambda N_{,\lambda}-\tfrac{1}{2}ie^{-1}\bar R^\mu e_{a\mu}\gamma^a\gamma_5 c+\tfrac{1}{2}\kappa\bar\psi_\lambda L^\lambda c\\&+\tfrac{1}{2}\kappa\bar\psi_\lambda e_a{}^\lambda\gamma^a(-i\gamma_5M+N)c\,,\\
\delta(L^\mu)=&\,c^\lambda L^\mu{}_{,\lambda}-c^\mu{}_{,\lambda}L^\lambda-\tfrac{3}{2}ie^{-1}\bar R^\nu(\delta_\nu^\mu-\tfrac{1}{3}e_{a\nu}e_b{}^\mu\gamma^a\gamma^b)\gamma_5 c\\&+\kappa\bar\psi_\lambda L^\lambda e_a{}^\mu\gamma^a c+\tfrac{1}{2}\kappa\bar\psi_\nu e^{a\nu}\gamma_a L^\mu c\\&+\tfrac{1}{2}\kappa\bar\psi_\nu g^{\mu\nu}(i\gamma_5M+N)c\\&+\tfrac{1}{4}i\kappa e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar\psi_\sigma g_{\nu\lambda}L^\lambda\gamma_5\gamma_a e^a{}_\rho c\,,\\
\delta(c_*)=&\,b\,,\quad\delta(c_*^{ab})=ib^{ab}\,,\quad\delta(c_{*\mu})=ib_\mu\,,\\
\delta(c)=&\,c^\lambda c_{,\lambda}+\tfrac{1}{2}\sigma_{ab}c^{ab}c+\tfrac{1}{2}\kappa\psi_\lambda\bar c\gamma^a e_a{}^\lambda c\,,\\
\delta(c_{ab})=&\,c^\lambda c_{ab,\lambda}+c_{ac}c^c{}_b+i\bar c\gamma^c ce_c{}^\lambda\omega_{\lambda ab}\\&+\tfrac{1}{3}i\kappa\bar c\gamma^c c\epsilon_{abcd}e^d{}_\lambda L^\lambda+\tfrac{2}{3}i\kappa\bar c(M-i\gamma_5N)\sigma_{ab}c\,,\\
\delta(c^\mu)=&\,c^\lambda c^\mu{}_{,\lambda}-i\bar c\gamma^\mu c\,,\\
\delta(b)=&\,\delta(b^{ab})=\delta(b_\mu)=0\,.\end{aligned}\tag{2.5}$$

Using Eq. (2.4), one can transform any field Φ into $\Phi+\epsilon\lambda\delta(\Phi)$ where ϵ is a real parameter and λ is an operator anticommuting with fermionic fields. In the above discussion the FP ghosts c_{ab}, c_*^{ab}, c^μ, and $c_{*\mu}$ are Hermitian operators subject to Fermi statistics, while c and c_* are Bose-Majorana fields. It is possible that one can simplify the formalism very much and rewrite Eq. (2.5) in simpler form by working without the Lagrange multipliers,[8] although the BRS algebra no longer closes.

Now let us calculate the propagators of the FP ghosts in order to confirm that each term of the propagators is proportional to $(k\cdot n)^{-m}$ which ensures the decoupling of the FP ghosts from gravitons and spin-$\frac{3}{2}$ fermions. In what follows we take the gravitational field $h^{a\mu}$ defined by

$$e^{a\mu}\equiv\eta^{a\mu}+\tfrac{1}{2}\kappa h^{a\mu}\tag{2.6}$$

and understand that the field tensor indices $\mu,\nu,\ldots,$ are freely raised and lowered by the Minkowski metric $\eta_{\mu\nu}$. Using the definition (2.6) in Eq. (2.3), we obtain the free propagators of the FP ghosts

$$\begin{array}{cc}\begin{array}{ccc}\quad c_{*\mu}\quad & \quad c_{*ab}\quad & c_*\end{array} & \\ \left(\begin{array}{ccc}\dfrac{2i\kappa}{(k\cdot n)}\left[\eta_{\mu\nu}-\dfrac{k_\mu n_\nu}{2(k\cdot n)}\right] & 0 & 0\\ \dfrac{-\kappa}{(k\cdot n)}(\eta_{b\nu}k_a-\eta_{a\nu}k_b) & \dfrac{i}{(k\cdot n)}\eta_{ac}\eta_{bd} & 0\\ 0 & 0 & \dfrac{i\kappa}{2(k\cdot n)}\end{array}\right) & \begin{array}{c}c_\nu\\ \\ c_{cd}\,.\\ \\ c\end{array}\end{array}\tag{2.7}$$

One can apparently see that each term of each propagator is proportional to $(k\cdot n)^{-m}$. Therefore, following the discussion given in the previous paper,[4] we have proved the decoupling of the FP ghosts from gravitons and spin-$\frac{3}{2}$ fermions.

III. PHYSICALLY IMPORTANT QUANTITIES FOR A SPIN-$\frac{3}{2}$ FERMION

A. Ward-Takahashi identities

Corresponding to three kinds of the invariances of the Lagrangian $L_{\rm SG}$, there exist three kinds of the Ward-Takahashi identities. The generating functional of the complete Green's function is defined by a path integral:

$$\begin{aligned}Z(J,K)=\int[de^{a\mu}][d\psi_\mu][dM][dN][dL^\mu]\\ \times\exp\Big[i\int d^4X(L_{\rm SG}+L'_{\rm GF}+J_{a\mu}e^{a\mu}\\+\bar J^\mu\psi_\mu+KM+K_5N+K_\mu L^\mu)\Big]\,,\end{aligned}\tag{3.1}$$

where

$$L'_{\mathrm{GF}} = -\frac{1}{8\alpha\kappa^2}\eta_{\mu\rho}n_\nu n_\sigma(\delta^\mu_a e^{a\nu} + \delta^\nu_a e^{a\mu} - 2\eta^{\mu\nu}) \times (\delta^\rho_b e^{b\sigma} + \delta^\sigma_b e^{b\rho} - 2\eta^{\rho\sigma}) - \frac{1}{8\beta}\eta^{ac}\eta^{bd}[\eta^{\mu\nu}n_\mu\partial_\nu(\eta_{a\lambda}e_b{}^\lambda - \eta_{b\lambda}e_a{}^\lambda)] \times [\eta^{\rho\sigma}n_\rho\partial_\sigma(\eta_{c\lambda}e_d{}^\lambda - \eta_{d\lambda}e_c{}^\lambda)] - \frac{\kappa}{2\gamma}\eta^{\mu\nu}\eta^{\rho\sigma}n_\mu\bar{\psi}_\nu n_\rho\psi_\sigma , \tag{3.2}$$

and L_{SG} is given by Eq. (2.1). The term L'_{GF} is obtained from L_{GF} given by Eq. (2.3) by functionally integrating over the Lagrange multiplier fields b_μ, b_{ab}, and b. The generating functional of the connected Green's functions is given in terms of $Z(J,K)$ by

$$W(J,K) = -i\ln Z(J,K).$$

Finally, we introduce the generating functional of the one-particle irreducible Green's functions defined by the Legendre transform of $W(J,K)$:

$$\Gamma(e,\psi,M,N,L) = W(J,K) - \int d^4x(J_{a\mu}e^{a\mu} + \bar{J}^\mu\psi_\mu + KM + K_5 N + K_\mu L^\mu), \tag{3.3}$$

$$\begin{aligned} e^{a\mu} &= \delta W/\delta J_{a\mu} , \\ \psi_\mu &= -\delta W/\delta \bar{J}^\mu, \quad \bar{\psi}_\mu = \delta W/\delta J^\mu , \\ M &= \delta W/\delta K , \\ N &= \delta W/\delta K_5 , \\ L^\mu &= \delta W/\delta K_\mu , \end{aligned} \tag{3.4}$$

$$\begin{aligned} J_{a\mu} &= -\delta\Gamma/\delta e^{a\mu} , \\ \bar{J}^\mu &= -\delta\Gamma/\delta\psi_\mu, \quad J^\mu = \delta\Gamma/\delta\bar{\psi}_\mu , \\ K &= -\delta\Gamma/\delta M , \\ K_5 &= -\delta\Gamma/\delta N , \\ K_\mu &= -\delta\Gamma/\delta L^\mu , \end{aligned} \tag{3.5}$$

where we have not discriminated the notational difference between quantum and classical fields. We adopt here and hereafter that $\delta/\delta F$ is the right derivative when F's are Majorana fields, ψ_μ, $\bar{\psi}_\mu$, J^μ, and $\bar{J}^\mu$.

Now in the following let us write three kinds of Ward-Takahashi (WT) identities.

(i) *General coordinate transformation* (GCT). The Lagrangian L_{SG} has the following invariance under GCT:

$$\begin{aligned} \delta_G(e^{a\mu}) &= (e^{a\mu}{}_{,\lambda} - e^{a\nu}\delta^\mu_\lambda\partial_\nu)\Omega^\lambda , \\ \delta_G(\psi_\mu) &= (\psi_{\mu,\lambda} + \psi_\lambda\partial_\mu)\Omega^\lambda , \\ \delta_G(M) &= M_{,\lambda}\Omega^\lambda , \\ \delta_G(N) &= N_{,\lambda}\Omega^\lambda , \\ \delta_G(L^\mu) &= (L^\mu{}_{,\lambda} - L^\nu\delta^\mu_\lambda\partial_\nu)\Omega^\lambda , \end{aligned} \tag{3.6}$$

where Ω^λ is a gauge function. Defining[9]

$$\tilde{\Gamma} = \Gamma + \int d^4x\left\{\frac{1}{8\alpha\kappa^2}\eta_{\mu\rho}n_\nu n_\sigma(\delta^\mu_a e^{a\nu} + \delta^\nu_a e^{a\mu} - 2\eta^{\mu\nu})(\delta^\rho_b e^{b\sigma} + \delta^\sigma_b e^{b\rho} - 2\eta^{\rho\sigma}) + \frac{1}{8\beta}\eta^{ac}\eta^{bd}[\eta^{\mu\nu}n_\mu\partial_\nu(\eta_{a\lambda}e_b{}^\lambda - \eta_{b\lambda}e_a{}^\lambda)][\eta^{\rho\sigma}n_\rho\partial_\sigma(\eta_{c\lambda}e_d{}^\lambda - \eta_{d\lambda}e_c{}^\lambda)] + \frac{\kappa}{2\gamma}\eta^{\mu\nu}\eta^{\rho\sigma}n_\mu\bar{\psi}_\nu n_\rho\psi_\sigma\right\}, \tag{3.7}$$

we can write down the WT identity

$$\frac{\delta\tilde{\Gamma}}{\delta e^{a\mu}}(e^{a\mu}{}_{,\rho} + e^{a\nu}\delta^\mu_\rho\overleftarrow{\partial}_\nu) + \frac{\delta\tilde{\Gamma}}{\delta\psi_\mu}(\psi_{\mu,\rho} - \psi_\rho\overleftarrow{\partial}_\mu) + \frac{\delta\tilde{\Gamma}}{\delta M}M_{,\rho} + \frac{\delta\tilde{\Gamma}}{\delta N}N_{,\rho} + \frac{\delta\tilde{\Gamma}}{\delta L^\mu}(L^\mu{}_{,\rho} + L^\nu\delta^\mu_\rho\overleftarrow{\partial}_\nu) = 0 . \tag{3.8}$$

In the above Eq. (3.8) $\tilde{\Gamma}$ yields the Ward-Takahashi identities for the diagram with loops, since it has subtracted the tree graphs.

(ii) *Local Lorentz transformation* (LLT). The Lagrangian L_{SG} has the following invariance under LLT:

$$\begin{aligned} \delta_L(e_a{}^\mu) &= e^{b\mu}\Omega_{ab} , \\ \delta_L(\psi_\mu) &= \tfrac{1}{2}\sigma^{ab}\psi_\mu\Omega_{ab} , \\ \delta_L(M) &= \delta_L(N) = \delta_L(L^\mu) = 0 , \end{aligned} \tag{3.9}$$

where Ω_{ab} is a gauge function. Defining[9]

$$\tilde{\Gamma}' = \Gamma + \int d^4x\frac{1}{8\beta}\eta^{ac}\eta^{bd}[\eta^{\mu\nu}n_\mu\partial_\nu(\eta_{a\lambda}e_b{}^\lambda - \eta_{b\lambda}e_a{}^\lambda)] \times [\eta^{\rho\sigma}n_\rho\partial_\sigma(\eta_{c\lambda}e_d{}^\lambda - \eta_{d\lambda}e_c{}^\lambda)] , \tag{3.10}$$

we can write the WT identity as

$$\frac{\delta\tilde{\Gamma}'}{\delta e_a{}^\mu}e^{b\mu} + \frac{\delta\tilde{\Gamma}'}{\delta\psi_\mu}\tfrac{1}{2}\sigma^{ab}\psi_\mu = 0 . \tag{3.11}$$

(iii) *Supersymmetry transformation* (SST). The Lagrangian L_{SG} has the following invariance under SST:

$$\delta_S(e^{a\mu}) = -\kappa g^{\mu\nu}\bar{\psi}_\nu\gamma^a\Omega\,,$$
$$\delta_S(\psi_\mu) = [2i\kappa^{-1}D_\mu + \gamma_5(g_{\mu\nu} - \tfrac{1}{3}e_{a\mu}e_{b\nu}\gamma^a\gamma^b)L^\nu - \tfrac{1}{3}ie_{a\mu}\gamma^a(M - i\gamma_5 N)]\Omega\,,$$
$$\delta_S(M) = [\tfrac{1}{2}e^{-1}\bar{R}^\mu e_{a\mu}\gamma^a + \tfrac{1}{2}i\kappa\bar{\psi}_\lambda L^\lambda\gamma_5 + \tfrac{1}{2}\kappa\bar{\psi}_\lambda e_a{}^\lambda\gamma^a(M + i\gamma_5 N)]\Omega\,, \tag{3.12}$$
$$\delta_S(N) = [-\tfrac{1}{2}ie^{-1}\bar{R}^\mu e_{a\mu}\gamma^a\gamma_5 + \tfrac{1}{2}\kappa\bar{\psi}_\lambda L^\lambda + \tfrac{1}{2}\kappa\bar{\psi}_\lambda e_a{}^\lambda\gamma^a(-i\gamma_5 M + N)]\Omega\,,$$
$$\delta_S(L^\mu) = [-\tfrac{3}{2}ie^{-1}\bar{R}^\nu(\delta_\nu^\mu - \tfrac{1}{3}e_{a\nu}e_b{}^\mu\gamma^a\gamma^b)\gamma_5 + \kappa\bar{\psi}_\lambda L^\lambda e_a{}^\mu\gamma^a + \tfrac{1}{2}\kappa\bar{\psi}_\nu e^{a\nu}\gamma_a L^\mu + \tfrac{1}{2}\kappa\bar{\psi}_\nu g^{\mu\nu}(i\gamma_5 M + N) + \tfrac{1}{4}i\kappa e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\nu g_{\rho\lambda}L^\lambda\gamma_5\gamma_a e^a{}_\sigma]\Omega\,,$$

where Ω is a gauge function and a Majorana spinor. In terms of $\bar{\Gamma}$ defined by Eq. (3.7), the following equation holds:

$$\sum_i \int d^4x \frac{\delta\bar{\Gamma}}{\delta\phi_i}\delta_S(\phi_i) = 0\,, \tag{3.13}$$

where the summation runs over all fields, $e^{a\mu}$, ψ_μ, M, N, and L^μ. The WT identity is obtained from Eq. (3.13) when one performs integration by parts and regards Ω as an arbitrary function.

Using the WT identities equations (3.8) and (3.13), one can easily obtain the following important results:

$$\partial_\mu(\delta^2\bar{\Gamma}/\delta h_{\mu\nu}\delta h_{\rho\sigma})|_0 = 0\,, \tag{3.14}$$
$$\partial_\mu(\delta^2\bar{\Gamma}/\delta\psi_\mu\delta\bar{\psi}_\nu)|_0 = 0\,, \tag{3.15}$$

where subscript 0 denotes that all external fields should vanish. The equations (3.14) and (3.15) mean that the radiative corrections to the kinetic terms of a graviton and a spin-$\frac{3}{2}$ fermion are transverse with respect to the momentum. One more thing to be noticed is that from Eqs. (2.3) and (3.14) a graviton has the same gauge-fixing term and the same WT identity for a two-point vertex function as those in Ref. 4 and hence a graviton has the same physical properties as those shown in Ref. 4.

B. Free spin-$\frac{3}{2}$ fermion propagator

In order to evaluate the free spin-$\frac{3}{2}$ fermion propagator, we must find rank-two tensors made of k_μ, n_μ, γ_μ, and $\eta_{\mu\nu}$. The kinetic term and the propagator of a spin-$\frac{3}{2}$ fermion are composed of these tensors. These tensors $S_{\mu\nu}$ are, however, constrained to be

$$S_{\mu\nu}(k)^t = C^{-1}S_{\nu\mu}(-k)C\,, \tag{3.16}$$

where C is a charge conjugation matrix $C^{-1}\gamma_a C = -\gamma_a{}^t$. This constraint is due to the fact that ψ_μ is a Majorana field:

$$\psi_\mu = C(\bar{\psi}_\mu)^t\,,$$
$$\mathrm{FT}\langle 0|T(\psi_\mu\bar{\psi}_\nu)|0\rangle(k)^t = C^{-1}\mathrm{FT}\langle 0|T(\psi_\nu\bar{\psi}_\mu)|0\rangle(-k)C\,. \tag{3.17}$$

There are fourteen independent such tensors listed in Table I. We also list in Tables II and III such tensors $(K_i)_{\mu\nu}$ and $(L_i)_{\mu\nu}$ that are made of tensors $(S_i)_{\mu\nu}$ listed in Table I and which satisfy[10]

$$n^\mu(K_i)_{\mu\nu} = (K_i)_{\mu\nu}n^\nu = 0 \quad (i = 1\text{–}8)\,, \tag{3.18}$$
$$k^\mu(L_i)_{\mu\nu} = (L_i)_{\mu\nu}k^\nu = 0 \quad (i = 1\text{–}8)\,, \tag{3.19}$$

respectively. The tensors $(L_i)_{\mu\nu}$ are obtained from $(K_i)_{\mu\nu}$ except for a constant number by exchanging k_μ and n_μ with each other in $(K_i)_{\mu\nu}$. By using these tensors $(K_i)_{\mu\nu}$ and $(L_i)_{\mu\nu}$, the kinetic term and the propagator of a spin-$\frac{3}{2}$ fermion may be made into simple forms. In terms of these tensors, the kinetic term of a spin-$\frac{3}{2}$ fermion in the momentum

TABLE I. Fourteen independent tensors which satisfy $(S_i)_{\mu\nu}(k)^t = C^{-1}(S_i)_{\nu\mu}(-k)C$.

S_1	$\eta_{\mu\nu}\not{k}$
S_2	$\eta_{\mu\nu}\frac{(k\cdot n)}{n^2}\not{n}$
S_3	$\frac{1}{k^2}k_\mu k_\nu\not{k}$
S_4	$\frac{1}{(k\cdot n)}k_\mu k_\nu\not{n}$
S_5	$\frac{1}{n^2}n_\mu n_\nu\not{k}$
S_6	$\frac{(k\cdot n)}{(n^2)^2}n_\mu n_\nu\not{n}$
S_7	$\frac{1}{(k\cdot n)}(k_\mu n_\nu + k_\nu n_\mu)\not{k}$
S_8	$\frac{1}{n^2}(k_\mu n_\nu + k_\nu n_\mu)\not{n}$
S_9	$k_\mu\gamma_\nu + k_\nu\gamma_\mu$
S_{10}	$\frac{(k\cdot n)}{n^2}(n_\mu\gamma_\nu + n_\nu\gamma_\mu)$
S_{11}	$\gamma_\mu\not{k}\gamma_\nu$
S_{12}	$\frac{(k\cdot n)}{n^2}\gamma_\mu\not{n}\gamma_\nu$
S_{13}	$\frac{1}{(k\cdot n)}(k_\nu\not{k}\gamma_\mu\not{n} + k_\mu\not{n}\gamma_\nu\not{k})$
S_{14}	$\frac{1}{n^2}(n_\nu\not{n}\gamma_\mu\not{k} + n_\mu\not{k}\gamma_\nu\not{n})$

TABLE II. Eight independent tensors which satisfy $n^\mu(K_i)_{\mu\nu}=0$. Here, $a=k^2n^2/(k\cdot n)^2$.

K_1	$S_1+aS_3-S_7$
K_2	$aS_3+2S_4-S_{11}-S_{13}$
K_3	S_1-S_5
K_4	$3S_5+S_8-S_9-S_{10}+S_{11}-S_{14}$
K_5	$S_2+S_4-S_8$
K_6	$S_4-S_8+S_{10}-S_{12}$
K_7	S_2-S_6
K_8	$2S_4-S_8-S_9+S_{10}$

space is written as

$$\frac{1}{2}\Lambda_{\mu\nu}=\frac{1}{2}\left[-\eta_{\mu\nu}\not{k}+(k_\mu\gamma_\nu+k_\nu\gamma_\mu)-\gamma_\mu\not{k}\gamma_\nu-\frac{\kappa}{\gamma}n_\mu n_\nu\right]$$
$$=\frac{1}{2}(-L_1+L_2)_{\mu\nu}-\frac{\kappa}{2\gamma}n_\mu n_\nu\,. \tag{3.20}$$

The free spin-$\frac{3}{2}$ fermion propagator $iG_{\mu\nu}$ is defined by

$$\Lambda_{\mu\nu}G^\nu{}_\rho=\eta_{\mu\rho}\,. \tag{3.21}$$

This is readily solved in terms of the tensors $(K_i)_{\mu\nu}$ to be

$$k^2G_{\mu\nu}=\left(-K_1+\frac{1}{N-2}K_2\right)_{\mu\nu}-\frac{\gamma}{\kappa}\frac{k^2}{(k\cdot n)^2}k_\mu k_\nu\,, \tag{3.22}$$

where N is the space-time dimension. Let us discuss the reason why $G_{\mu\nu}$ can be written only in terms of $(K_i)_{\mu\nu}$ in the case of $\gamma=0$. In this case the gauge condition is given by

$$n^\mu\psi_\mu=0\,,$$

and hence

$$n^\mu iG_{\mu\nu}=n^\mu\mathrm{FT}\langle 0|T(\psi_\mu\bar{\psi}_\nu)|0\rangle=0\,. \tag{3.23}$$

C. Exact spin-$\frac{3}{2}$ fermion propagator

As shown in Sec. III A, the radiative corrections to a two-point vertex function are transverse with respect to the momentum k_μ. Hence, the exact kinetic term $\frac{1}{2}\bar{\Lambda}_{\mu\nu}$ has a form

$$\bar{\Lambda}_{\mu\nu}=\left(-L_1+L_2+\sum_{i=1}^{8}\Lambda_iL_i\right)_{\mu\nu}-\frac{\gamma}{\kappa}n_\mu n_\nu\,, \tag{3.24}$$

TABLE III. Eight independent tensors which satisfy $k^\mu(L_i)_{\mu\nu}=0$.

L_1	$S_1+aS_5-S_7$
L_2	$aS_5-S_7+S_9-S_{11}$
L_3	S_1-S_3
L_4	$3S_4+S_7-S_9-aS_{10}+aS_{12}-S_{13}$
L_5	$S_2+aS_6-S_8$
L_6	$2S_5+aS_6-S_{12}-S_{14}$
L_7	aS_2-S_4
L_8	$2aS_5-S_7+S_9-aS_{10}$

where the coefficients Λ_i's are in principle calculable by perturbation. The exact spin-$\frac{3}{2}$ fermion propagator $i\bar{G}_{\mu\nu}$ is defined by

$$\bar{\Lambda}_{\mu\nu}\bar{G}^\nu{}_\rho=\eta_{\mu\rho}\,. \tag{3.25}$$

By noting the fact that γ is not affected by renormalization, the exact propagator can be written as

$$k^2\bar{G}_{\mu\nu}=\sum_{i=1}^{8}A_i(K_i)_{\mu\nu}-\frac{\gamma}{\kappa}\frac{k^2}{(k\cdot n)^2}k_\mu k_\nu\,. \tag{3.26}$$

We do not evaluate all the coefficients A_i explicitly because of their complexity. We will, however, give the explicit expressions for some of them, since they are necessary to calculate the physical polarization vectors.

D. Physical polarization vectors

There are eight polarization vectors for a spin-$\frac{3}{2}$ fermion, two of which are physical. Polarization vectors are defined as the eigenvectors of the exact propagator

$$i\bar{G}_{\mu\nu}\epsilon^\nu=B\epsilon_\mu\,, \tag{3.27}$$

where B is the eigenvalue of ϵ_μ. Two physical polarization vectors must express states of spin $\frac{3}{2}$ and helicities $\pm\frac{3}{2}$ and be defined by

$$k^\mu\epsilon_\mu=n^\mu\epsilon_\mu=\gamma^\mu\epsilon_\mu=0,\quad \epsilon_\mu{}^\dagger\epsilon^\mu=-E\,. \tag{3.28}$$

where E is the energy. Introducing two independent vectors l_μ and m_μ which satisfy

$$k\cdot l=n\cdot l=k\cdot m=n\cdot m=l\cdot m=0,\quad l^2=m^2=-1\,, \tag{3.29}$$

we obtain the physical polarization vectors as

$$\epsilon_\mu^\pm=\frac{E^{1/2}}{2}(l_\mu\pm im_\mu)u^\pm\,, \tag{3.30}$$

where massless spinors $u^\pm$ must satisfy

$$(\not{l}\pm i\not{m})u^\pm=0\,, \tag{3.31}$$

respectively, with normalization $(u^\pm)^\dagger u^\pm=2$. When one takes $n^\mu\propto(0,0,0,1)$ and in one Lorentz frame $k^\mu=(E,0,0,E)$, then one can take

$$l^\mu=(0,1,0,0),\quad m^\mu=(0,0,1,0)\,, \tag{3.32}$$

and therefore

$$u^+=(1,0,1,0),\quad u^-=(0,1,0,-1)\,. \tag{3.33}$$

These solutions mean that $u^\pm$ have $\pm\frac{1}{2}$ helicities and $l_\mu\pm im_\mu$ have ±1 helicities, respectively. The eigenvalues for two physical polarization vectors $\epsilon_\mu^\pm$ are the same and are given by

$$-iB=(A_1+A_3)\not{k}+(A_5+A_7)\frac{(k\cdot n)}{n^2}\not{n} \tag{3.34}$$

with

$$A_1+A_3=\frac{-1+\Lambda_1+\Lambda_3}{a}\{(-1+\Lambda_1+\Lambda_3)^2+(1/a)(\Lambda_5+a\Lambda_7)[2(-1+\Lambda_1+\Lambda_3)+(\Lambda_5+a\Lambda_7)]\}^{-1},$$
$$A_5+A_7=\frac{\Lambda_5+a\Lambda_7}{-1+\Lambda_1+\Lambda_3}(A_1+A_3),\quad a=k^2n^2/(k\cdot n)^2, \tag{3.35}$$

where A_i's are the coefficients in Eq. (3.26). In order for $\epsilon_\mu^\pm$ to be physical, it is necessary that $B\propto \not{k}/k^2$ only near the mass shell ($k^2=0$). The first term in Eq. (3.34) really satisfies this condition but the second does not. Therefore, the second term must vanish on the mass shell ($k^2=0$), or

$$\Lambda_5+a\Lambda_7\propto(k^2)^2, \tag{3.36}$$

which is easily derived when one observes Eq. (3.35). We cannot prove this by using a general method. Here, we only conjecture that this is true.

E. Physical polarization tensor

The physical polarization tensor is defined by

$$T_{\mu\nu}=\sum_{\lambda=\pm}\epsilon_\mu^{(\lambda)}\bar{\epsilon}_\nu^{(\lambda)}, \tag{3.37}$$

which satisfies

$$T_{\mu\nu}(k)^t=C^{-1}T_{\nu\mu}(-k)C, \tag{3.38a}$$
$$k^\mu T_{\mu\nu}=n^\mu T_{\mu\nu}=\gamma^\mu T_{\mu\nu}=0, \tag{3.38b}$$
$$T_\mu^\mu=-\tfrac{1}{2}\not{k}. \tag{3.38c}$$

In terms of the tensors $(K_i)_{\mu\nu}$ and $(L_i)_{\mu\nu}$ listed in Tables II and III, we can easily find $T_{\mu\nu}$ satisfying Eqs. (3.38):

$$T_{\mu\nu}=\frac{1}{2(a-1)(N-2)(N-3)}[(N-2)K_1-K_2-a(N-2)K_3-aK_4]_{\mu\nu}$$
$$=\frac{1}{2(a-1)(N-2)(N-3)}[(N-2-a)L_1+(a-1)L_2-a(N-3)L_3-L_4+aL_5-aL_6-L_7]_{\mu\nu}. \tag{3.39}$$

Note that the difference between $k^2G_{\mu\nu}$ and $2T_{\mu\nu}$ vanishes when k_μ is on the mass shell and $N=4$.

IV. SUMMARY AND CONCLUDING REMARKS

We have proposed a ghost-free axial gauge in supergravity in which gravitons and spin-$\frac{3}{2}$ fermions decouple from three kinds of the FP ghosts. In that gauge we have studied some physically important quantities for a spin-$\frac{3}{2}$ fermion such as free and exact propagators, two physical polarization vectors, and the physical polarization tensor.

Although the supersymmetric Lagrangian L_{SG} itself has a simple form, the whole Lagrangian density including the gauge-fixing and the FP ghost terms is very complicated. This is because of the complicated structure of the supersymmetry transformation and the corresponding BRS transformation. In a ghost-free axial gauge, however, the FP ghosts disappear from the Lagrangian. Therefore, a general structure of the theory in this gauge must be more easily seen than in a usual gauge in quantum supergravity. We must, however, also remark that the spin-$\frac{3}{2}$ and spin-2 propagators are much more complicated to calculate with than in the more conventional gauges.

The important problems that remain to be solved are the following: unitarity of an S matrix in this gauge without the FP ghosts[11], gauge (n_μ) independence of an S matrix[12], and $(k^2)^2$ dependence of the coefficients $\Lambda_5+a\Lambda_7$ (see Sec. III C). Though we have not succeeded in solving the last problem, it may be checked at least in a one-loop calculation.

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[9]By the definition of a graviton in Eq. (2.6), only the antisymmetric part of $h_{\mu\nu}$ contributes to the gauge-fixing term for the local Lorentz invariance, but this antisymmetric part of $h_{\mu\nu}$ has no kinetic term in the Lagrangian. Namely, this does not propagate. Hence, we can neglect this part.

[10]These tensors may be obtained by making appropriate products of $\not{k}$ and $\not{n}$ with the following tensors:

$$a_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{k^2}\gamma_\mu k_\nu \not{k}, \quad \tilde{a}_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{k^2}\not{k}\gamma_\mu k_\nu,$$

$$b_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{n^2}\gamma_\mu n_\nu \not{n}, \quad \tilde{b}_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{n^2}\not{n}\gamma_\mu n_\nu,$$

$$d_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{(k\cdot n)}k_\mu n_\nu, \quad e_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{n^2}n_\mu n_\nu,$$

$$f_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{k^2}k_\mu k_\nu,$$

which satisfy

$$\begin{aligned} k^\mu a_{\mu\nu} = a_{\mu\nu}\gamma^\nu = k^\mu \tilde{a}_{\mu\nu} = n^\mu b_{\mu\nu} = b_{\mu\nu}\gamma^\nu = n^\mu \tilde{b}_{\mu\nu} \\ = n^\mu d_{\mu\nu} = d_{\mu\nu}k^\nu = n^\mu e_{\mu\nu} = k^\mu f_{\mu\nu} = 0. \end{aligned}$$

For instance,

$$\tfrac{1}{2}(d_{\mu\rho}\gamma^\rho d_{\nu\sigma}\gamma^\sigma \not{n} + \not{n} d_{\nu\sigma}\gamma^\sigma d_{\mu\rho}\gamma^\rho) = \frac{n^2}{(k\cdot n)}K_6.$$

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