Photon-photon correlations in $e^+e^- \to F^*\bar{F}^* \to F\gamma_1\bar{F}\gamma_2$

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The F and F^* mesons may be observed in $e^+e^- \to F^*\bar{F}^* \to F\gamma_1\bar{F}\gamma_2$. Kinematic constraints relating the energies and directions of the decay photons can be used to discriminate against backgrounds. This method may also be helpful for the discovery of new baryons and heavier mesons. Detailed studies of the photons' angular correlations can determine the $F^*\bar{F}^*$ production amplitudes.

I. INTRODUCTION

The F^* meson ($J^P = 1^-$, $c\bar{s}$ state) is expected to decay exclusively into a photon and an F meson ($J^P = 0^-$, $c\bar{s}$ state). Thus, a possible signature for F^* production is a nearly monoenergetic photon, with an energy spread arising from the Doppler shift. If there is substantial $F^*\bar{F}^*$ pair production in e^+e^- annihilation (as there is at 4.028 GeV for $D^*\bar{D}^*$), it may be possible to identify such events by observing both decay photons. Such coincidence experiments may well have advantages over simple searches for monochromatic photons since backgrounds from pion decays would not be as large. In this paper, we consider what analysis can be made from the measurement of the energies and directions of the two photons in the process $e^+e^- \to F^*\bar{F}^* \to F\gamma_1\bar{F}\gamma_2$. Our conclusions apply as well to analogous processes where the $F^*$is replaced by B^* (the vector state of a b quark and an ordinary quark) and other heavy mesons or baryons with photonic decays.

The photon-photon correlations are of two sorts. The first is simply a consequence of kinematics. It may provide a useful means of eliminating background: Photons produced in other ways will not normally satisfy the kinematic restrictions which follow from the decay chain $F^*\bar{F}^* \to F\gamma_1\bar{F}\gamma_2$. The second source of correlation is dynamical. The spins of the F^* and $\bar{F}^*$ are correlated in a fashion dependent on the production amplitudes. The angular correlations of the decay photons reflect the initial spin correlations and thus provide a means of studying the production amplitudes themselves, should very-high-quality data become available.

The kinematics are discussed in Sec. II. Each event may be plotted in the E_1-E_2 plane, where E_1 and E_2 are the photon energies. Because of the Doppler effect, the points are spread throughout a square in this plane. If the angle between the photons is known, the point is further restricted to lie within an ellipse inside the square, the size and shape of the ellipse being determined by the angle between the photons. If the photons are nearly back-to-back the ellipse is narrow. This provides a means for discriminating against background, which is not constrained to lie within the ellipse. Some kinematical details are given in Appendix A.

In Sec. III we discuss the production amplitudes using a nonrelativistic analysis, suitable near threshold. The production amplitudes are related to the observable angular distributions. A simple model for the production amplitudes is discussed in Appendix B.

II. KINEMATICS

The process is described in Fig. 1. We treat the F^* and F as nonrelativistic. The (unobserved) direction of the F^* in the laboratory (which is the e^+e^- c.m.) is $\hat{z}$ and the directions of γ_1 and γ_2 are $\hat{k}_1$ and $\hat{k}_2$. The c.m. energy is ΔE ($= \sqrt{s} - 2m_{F^*}$) above threshold, and the velocity of the F^* in the laboratory is

$$\beta \cong \left(\Delta E / m_{F^*}\right)^{1/2}. \tag{1}$$

The photon's energy in the F^* ($\bar{F}^*$) rest frame is

$$E_0 = (m_{F^*} - m_F)\left(1 - \frac{m_{F^*} - m_F}{2m_{F^*}}\right). \tag{2}$$

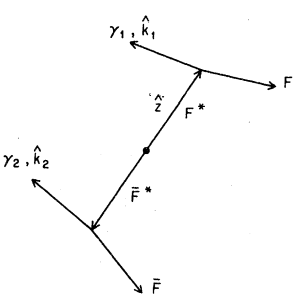

FIG. 1. Schematic representation of $e^+e^- \to F^*\bar{F}^* \to F\gamma_1\bar{F}\gamma_2$ in the center of mass.

The energies of the photons in the laboratory are Doppler shifted due to the F^* ($\bar{F}^*$) velocity. To first order in β they are given by

$$E_1 = E_0(1+\beta\hat{k}_1\cdot\hat{z}),$$
$$E_2 = E_0(1-\beta\hat{k}_2\cdot\hat{z}). \tag{3}$$

We shall ignore the difference between the direction of the photons in the F^* ($\bar{F}^*$) rest frame and their direction in the laboratory. This difference is of order β for the cosine of the relevant angles. Using Eq. (3), it induces a correction of order β^2 in E_1 and E_2.

Just above threshold ($\beta \cong 0$), the signal for the F^* and the F would be a pair of monoenergetic photons with $E=E_0$. However, for finite ΔE, the photon's energies are smeared due to the Doppler shift and the events populate the following square in the E_1-E_2 plane

$$E_0(1-\beta) \leq E_1,\quad E_2 \leq E_0(1+\beta). \tag{4}$$

Assuming a uniform distribution in the E_1-E_2 plane of 2γ background events, the amount of background in the signal square [Eq. (4)] is proportional to ΔE.

Let us now see what additional information we can get by taking into account the angle α between the two photons. The three unit vectors $\hat{z}$, $\hat{k}_1$, and $\hat{k}_2$ are kinematically uncorrelated, and they can independently point in any direction. However, the cosines of the three angles

$$x=\hat{k}_1\cdot\hat{z},$$
$$y=\hat{k}_2\cdot\hat{z}, \tag{5}$$
$$z=\hat{k}_1\cdot\hat{k}_2$$

are correlated. For example, if $x=1$ and $y=1$, z is necessarily 1. Since using Eq. (3), the x-y plane corresponds to the E_1-E_2 plane (up to a scaling factor, a shift of origin and $y \leftrightarrow -y$), this introduces a correlation between the three measurable quantities E_1, E_2 and $\cos\alpha = \hat{k}_1\cdot\hat{k}_2$.

When we let the three unit vectors go in all possible directions, the corresponding points occupy a volume in the x-y-z space, which has the shape of an inflated tetrahedron (Fig 2.) It is contained in the $-1 \leq x,\ y,\ z \leq 1$ cube, and it occupies $\pi^2/16 \cong 0.62$ of its volume. The boundary of this volume is given (see Appendix A) by

$$x^2+y^2+z^2-2xyz=1. \tag{6}$$

One of the main diagonals of each face of the cube belongs to this boundary, and these six diagonals have a tetrahedral shape.

For a given angle between the two photons, $\alpha(z=\cos\alpha)$, Eq. (6) describes an ellipse in the

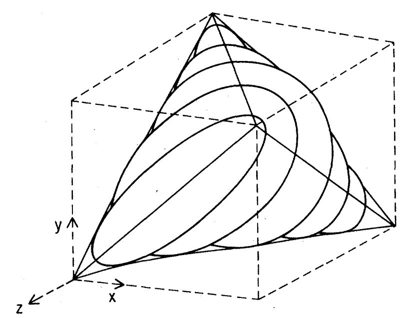

FIG. 2. The allowed region for the variables $x=\hat{k}_1\cdot\hat{z}$, $y=\hat{k}_2\cdot\hat{z}$, and $z=\hat{k}_1\cdot\hat{k}_2$. If the origin is at the center of the cube, the surface is given by Eq. (6). The slices shown have fixed values of z, the cosine of the angle between the photons. The variables x and y are linearly related to the observed photon energies. See Eq. (3).

x-y plane whose axes are at 45° to the x and y axes,

$$\frac{(x-y)^2}{2a^2}+\frac{(x+y)^2}{2b^2}=1, \tag{7}$$

where

$$a^2=2\sin^2\frac{\alpha}{2},\quad b^2=2\cos^2\frac{\alpha}{2}. \tag{8}$$

For $\alpha=0$, the ellipse degenerates to the line $x=y$ ($E_1=-E_2$), for $\alpha=90°$ we get a circle, and for $\alpha=180°$ (back-to-back photons) we get the line $x=-y$ ($E_1=E_2$). The four lines of the original square [Eq. (4)], $-1\leq x, y\leq 1$, are tangent to all the ellipses. The area of each ellipse ($S=\pi ab$) in the x-y plane is

$$S=\pi\sin\alpha. \tag{9}$$

We assume that the distribution of the background events in $\cos\alpha$ is uniform, and that its E_1-E_2 distribution is not correlated to α. If we further assume that the distribution of the signal events in $\cos\alpha$ is uniform (in the next section we shall express the actual distribution in $\cos\alpha$ in terms of the production amplitudes), we get a gain factor G for the signal/background ratio (the gain is relative to the analysis in which we do not take α into account) which is equal to the square area/ellipse area,

$$G=4/\pi\sin\alpha. \tag{10}$$

The most efficient way of extracting the information associated with the correlation between E_1, E_2, and α depends on the details of the experiment. For example, if the energy resolution σ is larger than the size of the square (which depends on the *a priori* unknown ΔE), our analysis is not useful. If, on the other hand, $2E_0\beta/\sigma \gg 1$ and the statistics are large enough, we can look at the data in bins of $\cos\alpha$.

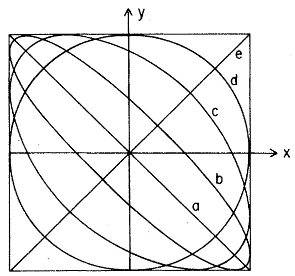

FIG. 3. Some ellipses determined by fixing $z=\hat{k}_1\cdot\hat{k}_2$. For a given value of z, the value of x and y must fall inside the ellipse. Using Eq. (3), knowing the value of z requires the values of the photon energies, E_1 and E_2, to fall inside a similar ellipse. The values of z for the ellipses shown are: (a) $z=-1$, (b) $z=-0.9$, (c) $z=-0.5$, (d) $z=0$, and (e) $z=1$.

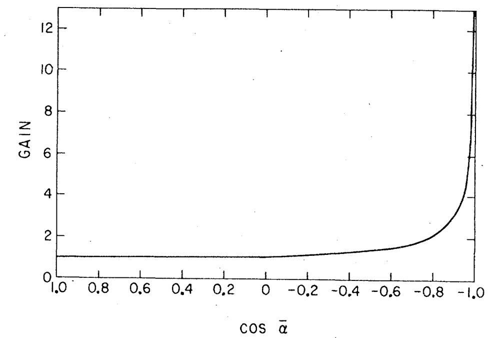

FIG. 4. The gain factor obtained by considering only photon pairs with $\cos\bar\alpha \leq \cos\alpha \leq -1$. The gain is achieved because the photon energies must lie inside a narrow ellipse. See Fig. 3.

As shown in Fig. 3, when the photons are parallel or antiparallel, the ellipses degenerate to straight lines. Thus, good discrimination against background can be achieved by looking at pairs of photons with α near 0 or π. Since experimentally it is impossible to separate the showers of two photons that go in the same direction, we can only use the back-to-back bin to maximize G. In Fig. 4 we show the gain factor for the bin $-1 \leq \cos\alpha \leq \cos\bar\alpha$, as a function of $\cos\bar\alpha$. Note that assuming a uniform distribution in $\cos\alpha$, each bin contains a fraction $\Delta\cos\alpha/2$ of the total number of the events (both for background and signal), where $\Delta\cos\alpha$ is the bin size in $\cos\alpha$. For the bin $-1\leq\cos\alpha\leq-0.9$ ($154° \leq \alpha \leq 180°$), we use 5% of the events and get a gain factor of 3 [to get $G=4$ (2), we need to take 2.5% (10%) of the events].

III. PRODUCTION AMPLITUDES

We shall define the production amplitudes using a nonrelativistic analysis and express the various angular correlations in terms of these amplitudes.

The γ^* has $J^{PC}=1^{--}$. The $F^*\bar{F}^*$ pair has $\vec{J}=\vec{L}+\vec{S}$, $P=(-1)^L$, and $C=(-1)^{L+S}$. Thus L is odd and S is even. For $S=0$, $L=1$. For $S=2$, $L=1$ or 3. Near threshold, the $L=3$ wave is expected to be small, and we neglect it.

The amplitude is linear in the polarization vectors $\vec\epsilon$, $\vec{\bar\epsilon}$, and $\vec\eta$ of the F^*, $\bar{F}^*$, and γ^*. (We use the complex conjugate of the polarization vectors for outgoing particles.) The $L=1$ amplitude contains a single power of $\vec{p}$, the F^* momentum. We write

$$M=A\vec{\bar\epsilon}^*\cdot\vec\epsilon^*\vec\eta\cdot\vec{p}+B(\tfrac{1}{2}\vec{\bar\epsilon}^*\cdot\vec\eta\vec\epsilon^*\cdot\vec{p}+\tfrac{1}{2}\vec{\bar\epsilon}^*\cdot\vec{p}\,\vec\epsilon^*\cdot\vec\eta-\tfrac{1}{3}\vec{\bar\epsilon}^*\cdot\vec\epsilon^*\vec\eta\cdot\vec{p})\,, \tag{11}$$

where A is the $S=0$ amplitude and B is the $S=2$ amplitude.

The amplitude for the $F^*\to F\gamma_1$ decay is given by $\vec\epsilon\cdot\vec{B}$ where $\vec{B}$ represents the magnetic field of the emitted photon, $\vec{B}=\vec{k}_1\times\vec\xi^*$, and $\vec{k}_1$ and $\vec\xi$ are its momentum and polarization ($\vec{\bar{B}}$, $\vec{k}_2$, $\vec{\bar\xi}$ refer to $\bar{F}^*\to\bar{F}\gamma_2$).

The full amplitude is therefore

$$T=\sum_{\epsilon,\bar\epsilon} M\vec\epsilon\cdot\vec{B}\vec{\bar\epsilon}\cdot\vec{\bar{B}}\,. \tag{12}$$

Using $\sum_\epsilon \epsilon_i^*\epsilon_j=\delta_{ij}$, we get T by substituting $\vec\epsilon^*\to\vec{B}$ and $\vec{\bar\epsilon}^*\to\vec{\bar{B}}$ in Eq. (11).

Assuming unpolarized e^+ and e^- beams, and using the $e^+e^-\gamma^*$ coupling (for massless $e^\pm$), we find that the γ^* is an equal mixture of the two transverse polarizations relative to the beam direction $\hat{n}$. We average over the γ^* polarizations in the expression for the cross section $|T|^2$, using

$$\sum_{\text{transverse polarizations}} \eta_i^*\eta_j=\delta_{ij}-\hat{n}_i\hat{n}_j\,. \tag{13}$$

If we ignore the beam direction (average over all possible directions $\hat{n}$) the right-hand side of Eq. (13) becomes $\frac{2}{3}\delta_{ij}$.

When we sum over the two polarizations of γ_1 we get

$$\sum_{\text{polarizations}} \hat{B}_i\hat{B}_j^*=\delta_{ij}-\hat{k}_i\hat{k}_j \tag{14}$$

($\hat{k}$ actually stands for $\hat{k}_1$). A similar expression holds for γ_2.

The coefficients of the various kinematic terms in $|T|^2$ are expressed in terms of the A and B amplitudes [Eq. (11)] in Table I. Note that although the F^* direction $\hat{p}$ is not measurable, $\hat{k}_1\cdot\hat{p}$ and

TABLE I. Coefficients of the various terms in the angular distribution of the photons in $e^+e^- \to F^*\bar{F}^* \to F\gamma_1\bar{F}\gamma_2$. The directions of γ_1 and γ_2 are $\hat{k}_1$ and $\hat{k}_2$. The direction of the beam is $\hat{n}$ and the direction of the F^* is $\hat{p}$.

	$\lvert A\rvert^2$	$\lvert B\rvert^2$	(AB^*+BA^*)
1	1	$\frac{4}{9}$	$\frac{2}{3}$
$(\hat{k}_1\cdot\hat{k}_2)^2$	1	$\frac{1}{9}$	$-\frac{1}{3}$
$(\hat{n}\cdot\hat{k}_1)^2$	0	$\frac{1}{4}$	0
$(\hat{n}\cdot\hat{k}_2)^2$	0	$\frac{1}{4}$	0
$(\hat{p}\cdot\hat{n})^2$	-1	$\frac{1}{18}$	$-\frac{2}{3}$
$(\hat{k}_1\cdot\hat{p})^2$	0	$-\frac{1}{12}$	-1
$(\hat{k}_2\cdot\hat{p})^2$	0	$-\frac{1}{12}$	-1
$(\hat{p}\cdot\hat{n})^2(\hat{k}_1\cdot\hat{k}_2)^2$	-1	$-\frac{1}{9}$	$\frac{1}{3}$
$(\hat{k}_2\cdot\hat{p})^2(\hat{k}_1\cdot\hat{n})^2$	0	$-\frac{1}{4}$	0
$(\hat{k}_1\cdot\hat{p})^2(\hat{k}_2\cdot\hat{n})^2$	0	$-\frac{1}{4}$	0
$(\hat{p}\cdot\hat{n})(\hat{p}\cdot\hat{k}_1)(\hat{n}\cdot\hat{k}_1)$	0	$-\frac{1}{6}$	1
$(\hat{p}\cdot\hat{n})(\hat{p}\cdot\hat{k}_2)(\hat{n}\cdot\hat{k}_2)$	0	$-\frac{1}{6}$	1
$(\hat{p}\cdot\hat{k}_1)(\hat{p}\cdot\hat{k}_2)(\hat{k}_1\cdot\hat{k}_2)$	0	$-\frac{1}{6}$	1
$(\hat{p}\cdot\hat{n})(\hat{p}\cdot\hat{k}_2)(\hat{n}\cdot\hat{k}_1)(\hat{k}_2\cdot\hat{k}_1)$	0	$\frac{1}{3}$	$-\frac{1}{2}$
$(\hat{p}\cdot\hat{n})(\hat{p}\cdot\hat{k}_1)(\hat{n}\cdot\hat{k}_2)(\hat{k}_1\cdot\hat{k}_2)$	0	$\frac{1}{3}$	$-\frac{1}{2}$
$(\hat{p}\cdot\hat{k}_1)(\hat{p}\cdot\hat{k}_2)(\hat{k}_1\cdot\hat{n})(\hat{k}_2\cdot\hat{n})$	0	$-\frac{1}{2}$	0

$\hat{k}_2\cdot\hat{p}$ are related to the measurable E_1 and E_2 by Eq. (3). Ignoring the beam direction, we get the terms of Table II.

If we integrate over all the F^* directions, using $\int d\Omega_{\hat{p}}\hat{p}_i\hat{p}_j \to \frac{1}{3}\delta_{ij}$, we get

$$|T|^2 \propto \{|A|^2[1+(\hat{k}_1\cdot\hat{k}_2)^2]+\tfrac{1}{18}|B|^2[13+(\hat{k}_1\cdot\hat{k}_2)^2]\}\,. \tag{15}$$

To get the total rate, we integrate over $\hat{k}_1$ and $\hat{k}_2$ and get

$$|T|^2 \propto (|A|^2+\tfrac{5}{9}|B|^2)\,. \tag{16}$$

We see from Eq. (15) that the $\mathcal{S}=0$ amplitude leads to a rather anisotropic distribution in α (the angle between the two photons), while the $\mathcal{S}=2$ amplitude is nearly isotropic.

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APPENDIX A: KINEMATICS

Let $\alpha_1(\alpha_2)$ be the angle between $\hat{k}_1(\hat{k}_2)$ and $\hat{z}$. For fixed values of α_1 and α_2, the extreme values of $z=\cos\alpha=\hat{k}_1\cdot\hat{k}_2$ are achieved when the three vectors are coplanar. Then

$$\alpha=\alpha_1\pm\alpha_2 \tag{A1}$$

TABLE II. Coefficients of the various terms in the angular distribution of the photons in $e^+e^- \to F^*\bar{F}^* \to F\gamma_1\bar{F}\gamma_2$, ignoring the beam direction. The directions of γ_1, γ_2, and F^* are $\hat{k}_1$, $\hat{k}_2$, and $\hat{p}$.

	$\lvert A\rvert^2$	$\lvert B\rvert^2$	(AB^*+BA^*)
1	2	$\frac{17}{9}$	$\frac{4}{3}$
$(\hat{k}_1\cdot\hat{k}_2)^2$	2	$\frac{2}{9}$	$-\frac{2}{3}$
$(\hat{k}_1\cdot\hat{p})^2$	0	$-\frac{2}{3}$	-2
$(\hat{k}_2\cdot\hat{p})^2$	0	$-\frac{2}{3}$	-2
$(\hat{k}_1\cdot\hat{p})(\hat{k}_2\cdot\hat{p})(\hat{k}_1\cdot\hat{k}_2)$	0	$-\frac{1}{3}$	2

and

$$z=xy\pm(1-x^2)^{1/2}(1-y^2)^{1/2}\,, \tag{A2}$$

which leads to Eq. (6).

If we assume a uniform and independent distribution of the three vectors, the density of events in the volume of Eq. (6) is

$$\rho \propto 1/(1-x^2-y^2-z^2+2xyz)^{1/2}\,, \tag{A3}$$

which has an integrable singularity at the boundary of our volume. This means that for a fixed α the density is higher at the boundary of the ellipse—the outer $1/n^2$ fraction of its area contains $1/n$ of the events. However, this effect may be washed out when we integrate over $\cos\alpha$ in a bin. For example, for $-1 \le \cos\alpha \le 1$, the density in the square is uniform, since α_1 and α_2 are kinematically independent when we do not constrain α.

APPENDIX B: MODEL FOR PRODUCTION AMPLITUDES

A simple model for heavy-meson pair creation in e^+e^- annihilation has been given by De Rújula, Georgi, and Glashow.[1] In this model the virtual photon couples to the heavy-quark pair, $Q\bar{Q}$. Subsequently, a light-quark pair $q\bar{q}$ arises from the vacuum. The basic assumption is that there is no correlation between the light-quark and the heavy-quark spins. This is justified by arguing that the interactions of heavy-quark spins will be inversely proportional to the quark masses, so the heavy-quark spins will be effectively decoupled from the light quarks.

Let us denote the spins of Q, $\bar{Q}$, q, and $\bar{q}$ by $\vec{S}_1$, $\vec{S}_2$, $\vec{s}_2$, and $\vec{s}_1$. Further, let us define

$$\begin{aligned}
&\vec{S}=\vec{S}_1+\vec{S}_2\,,\\
&\vec{s}=\vec{s}_1+\vec{s}_2\,,\\
&\vec{\Sigma}_1=\vec{S}_1+\vec{s}_1,\\
&\vec{\Sigma}_2=\vec{S}_2+\vec{s}_2\,,\\
&\vec{\mathcal{S}}=\vec{S}+\vec{s}=\vec{\Sigma}_1+\vec{\Sigma}_2.
\end{aligned} \tag{B1}$$

For the $Q\overline{Q}$ pair, $C=(-1)^{L+S}$ and $P=(-1)^{L+1}$, so L is even and S is odd, i.e., $S=1$. Since $\vec{S}$ is decoupled from the light quarks, it is conserved in the process in which $q\overline{q}$ is created.

The $q\overline{q}$ pair may be thought of as being initially in a state with orbital angular momentum $\vec{l}$ and spin angular momentum $\vec{s}=\vec{s}_1+\vec{s}_2$. The quantities may be modified by interaction with the $Q\overline{Q}$. After interacting, the $Q\overline{Q}$ has quantum numbers L' and $S'=S$, while the $q\overline{q}$ has quantum numbers l' and s'. Now for the meson-meson system, $P=-1=(-1)^{L'+l'}$ and $C=-1=(-1)^{L'+l'+S+s'}$. Thus $S+s'$ is even and since $S=1$, $s'=1$ also.

When the Q and $\overline{q}$ fuse, the result may be a pseudoscalar P or a vector V. Altogether, there are five possible final states: $P\overline{P}$, $P\overline{V}$, $\overline{P}V$, $(V\overline{V})_{s=0}$, and $(V\overline{V})_{s=2}$. Let the relative probabilities of these states be a, b, c, d, and e, so that $a+b+c+d+e=1$. By charge conjugation invariance, $b=c$. We can determine these probabilities from the assumption that the light and heavy-quark spins are uncorrelated, that is, for any components, i and j $(i, j=x, y, z)$:

$$\langle S_1^i s_1^j\rangle=\langle S_1^i s_2^j\rangle=\langle S_2^i s_1^j\rangle=\langle S_2^i s_2^j\rangle=0. \quad \text{(B2)}$$

Since $s=1$, $s^2=2=\langle s_1{}^2+2\vec{s}_1\cdot\vec{s}_2+s_2{}^2\rangle$, so $\langle\vec{s}_1\cdot\vec{s}_2\rangle=\frac{1}{4}$, and

$$\langle s_1^i s_2^j\rangle=\tfrac{1}{12}\delta_{ij}. \quad \text{(B3)}$$

Similarly,

$$\langle S_1^i S_2^j\rangle=\tfrac{1}{12}\delta_{ij}, \quad \text{(B4)}$$

if we ignore the correlation with the beam direction. Using these relations, we find

$$\langle\vec{\Sigma}_1{}^2\rangle=\tfrac{3}{2}, \quad \text{(B5)}$$

$$\langle\vec{\Sigma}_1{}^2\vec{\Sigma}_2{}^2\rangle=\langle(\tfrac{3}{2}+2\vec{S}_1\cdot\vec{s}_1)(\tfrac{3}{2}+2\vec{S}_2\cdot\vec{s}_2)\rangle=\tfrac{7}{3}, \quad \text{(B6)}$$

$$\langle\vec{\Sigma}_1\cdot\vec{\Sigma}_2\rangle=\tfrac{1}{2}. \quad \text{(B7)}$$

On the other hand, it is easy to see that we can express these in terms of the relative probabilities as

$$\langle\Sigma_1{}^2\rangle=2c+2d+2e, \quad \text{(B8)}$$

$$\langle\Sigma_1{}^2\Sigma_2{}^2\rangle=4d+4e, \quad \text{(B9)}$$

$$\langle\vec{\Sigma}_1\cdot\vec{\Sigma}_2\rangle=-2d+e. \quad \text{(B10)}$$

Using Eqs. (B5)–(B9), we find

$$a=\tfrac{1}{12},\quad b+c=\tfrac{4}{12},\quad d+e=\tfrac{7}{12}. \quad \text{(B11)}$$

This is the well-known prediction of De Rújula, Georgi, and Glashow. If we use Eq. (B10) as well, we find

$$d=\tfrac{1}{36},\quad e=\tfrac{20}{36}. \quad \text{(B12)}$$

Thus, the simple model predicts a dominance of the $S=2$ final state. Unfortunately, this gives a rather isotropic distribution of photons.

The model described here is not especially successful in explaining the data on D and D^* production.[2] It does show, however, how dynamical models can be related to the production amplitudes for $F^*\overline{F}^*$, and thus, how, in principle, these models can be tested with photon-photon correlations.

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[1]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. 37, 398 (1976).

[2]See, for example, A. Barbaro-Galtieri, LBL Report No. LBL-8537, presented at the XVI International School of Subnuclear Physics, Erice, Italy, 1978 (unpublished).