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Can radiative corrections to parity-violating neutral currents distinguish between the Weinberg-Salam and left-right-symmetric models?

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We compute the radiative corrections to parity-violating neutral currents and give predictions for parity-violating effects in the Weinberg-Salam model and in the left-right-symmetric models based on the gauge groups $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ and $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$. We show that radiative corrections can possibly distinguish between the Weinberg-Salam model and the $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ model on the one side and the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model on the other side.

Recently, a SLAC experiment[1] on deep-inelastic scattering of polarized electrons on deuteron and proton targets has established the existence of parity-violating neutral currents. They measured the asymmetries

$$A^-(d)/|q^2| = (9.5 \pm 1.6) \times 10^{-5},$$

$$A^-(p)/|q^2| = (9.7 \pm 2.7) \times 10^{-5},$$

where

$$A^- = \frac{\sigma(e_L^-) - \sigma(e_R^-)}{\sigma(e_L^-) + \sigma(e_R^-)}$$

is the asymmetry between the cross sections of left-handed and right-handed polarized electrons. In the tree approximation and using the quark-parton model, these asymmetries imply in the Weinberg-Salam[2] (WS) model

$$\sin^2 \vartheta_W = 0.21 \pm 0.03 \text{ (deuteron)}$$

and

$$\sin \vartheta_H = 0.19 \pm 0.06 \text{ (proton)}.$$

In this approximation, however, the left-right-symmetric gauge models based on $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ (Ref. 3) or $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ (Refs. 4, 5) give essentially the same predictions,[6] provided that spontaneous left-right symmetry breaking in the charged and in the neutral sector has the same origin and that the parity-violating parameter $Q_\lambda = m_{W_R^\pm}{}^2/m_{W_L^\pm}{}^2$ is sufficiently large. [There is a common factor $(1 - 1/Q_\lambda)$ for all parity-violating effects in the left-right-symmetric gauge models).[7]]

However, the WS model and the left-right-symmetric models answer the question of the origin of parity violation in weak interactions in quite different ways: In the WS model, parity violation is due to the gauge group structure, which has been motivated by the assumption that neutrinos are massless left-handed particles. In the left-right-symmetric models, parity violation is a low-energy effect due to the spontaneous symmetry breaking. Thus, experimental distinction between the two classes of models would be very important for our understanding of parity violation in weak interactions.

For the discussion of parity violation the essential difference between the two classes of models is the existence of the superheavy right-handed bosons in the left-right-symmetric gauge models. These superheavy bosons manifest themselves through radiative corrections.[8] The most important parity-violating effect[9] is the different coupling-constant renormalization of g_L and g_R [the effective coupling constants of the neutral bosons of $SU(2)_L$ and $SU(2)_R$] due to different masses of the left-handed and right-handed charged bosons. This effect may be especially large[10] (~20% of the WS predictions), if one of the neutral bosons is relatively light. We find that for large values of $Q_\lambda = m_{W_R}{}^2/m_{W_L}{}^2$ the predictions of the Weinberg-Salam model and the $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ model on parity-violating neutral-current effects are very similar. In the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model, however, radiative corrections enhance the predictions on $A^-(d)$ compared with the WS model and give smaller parity-violating effects in heavy atoms. The magnitude of these corrections depends on the mass of the lightest massive neutral boson. All parity-violating neutral-current effects depend strongly on the Weinberg angle $\sin^2 \vartheta_W$. The renormalized value of $\sin^2 \vartheta_W$ may be determined by neutrino neutral-current experiments or by further experiments on the parity-violating interaction in atoms, especially in hydrogen and deuterium. Of course, radiative corrections have to be included for the study of these processes. Once $\sin^2 \vartheta_W$ is fixed, asymmetry measurements like the SLAC experiment could eventually decide whether parity violation in weak interactions is due to the gauge group structure or it is a low-energy effect due to spontaneous symmetry breaking.

The rest of this paper is organized as follows: In Sec. I we give general formulas for parity-violating fourth-order contributions. In Sec. II we make predictions on parity-violating effects and on neutral-current neutrino interactions for the WS, the $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$, and the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ models.

I. FOURTH-ORDER CONTRIBUTIONS TO PARITY-VIOLATING NEUTRAL CURRENTS[10,11]

We define an effective Lagrangian for the interaction between electrons and quarks in the neutral sector,

$$\begin{aligned} -\mathcal{L}_{\rm eff}^{\rm NC} &= g_{VV}^{(u)}\bar{e}\gamma_\mu e\bar{u}\gamma_\mu u + g_{AA}^{(u)}\bar{e}\gamma_\mu\gamma_5 e\bar{u}\gamma_\mu\gamma_5 u \\ &\quad + g_{VA}^{(u)}\bar{e}\gamma_\mu e\bar{u}\gamma_\mu\gamma_5 u + g_{AV}^{(u)}\bar{e}\gamma_\mu\gamma_5 e\bar{u}\gamma_\mu u \\ &\quad + (u \to d), \end{aligned} \tag{1}$$

where $g_{VV}^{(u)}$, etc. are functions of kinematic variables such as q^2, $s, \ldots$. In the tree approximation, these functions are given in the t channel by

$$\begin{aligned} g_{VV}^{(u)} &= \frac{Q_e Q_u e^2}{q^2} + \sum_{i=1}^{N} \frac{V_e^{(i)} V_u^{(i)}}{q^2 - m_{Z_i}^2}, \\ g_{AA}^{(u)} &= \sum_{i=1}^{N} \frac{A_e^{(i)} A_u^{(i)}}{q^2 - m_{Z_i}^2}, \\ g_{VA}^{(u)} &= \sum_{i=1}^{N} \frac{V_e^{(i)} A_u^{(i)}}{q^2 - m_{Z_i}^2}, \\ g_{AV}^{(u)} &= \sum_{i=1}^{N} \frac{A_e^{(i)} V_u^{(i)}}{q^2 - m_{Z_i}^2}, \end{aligned} \tag{2}$$

where N is the total number of massive neutral gauge bosons of the theory and $V^{(i)}$ and $A^{(i)}$ are the vector and axial-vector couplings of the boson Z_i to the corresponding fermion.

Let us now discuss the different types of parity-violating fourth-order corrections to these functions.

A. Effective vertices of the Z bosons

In Fig. 1 we show some contributions to the effective Z_i vertices. The simplest way to treat these corrections in the left-right-symmetric gauge models is to work with an effective Lagrangian based on effective coupling constants.[12] In the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model, for example, we have

$$\begin{aligned} \mathcal{L}^{\rm NC} &= \tfrac{1}{2} g_L \bar{\psi}_L \gamma_\mu \tau_3 \psi_L W_{3L}^\mu + \tfrac{1}{2} g_R \bar{\psi}_R \gamma_\mu \tau_3 \psi_R W_{3R}^\mu \\ &\quad + \tfrac{1}{2} \tilde{g}_L' \bar{\psi}_L \gamma_\mu Y \psi_L U_L^\mu + \tfrac{1}{2} \tilde{g}_R' \bar{\psi}_R \gamma_\mu Y \psi_R U_R^\mu, \end{aligned}$$

where g_L, g_R, $\tilde{g}_L'$, $\tilde{g}_R'$ are the effective coupling constants. Neglecting fermion masses and exter-

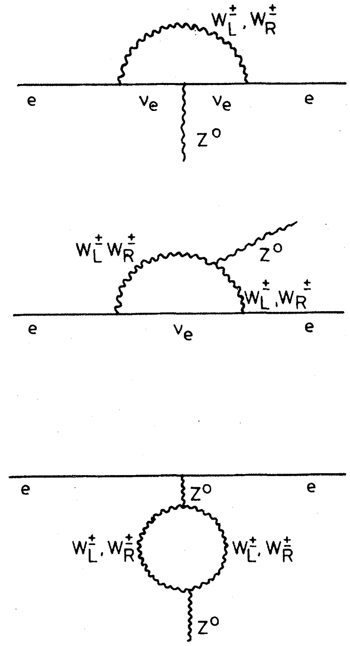

FIG. 1. Examples of graphs which lead to parity-violating effective Z vertices.

nal momenta one finds from renormalization-group calculations[13]

$$g_L^2 - g_R^2 \approx \frac{22}{48\pi^2} g_L^4 \ln \frac{m_{W_R^\pm}^2}{m_{W_L^\pm}^2}$$

and

$$\tilde{g}_L' = \tilde{g}_R' = \tilde{g}' . \tag{3}$$

Here we also have neglected contributions due to Higgs scalars. Note, however, that neutral physical Higgs scalars do not contribute to the coupling-constant renormalization. At low energies and for $Q_\lambda = m_{WR}^2/m_{WL}^2$ sufficiently large, the difference between g_L and g_R gives the leading parity-violating correction in the left-right-symmetric gauge models. In the Weinberg-Salam model, all coupling-constant renormalizations are absorbed in the renormalized value of $\sin^2\vartheta_W$.

B. Parity-violating corrections to the effective photon vertices (charge-radius effect)

In Fig. 2 we show parity-violating contributions to the effective photon vertices. Since the photon is massless, we cannot neglect terms of order $q^2/m_{W(Z)}^2$ in evaluating the contribution of these graphs to the effective vertices. We only keep leading contributions and we find from Fig. 2(a)

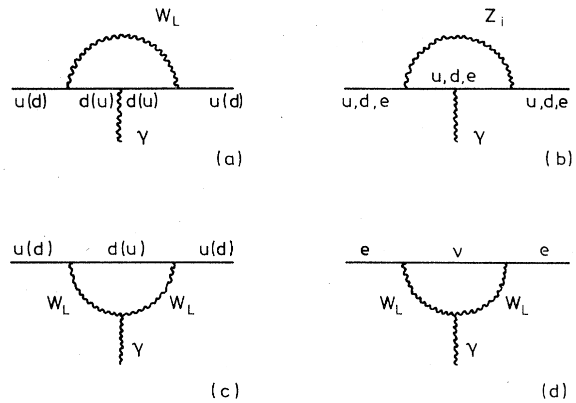

FIG. 2. Some graphs contributing to the effective parity-violating γ vertices (charge-radius effect).

$$\Delta g_{VA}^{(u)} = Q_e Q_d \frac{e^2 g_L{}^2}{96\pi^2} \frac{\ln(m_{W_L}{}^2/m_0{}^2)}{m_{W_L}{}^2},$$

$$\Delta g_{VA}^{(d)} = Q_e Q_u \frac{e^2 g_L{}^2}{96\pi^2} \frac{\ln(m_{W_L}{}^2/m_0{}^2)}{m_{W_L}{}^2}, \tag{4}$$

and from Fig. 2(b) for the boson Z_i

$$\Delta g_{VA}^{(u)} = Q_e Q_u \frac{e^2 V_u^{(i)} A_u^{(i)}}{12\pi^2} \frac{\ln(m_{Z_i}{}^2/m_0{}^2)}{m_{Z_i}{}^2},$$

$$\Delta g_{AV}^{(i)} = Q_e Q_u \frac{e^2 V_e^{(i)} A_e^{(i)}}{12\pi^2} \frac{\ln(m_{Z_i}{}^2/m_0{}^2)}{m_{Z_i}{}^2}, \tag{5}$$

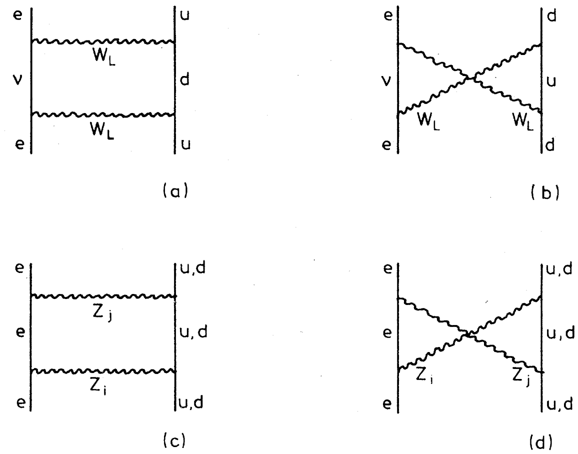

FIG. 3. Box diagram with exchange of two charged or two neutral gauge bosons.

and similarly for d. ($m_0 \approx 1-2$ GeV is the scale at which quarks behave as though they were essentially free.) The graphs of Figs. 2(c), 2(d) give no logarithmic factors and may be neglected here.

C. Box diagrams

Neglecting masses and momenta of the fermions, the contribution of diagrams with exchange of two charged $W_L^\pm$ [Figs. 3(a), 3(b)] becomes

$$\Delta g_{VV}^{(u)} = \Delta g_{AA}^{(u)} = \Delta g_{VA}^{(u)} = \Delta g_{AV}^{(u)} = \frac{g_L{}^4}{64\pi^2 m_{W_L}{}^2}$$
$$\Delta g_{VV}^{(d)} = \Delta g_{AA}^{(d)} = \Delta g_{VA}^{(d)} = \Delta g_{AV}^{(d)} = -\frac{g_L{}^4}{256\pi^2 m_{W_L}{}^2}. \tag{6}$$

The two diagrams in Figs. 3(c) and 3(d) with exchange of two neutral bosons give the following contribution to the g's:

$$\Delta g_{VV}^{(u)} = B_1 B_2 f, \quad \Delta g_{AA}^{(u)} = A_1 A_2 f,$$
$$\Delta g_{VA}^{(u)} = B_1 A_2 f, \quad \Delta g_{AV}^{(u)} = A_1 B_2 f + (u \leftrightarrow d), \tag{7}$$

$$A_1 = V_e^{(i)} V_e^{(g)} + A_e^{(i)} A_e^{(g)}, \quad A_2 = V_u^{(i)} V_u^{(g)} + A_u^{(i)} A_u^{(g)},$$
$$B_1 = V_e^{(i)} A_e^{(g)} + A_e^{(i)} V_e^{(g)}, \quad B_2 = V_u^{(i)} A_u^{(g)} + A_u^{(i)} V_u^{(g)} \tag{8}$$

$$f = \begin{cases} \dfrac{3}{16\pi^2} \dfrac{\ln(m_{Z_i}{}^2/m_{Z_g}{}^2)}{m_{Z_i}{}^2 - m_{Z_g}{}^2} & \text{for the exchange of two different massive bosons } Z_i \text{ and } Z_g, \\ \dfrac{3}{16\pi^2} \dfrac{1}{m_{Z_i}{}^2} & \text{for the exchange of two identical bosons } Z_i, \\ \dfrac{3}{16\pi^2} \dfrac{\ln(m_{Z_i}{}^2/m_0{}^2)}{m_{Z_i}{}^2} & \text{for the exchange of one massive boson } Z_i \text{ and one photon.} \end{cases} \tag{9}$$

II. PREDICTIONS ON PARITY-VIOLATING EFFECTS FOR DIFFERENT MODELS

We will discuss three different gauge models: the standard WS model and the left-right-symmetric models based on $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ and $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$. Here we consider manifest left-right symmetric models, where the left-right symmetry of the underlying Lagrangian is not broken by the fermion representations but arises as a consequence of spontaneous symmetry breaking. We choose for each model a minimal set of Higgs scalars, assuming that parity violation in the charged and in the neutral sector has the same origin. Then we are left with only one free parameter ($\sin^2\vartheta_W$) in the Weinberg-Salam model. In the $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ model we need two Higgs doublets φ_L and φ_R assigned to the representations $(\frac{1}{2}, 0, 1)$ and $(0, \frac{1}{2}, 1)$ with vacuum expectation values $\langle \phi_L \rangle = \binom{0}{\lambda_L}$ and $\langle \phi_R \rangle = \binom{0}{\lambda_R}$. We therefore have the additional parity-violating parameter $Q_\lambda = |\lambda_R|^2/|\lambda_L|^2 = m_{WR^\pm}{}^2/m_{WL^\pm}{}^2$ in this theory. In the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model we have the Higgs multiplets $\phi_L(\frac{1}{2}, 0, 1, 0)$, $\phi_R(0, \frac{1}{2}, 0, 1)$, and $\chi(0, 0, 1, -1)$ with vacuum expectation values $\langle \phi_L \rangle = \binom{0}{\lambda_L}$, $\langle \phi_R \rangle = \binom{0}{\lambda_R}$, and $\langle \chi \rangle = \chi$. The singlet χ is needed in order to break the discrete symmetry between the vector and the axial-vector part of the Lagrangian, thus avoiding a massless axial-vector gauge boson. In this model we are left with three parameters ($\sin^2\vartheta_W$, $Q_\lambda = |\lambda_R|^2/|\lambda_L|^2 = m_{WR^\pm}{}^2/m_{WL^\pm}{}^2$ and $Q_\chi = (|\chi|^2/|\lambda_L|^2)$ which have to be determined by experiments. Note that with our minimal choice of Higgs multiplets we have three neutral physical Higgs scalars [in the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model] and there are no charged physical Higgs scalars contributing to the coupling-constant renormalization.[14]

From weak decays we have the experimental lower bound $Q_\lambda \geq 10$. In the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model, neutrino physics and parity-violating neutral currents are completely independent of Q_χ in the tree approximation.[4,15] Thus, Q_χ remains a free parameter which essentially determines the mass of the lightest boson Z_1. We show in Fig. 4 the dependence of the masses of the physical bosons Z_1, Z_2, Z_3 on Q_χ. For $Q_\chi \to \infty$ we recover the $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ model.

A recent experiment at SLAC (Ref. 16) on the weak forward-backward-asymmetry $A^{\mu\mu}$ in $e^+e^- \to \mu^+\mu^-$ scattering ($-0.025 \leq A^{\mu\mu} \leq 0.017$) puts a lower bound for the mass of the lightest neutral boson in this model. For $\sin^2\vartheta_W = 0.25$, $Q_\lambda = 10$, we find $Q_\chi \geq 0.6$ which corresponds to $m_{Z1} \geq 52$ GeV. (For comparison, the value $Q_\chi = 0.4$ would correspond to an asymmetry $A^{\mu\mu} = 0.035$ at $\sqrt{s} = 6.8$ GeV.)

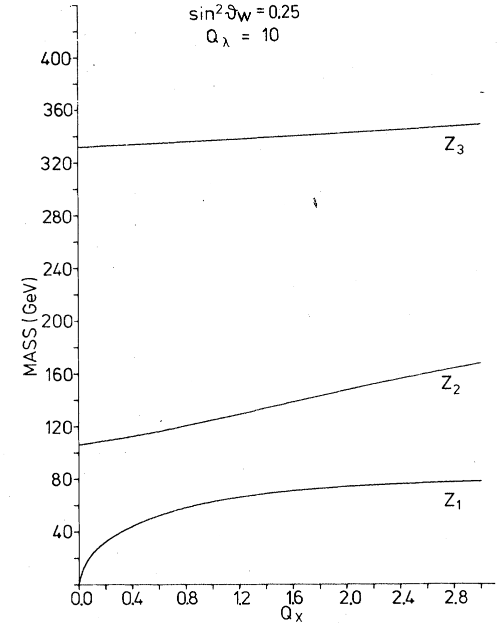

FIG. 4. Masses of the physical bosons Z_1, Z_2, and Z_3 in the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model in dependence on Q_χ ($\sin^2\vartheta_W = 0.25$, $Q_\lambda = 10$).

We now define the following neutral fields:

For the WS *model*

$$Z^\mu_{(\mathrm{WS})} = \cos\vartheta_W W^\mu_{3L} - \sin\vartheta_W U^\mu_{(\mathrm{WS})},$$
$$m_{Z(\mathrm{WS})} = m_{WL}/\cos\vartheta_W . \tag{10}$$

For the $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ *model*

$$\bar{Z}^\mu_V = \cos\vartheta_V(\sin\lambda W^\mu_{3R} + \cos\lambda W^\mu_{3L}) - \sin\vartheta_V U^\mu_{(L+R)},$$
$$\bar{Z}^\mu_A = \cos\lambda W^\mu_{3R} - \sin\lambda W^\mu_{3L} . \tag{11}$$

For the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ *model*

$$Z^\mu_V = \cos\vartheta_V(\sin\lambda W^\mu_{3L} + \cos\lambda W^\mu_{3R}) - \frac{\sin\vartheta_V}{\sqrt{2}}(U^\mu_L + U^\mu_R),$$
$$Z^\mu_{A1} = \cos\lambda W^\mu_{3R} - \sin\lambda W^\mu_{3L}, \tag{12}$$
$$Z^\mu_{A2} = \frac{1}{\sqrt{2}}(U^\mu_L - U^\mu_R),$$

where

$$g_L\cos\lambda\sin\vartheta_V = g_R\sin\lambda\sin\vartheta_V = (\tilde{g}'/\sqrt{2})\cos\vartheta_V = g'\cos\vartheta_V = g'_{(\mathrm{WS})}\cos\vartheta_W = g_L\sin\vartheta_W = e . \tag{13}$$

Here g_L and g_R are the effective coupling constants (including renormalization corrections) of $SU(2)_L$ and $SU(2)_R$, respectively, and $\tilde{g}'$ [$U(1)_L$, $U(1)_R$],

g' $[U(1)_{L+R}]$, $g'_{(WS)}$ $[U(1)_{(WS)}]$ the corresponding effective U(1) coupling constants. The relation between the renormalized mixing angle of the left-right-symmetric models ($\sin^2\vartheta_V$) and the renormalized mixing angle of their Weinberg-Salam-$[SU(2)_L \times U(1)]$ subgroup ($\sin^2\vartheta_W$) is given by $\sin^2\vartheta_W = \sin^2\vartheta_V \cos^2\lambda$. Note that the relation $\sin^2\vartheta_W = \frac{1}{2}\sin^2\vartheta_V$ holds only in lowest order. If we include radiative corrections, $\cos^2\lambda$ will depend on the value of Q_λ and we have

$$\sin^2\vartheta_W = \tfrac{1}{2}\sin^2\vartheta_V - \frac{11}{12\pi}\alpha \ln Q_\lambda .$$

In order to give a correct comparison of the Weinberg-Salam model with the left-right-symmetric models, the predictions will be given with dependence on $\sin^2\vartheta_W$.

With our minimal choice of Higgs multiplets and using the effective coupling constants we find the following mass matrices for the above-defined neutral fields:

$$M^2 = \begin{array}{c c} & \begin{array}{cc} \bar{Z}_A & \bar{Z}_V \end{array} \\ \begin{array}{c} \bar{Z}_A \\ \\ \bar{Z}_V \end{array} & \begin{pmatrix} Q_\lambda \cos^2\lambda \cot^2\lambda + \sin^2\lambda & \dfrac{\cos\lambda \sin\lambda}{\cos\vartheta_V}(\cot^2\lambda Q_\lambda - 1) \\ \dfrac{\cos\lambda \sin\lambda}{\cos\vartheta_V}(\cot^2\lambda Q_\lambda - 1) & \dfrac{\cos^2\lambda}{\cos^2\vartheta_V}(Q_\lambda + 1) \end{pmatrix} \end{array} \times m_{W_L}{}^2 \tag{14}$$

$[SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ model] and

$$M^2 = \begin{array}{c c} & \begin{array}{ccc} Z_{A1} & Z_V & Z_{A2} \end{array} \\ \begin{array}{c} Z_{A1} \\ \\ Z_V \\ \\ Z_{A2} \end{array} & \begin{pmatrix} Q_\lambda \cos^2\lambda \cot^2\lambda + \sin^2\lambda & \dfrac{\cos\lambda \sin\lambda}{\cos\vartheta_V}(\cot^2\lambda Q_\lambda - 1) & \sin\lambda \cos\lambda \tan\vartheta_V(\cot^2\lambda Q_\lambda + 1) \\ \dfrac{\cos\lambda \sin\lambda}{\cos\vartheta_V}(\cot^2\lambda Q_\lambda - 1) & \dfrac{\cos^2\lambda}{\cos^2\vartheta_V}(Q_\lambda + 1) & \dfrac{\cos^2\lambda \sin\vartheta_V}{\cos^2\vartheta_V}(Q_\lambda - 1) \\ \sin\lambda \cos\lambda \tan\vartheta_V(\cot^2\lambda Q_\lambda + 1) & \dfrac{\cos^2\lambda \sin\vartheta_V}{\cos^2\vartheta_V}(Q_\lambda - 1) & \cos^2\lambda \tan^2\vartheta_V(1 + Q_\lambda + 4Q_\chi) \end{pmatrix} \end{array} \times m_{W_L}{}^2 \tag{15}$$

$[SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model].

In Table I, we give the coupling constants of these fields to the fermions. The effective four-fermion Lagrangian (1) is now calculated by diagonalization of the mass matrices (14) and (15).

Let us now discuss predictions for some experiments on parity-violating neutral-current interactions, especially if they permit one to distinguish between the Weinberg-Salam model and the left-right-symmetric models if we include radiative corrections.

A. Deep-inelastic scattering of polarized electrons on nucleons

We first discuss the asymmetry in deep-inelastic scattering of left-handed and right-handed polarized electrons on nucleons, which has been measured at SLAC.[1]

We define the following electron-quark cross sections[17]:

$$\begin{aligned} d\sigma_{LL}^{(i)} &\sim |g_{VV}^{(i)} + g_{AA}^{(i)} + g_{VA}^{(i)} + g_{AV}^{(i)}|^2 : && \text{scattering of a left-handed electron on a left-handed quark,} \\ d\sigma_{RR}^{(i)} &\sim |g_{VV}^{(i)} + g_{AA}^{(i)} - g_{VA}^{(i)} - g_{AV}^{(i)}|^2 : && \text{scattering of a right-handed electron on a right-handed quark,} \\ d\sigma_{LR}^{(i)} &\sim |g_{VV}^{(i)} - g_{AA}^{(i)} - g_{VA}^{(i)} + g_{AV}^{(i)}|^2(1-y)^2 : && \text{scattering of a left-handed electron on a right-handed quark,} \\ d\sigma_{RL}^{(i)} &\sim |g_{VV}^{(i)} - g_{AA}^{(i)} + g_{VA}^{(i)} - g_{AV}^{(i)}|^2(1-y)^2 : && \text{scattering of a right-handed electron on a left-handed quark,} \end{aligned} \tag{16}$$

where $y = (E - E')/E$ is the usual kinematic parameter. Note that for corresponding lepton-antiquark (antilepton-quark) scattering, g_{VV} and g_{AV} (g_{VV} and g_{VA}) change their sign.

The parity-violating asymmetries for deep-inelastic scattering of polarized charged leptons on unpolarized nucleons,

TABLE I. Coupling constants of the neutral gauge fields with the fermions.

	$Z_{(\mathrm{WS})}$	Z_V $(\bar{Z}_V)$	Z_{A1} $(\bar{Z}_A)$	Z_{A2}
$V_e\Big/\frac{g_L}{2\sqrt{2}}$	$-\frac{1}{\sqrt{2}\cos\vartheta_W}(1-4\sin^2\vartheta_W)$	$-\frac{\sqrt{2}\cos\lambda}{\cos\vartheta_V}(1-2\sin^2\vartheta_V)$	$\frac{\sin^2\lambda-\cos^2\lambda}{\sqrt{2}\sin\lambda}$	0
$A_e\Big/\frac{g_L}{2\sqrt{2}}$	$-\frac{1}{\sqrt{2}\cos\vartheta_W}$	0	$\frac{1}{\sqrt{2}\sin\lambda}$	$-\frac{\sqrt{2}\cos\lambda\sin\vartheta_V}{\cos\vartheta_V}$
$V_u\Big/\frac{g_L}{2\sqrt{2}}$	$\frac{1}{\sqrt{2}\cos\vartheta_W}(1-\frac{8}{3}\sin^2\vartheta_W)$	$\frac{\sqrt{2}\cos\lambda}{\cos\vartheta_V}(1-\frac{4}{3}\sin^2\vartheta_V)$	$-\frac{\sin^2\lambda-\cos^2\lambda}{\sqrt{2}\sin\lambda}$	0
$A_u\Big/\frac{g_L}{2\sqrt{2}}$	$\frac{1}{\sqrt{2}\cos\vartheta_W}$	0	$-\frac{1}{\sqrt{2}\sin\lambda}$	$\frac{\sqrt{2}\cos\lambda\sin\vartheta_V}{3\cos\vartheta_V}$
$V_d\Big/\frac{g_L}{2\sqrt{2}}$	$-\frac{1}{\sqrt{2}\cos\vartheta_W}(1-\frac{4}{3}\sin^2\vartheta_W)$	$-\frac{\sqrt{2}\cos\lambda}{\cos\vartheta_V}(1-\frac{2}{3}\sin^2\vartheta_V)$	$\frac{\sin^2\lambda-\cos^2\lambda}{\sqrt{2}\sin\lambda}$	0
$A_d\Big/\frac{g_L}{2\sqrt{2}}$	$-\frac{1}{\sqrt{2}\cos\vartheta_W}$	0	$\frac{1}{\sqrt{2}\sin\lambda}$	$\frac{\sqrt{2}\cos\lambda\sin\vartheta_V}{3\cos\vartheta_V}$

$$A^{(\mp)}_{(N)}(x,y)=\frac{d\sigma(l_L^{(\mp)},N)-d\sigma(l_R^{(\mp)},N)}{d\sigma(l_L^{(\mp)},N)+d\sigma(l_R^{(\mp)},N)},$$

are calculated in the parton model

$$d\sigma(l_L^-,N)=\sum_i f_i(x)(d\sigma_{LL}^{(i)}+d\sigma_{LR}^{(i)}),$$
$$d\sigma(l_R^-,N)=\sum_i f_i(x)(d\sigma_{RR}^{(i)}+d\sigma_{RL}^{(i)}). \tag{17}$$

The sum is over parton types i, including quarks and antiquarks. The $f_i(x)$ are the usual parton distribution functions. Note that, neglecting antiquarks, $A^{(\mp)}(d)$ is completely independent of the $f_i(x)$, since the deuteron is an isospin singlet. (Predictions are given for $x=0.2$.) Thus, predictions for $A^{(\mp)}(d)$ should be quite independent of possible modifications of the parton model, as long as the contribution of the quark-antiquark sea is not too important. In Fig. 5 we give $A^-(d)/|q^2|$ for various choices of $\sin^2\vartheta_W$, Q_λ, and Q_χ ($y=0.21$, $q^2=-1.6$ GeV^2). For very large values of Q_λ and Q_χ the Weinberg-Salam model and the left-right-symmetric models give essentially the same result [Fig. 5(a)]. For $Q_\lambda=100$ ($Q_\chi=100$) the predictions of the two classes of models differ by ~1% ($\cong 1/Q_\lambda$) and the difference of the corresponding corrections due to radiative effects is of order $\sim G_F(\alpha/Q_\lambda)\ln Q_\lambda$ which gives numerically about 0.1–0.2%. In the $SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ model, however, radiative corrections are very important for small values of the parameter Q_χ [see Figs. 5(b), 5(c), and 5(d)], since they depend on the masses of the neutral bosons. For large values of Q_λ these corrections are proportional to $\ln Q_\lambda$ [see Fig. 5(c) and Fig. 5(d)]. Radiative corrections tend to increase the parity-violating asymmetry $A^-(d)$. Note that for $\sin^2\vartheta_W \geq 0.25$ the $SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ model with small values of Q_χ seems to be favored by the SLAC experiment, but a more precise determination of $A^-(d)$ will be necessary.

Similar asymmetries can be defined for the scattering of electrons on protons or neutrons [$A^-(p)$, $A^-(n)$] and for the corresponding positron scattering [$A^+(d), A^+(p), A^+(n)$]. In Fig. 6 we compare the asymmetries $A^-(p)/|q^2|$, $A^-(n)/|q^2|$, $A^-(d)/|q^2|$, $A^+(p)/|q^2|$, $A^+(n)/|q^2|$, and $A^+(d)/|q^2|$, for the WS and the $SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ model (using the parton distribution function of Ref. 18 and the kinematical variables $x=0.2$, $y=0.21$, $q^2=-1.6$ GeV^2).

B. Parity violation in heavy atoms

Other experiments on parity-violating neutral currents have been performed by measuring parity-violating transitions in heavy atoms. A useful quantity for parity-violating effects in heavy atoms is the weak charge[19]

$$Q_W=2[(2g_{AV}^{(u)}+g_{AV}^{(d)})Z+(g_{AV}^{(u)}+2g_{AV}^{(d)})N]/(G_F/\sqrt{2}). \tag{18}$$

Since atomic physics calculations are somewhat ambiguous we give in Fig. 7 the dependence of the ratio Q_W/Q_W (WS) on Q_χ (for bismuth) rather than the absolute value of the parity-violating optical rotation. Here Q_W (WS)$=(1-4\sin^2\vartheta_W)Z-N$ is the weak charge in the Weinberg-Salam model in the tree approximation. Note that radiative cor-

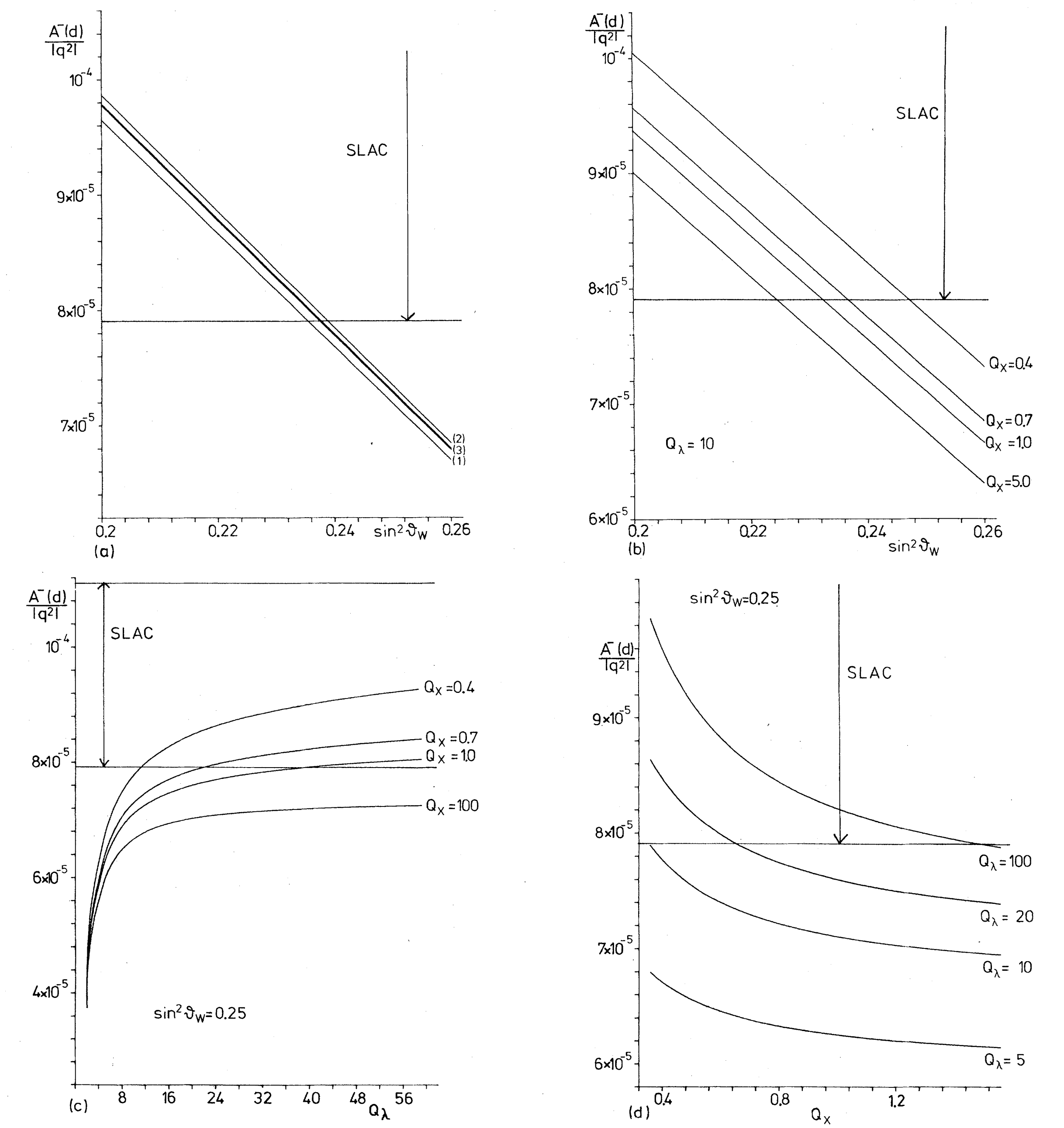

FIG. 5. (a) Dependence of $A^-(d)$ on $\sin^2\vartheta_W$ in the Weinberg-Salam, the $SU(2)_L\times SU(2)_R\times U(1)_{L+R}$, and the $SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ models for large values of Q_λ and Q_χ ($Q_\lambda=100$, $Q_\chi=100$). The curves are given for the WS model in the tree approximation (1) and including radiative corrections (2) and for the left-right-symmetric models including radiative corrections (3). (b) Dependence of $A^-(d)$ on $\sin^2\vartheta_W$ in the $SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ models for different values of Q_χ ($Q_\lambda=10$). (c) Dependence of $A^-(d)$ on Q_λ for different values of Q_χ ($\sin^2\vartheta_W=0.25$) [$SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ model]. (d) Dependence of $A^-(d)$ on Q_χ for different values of Q_λ ($\sin^2\vartheta_W=0.25$) [$SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ model].

rections diminish the parity-violating effects, in contrast with the SLAC experiment. The reason is the negative interference between $g_{AV}^{(u)}$ and $g_{AV}^{(d)}$. Unfortunately, the experimental and theoretical situation is not very clear at the moment.

C. Parity violation in hydrogen and deuterium

Experiments on parity violation in hydrogen and deuterium are very relevant for gauge theory predictions since atomic physics uncertainties are ab-

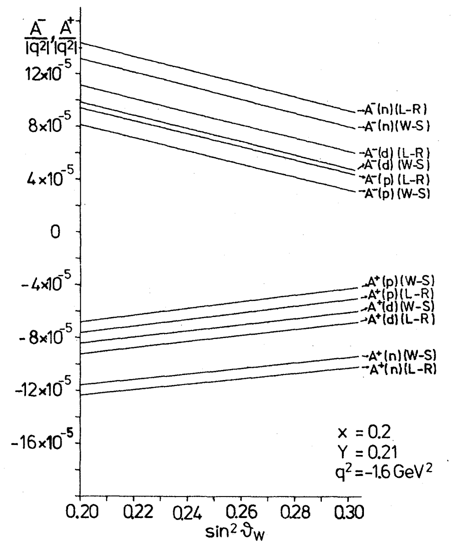

FIG. 6. Comparison of $A^-(d)$, $A^+(d)$, $A^-(p)$, $A^+(p)$, $A^-(n)$, and $A^+(n)$ in the WS model and in the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model ($Q_\lambda = 100$, $Q_\chi = 0.7$).

sent. The relevant parameters[20] are

$$
\begin{aligned}
&C_{1p} = (2 g_{AV}^{(u)} + g_{AV}^{(d)})/(G_F/\sqrt{2}),\\
&C_{1n} = (g_{AV}^{(u)} + 2 g_{AV}^{(d)})/(G_F/\sqrt{2}),\\
&C_{2p} = [2F g_{VA}^{(u)} + (F-D) g_{VA}^{(d)}]/(G_F/\sqrt{2}),\\
&C_{2n} = [(F-D) g_{VA}^{(u)} + 2F g_{VA}^{(d)}]/(G_F/\sqrt{2}),\\
&(F \approx 0.425\,,\ \ D \approx 0.825)\,.
\end{aligned}
\tag{19}
$$

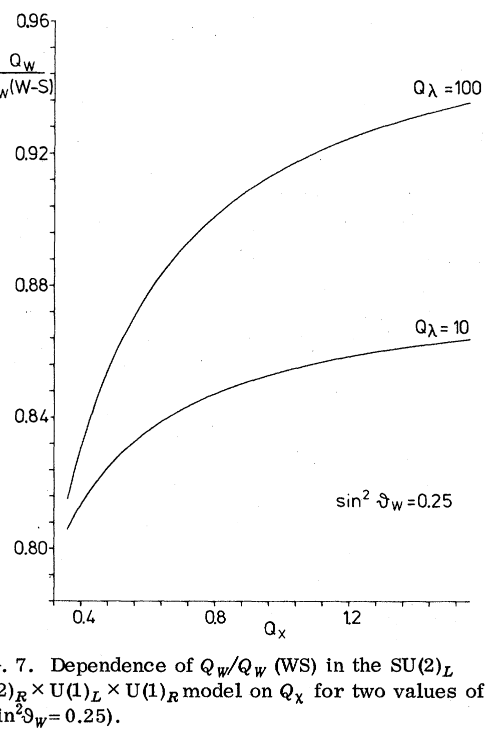

FIG. 7. Dependence of Q_W/Q_W (WS) in the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model on Q_χ for two values of Q_λ ($\sin^2\vartheta_W = 0.25$).

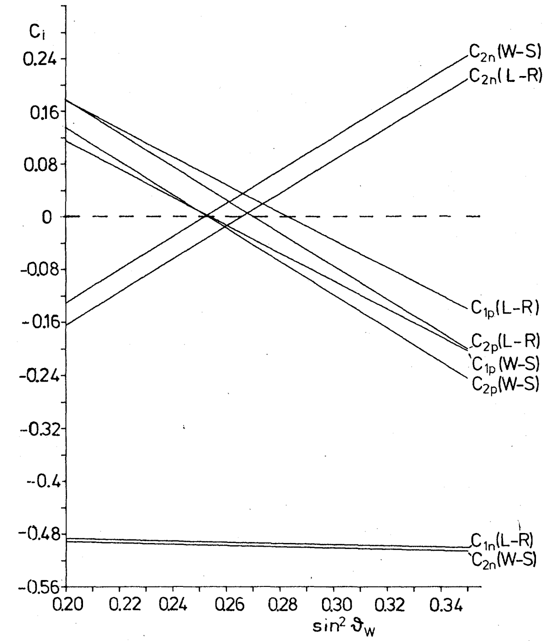

FIG. 8. Dependence of C_{1p}, C_{2p}, C_{1n}, C_{2n} on $\sin^2\vartheta_W$ in the WS model and in the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model ($Q_\lambda = 100$, $Q_\chi = 0.7$).

They may all be determined by different experiments. In Fig. 8 we show the dependence of these parameters on $\sin^2\vartheta_W$ for the WS and the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model. The values of C_{1p}, C_{2p}, C_{2n} are particularly interesting, since they vanish in the WS model for $\sin^2\vartheta_W = 0.25$. Measuring these quantities will help to determine the value of $\sin^2\vartheta_W$.

D. Radiative corrections to the neutral-current neutrino interactions

Since parity-violating neutral-current effects depend strongly on the value of $\sin^2\vartheta_W$, we have also calculated radiative corrections to the neutral-current neutrino interactions. They have to be included for the determination of $\sin^2\vartheta_W$ by pure neutral-current neutrino experiments. We find with our choice of a spontaneous-symmetry-breaking pattern that these corrections are rather small.[21] Thus the Weinberg-Salam model and the left-right-symmetric models give very similar predictions for neutral-current neutrino experiments, even if we include radiative corrections. The calculation of these radiative corrections is similar to that of Sec. I. However, the interactions of the left-handed neutrinos are completely independent of the renormalized value of g_R. (This can be derived directly from a general theorem of Ref. 15.) For a given renormalized value of $\sin^2\vartheta_W$ only charge radius and box diagrams

TABLE II. Effective neutrino coupling constants with electrons and u and d quarks in the tree approximation and including radiative corrections.

	WS model, $SU(2)_L\times SU(2)_R\times U(1)_{L+R}$ model, and $SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ model Tree approximation			WS model and $SU(2)_L\times SU(2)_R\times U(1)_{L+R}$ model ($Q_\lambda=100$) Radiative corrections included			$SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ model ($Q_\lambda=100$, $Q_\chi=0.7$) Radiative corrections included		
$\sin^2\vartheta_W$	0.2	0.25	0.3	0.2	0.25	0.3	0.2	0.25	0.3
$g_V(e)$	−0.1	0	0.1	−0.1129	−0.0113	0.0898	−0.1128	−0.0113	0.0896
$g_A(e)$	−0.5	−0.5	−0.5	−0.5065	−0.5052	−0.5044	−0.5061	−0.5048	−0.5040
$g_V(u)$	0.2333	0.1667	0.1	0.2387	0.1719	0.1051	0.2388	0.1720	0.1052
$g_A(u)$	0.5	0.5	0.5	0.5006	0.5005	0.5004	0.5009	0.5008	0.5007
$g_V(d)$	−0.3667	−0.333	−0.3	−0.3758	−0.3410	−0.3067	−0.3758	−0.3410	−0.3067
$g_A(d)$	−0.5	−0.5	−0.5	−0.5068	−0.5055	−0.5046	−0.5068	−0.5055	−0.5046

similar to Fig. 2 and Fig. 3 contribute to the radiative corrections to the neutral-current neutrino interactions. Using the effective Lagrangian for left-handed neutrinos

$$-\mathcal{L}_{\text{eff}}^{\text{NC}}=\frac{G_F}{\sqrt{2}}\,\bar{\nu}_L\gamma_\mu\nu_L[\bar{e}\gamma_\mu(g_V^{(e)}+g_A^{(e)}\gamma_5)e+(e\to u)+(e\to d)],$$

we give in Table II the values of the effective couplings for the three models in the tree approximation and including radiative corrections.

III. DISCUSSION AND CONCLUSION

What is now the answer to our question: Can radiative corrections to parity-violating neutral currents distinguish between the Weinberg-Salam model and left-right-symmetric models? We have seen [Fig. 5(a)] that the Weinberg-Salam model and the $SU(2)_L\times SU(2)_R\times U(1)_{L+R}$ model give essentially the same predictions on parity-violating neutral currents. This holds also for the $SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ model for very large values of the parameter Q_χ (say $Q_\chi=100$). In this case all predictions on parity-violating neutral currents in the left-right-symmetric models are smaller by a common factor $(1-m_{W_L^\pm}{}^2/m_{W_R^\pm}{}^2)$ compared with the Weinberg-Salam model.

In the $SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ model, however, radiative corrections are especially large for small values of Q_χ and large values of $Q_\lambda=m_{W_R}{}^2/m_{W_L}{}^2$. In this case the main difference (concerning parity-violating neutral-current predictions) between the Weinberg-Salam model and the $SU(2)_L\times SU(2)_R\times U(1)_{L+R}$ model on the one hand and the $SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ model on the other hand is due to radiative corrections. These corrections tend to increase the predictions on the parity-violating asymmetry in electron-deuteron scattering $[A^-(d)]$ and they give smaller parity-violating effects in heavy atoms [compared with the WS model and the $SU(2)_L\times SU(2)_R\times U(1)_{L+R}$ model]. For small values of Q_χ radiative corrections to parity-violating neutral currents can therefore distinguish between the WS model and the $SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ model.

However, an exact determination of $\sin^2\vartheta_W$ is crucial for a definite conclusion, since all predictions on parity-violating neutral-current effects depend strongly on $\sin^2\vartheta_W$. This may be done by improved neutral-current neutrino experiments or by measuring parity-violating effects in hydrogen and deuterium. We see from Fig. 8 that the sign of the parity-violating quantities C_{1p}, C_{1n}, and C_{2p} can decide in the Weinberg-Salam model if $\sin^2\vartheta_W$ is larger or smaller than $\frac{1}{4}$. In the $SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ model ($Q_\chi=0.7$, $Q_\lambda=100$) these quantities have the opposite sign as in the WS model for $0.25\le\sin^2\vartheta_W\le 0.27$.

In conclusion, the comparison of the SLAC experiment on deep-inelastic scattering of polarized electrons on deuteron with neutral-current neutrino experiments or with experiments on parity violation in atoms (especially in hydrogen and deuterium) could help to answer the question of the origin of parity violation in weak interactions.

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