Study of the Reaction $\pi^- p \rightarrow \pi^-\pi^-\pi^-\pi^+\pi^+ p$ at 3.9 GeV/c†

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We have obtained 459 events of the type $\pi^- p \rightarrow \pi^-\pi^-\pi^-\pi^+\pi^+ p$ at 3.9 GeV/c. The cross section is found to be 0.115±0.011 mb. Strong Δ^{++}(1236) production is observed, along with the production of ρ^0 in association with the Δ^{++}(1236). A statistically significant A_1^- signal is present. An anomalous enhancement is observed at 1.4-GeV $\pi^+\pi^+p$ mass which is mainly caused by the strong Δ^{++}(1236) production. No meaningful structure is seen in any of the four- or five-body invariant-mass spectra. The data have been fitted with a model of incoherent resonance production.

I. INTRODUCTION

Comparatively few investigations have been made on reactions in which six or more particles are produced in the final state in πN interactions.[1–6] Many-body final states are of some interest, however, since they allow one to search for decays of resonant states into four or more particles and to study meson and baryon systems in multiply charged configurations. In addition, it is important to study the production properties of the well-known two-body resonances in final states with several particles.

In this paper we present the results on a study of 459 events of the six-body reaction

$$\pi^- p \rightarrow \pi^-\pi^-\pi^-\pi^+\pi^+ p \tag{1}$$

at 3.9 GeV/c. This is a part of an extensive investigation which has been made on $\pi^- p$ reactions with three to six pions plus a nucleon in the final state at this momentum.[7,8] The data have been fitted to a model of incoherent resonance production using a maximum-likelihood technique.

In the following sections we shall briefly describe the experimental method and results, and discuss the fit we have made to the data.

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II. EXPERIMENTAL PROCEDURE

The film studied consisted of 120 000 pictures obtained with the 72-in. hydrogen bubble chamber at the bevatron of the Lawrence Radiation Laboratory in Berkeley. Scanning yielded about 1800 six-prong events, which were measured on scanning and measuring projectors (SMP's) attached on-line to an IBM 7040 computer. The measured events were analyzed using the programs PANAL and PACKAGE. Only those events having a χ^2 less than 24 for the four-constraint hypothesis (1) were accepted. Events fitting more than one hypothesis were resolved by a physicist on the scanning table. Our final sample contains 459 events.

The cross section for reaction (1) was found to be 0.115±0.011 mb. The beam contamination was found to be negligibly small.

III. EXPERIMENTAL RESULTS

The experimental results are shown in Figs. 1–5. The solid curves represent the predictions of pure phase space.

The most prominent feature of the data is the strong Δ^{++} production as seen in Fig. 1(a). We observe the peak at 1.195 GeV.[9] A shoulder on the $M(\pi^+\pi^-)$ plot in Fig. 1(d) at the ρ^0 mass is evident. There is an excess of 113±29 events over the phase-space prediction at the mass interval 0.675–0.825 GeV as given in Table I. We show later that there is considerable ρ^0 production in the data. The $M(\pi^+\pi^-\pi^-)$ plot in Fig. 2(b) is inconsistent with phase space, and shows evidence for A_1^- production as a 3-standard-deviation effect in the region 1.0–1.125 GeV.

The other mass distributions shown in Figs. 1–3 are seen to closely resemble phase space in their shapes, with the exception of the $M(\pi^+\pi^+p)$ distribution, in Fig. 1(f). No significant structure is observed in the four- and five-body spectra of Figs. 2(d)–2(f) and Fig. 3.

A broad enhancement at 1.4 GeV and about 0.15 GeV wide is seen in the $\pi^+\pi^+p$ configuration. The pure

[9] The derivation of this value will be explained later. In the study on the reaction $\pi^- p \rightarrow \pi^-\pi^-\pi^+ p$ at the same momentum, a mass of 1.221±0.005 GeV and width of 0.091±0.008 GeV are obtained for Δ^{++} [P. G. Wohlmut, University of Hawaii (private communication)].

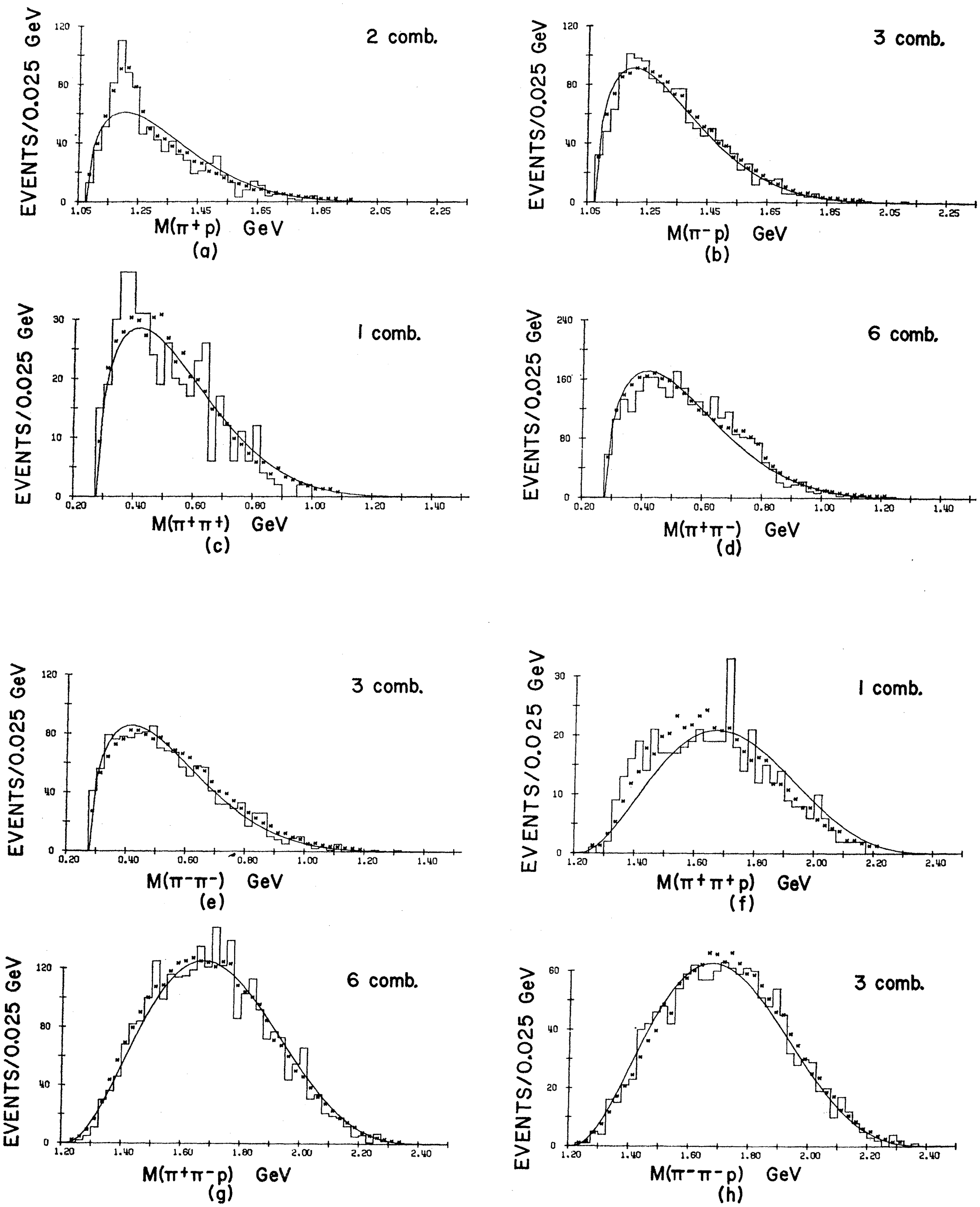

FIG. 1. Invariant-mass distributions. The solid curves are pure phase space. The asterisks are predictions of the resonance-model fit.

phase-space prediction gives a poor fit (χ^2 confidence level less than 0.001).

It is observed that the momentum-transfer distributions in Fig. 4 and angular distributions in Fig. 5 show no striking peripheralism, although there is clearly a small amount indicated in the plot of $-t(p)$ in Fig. 4(a) and the proton angular distribution in Fig. 5(a).

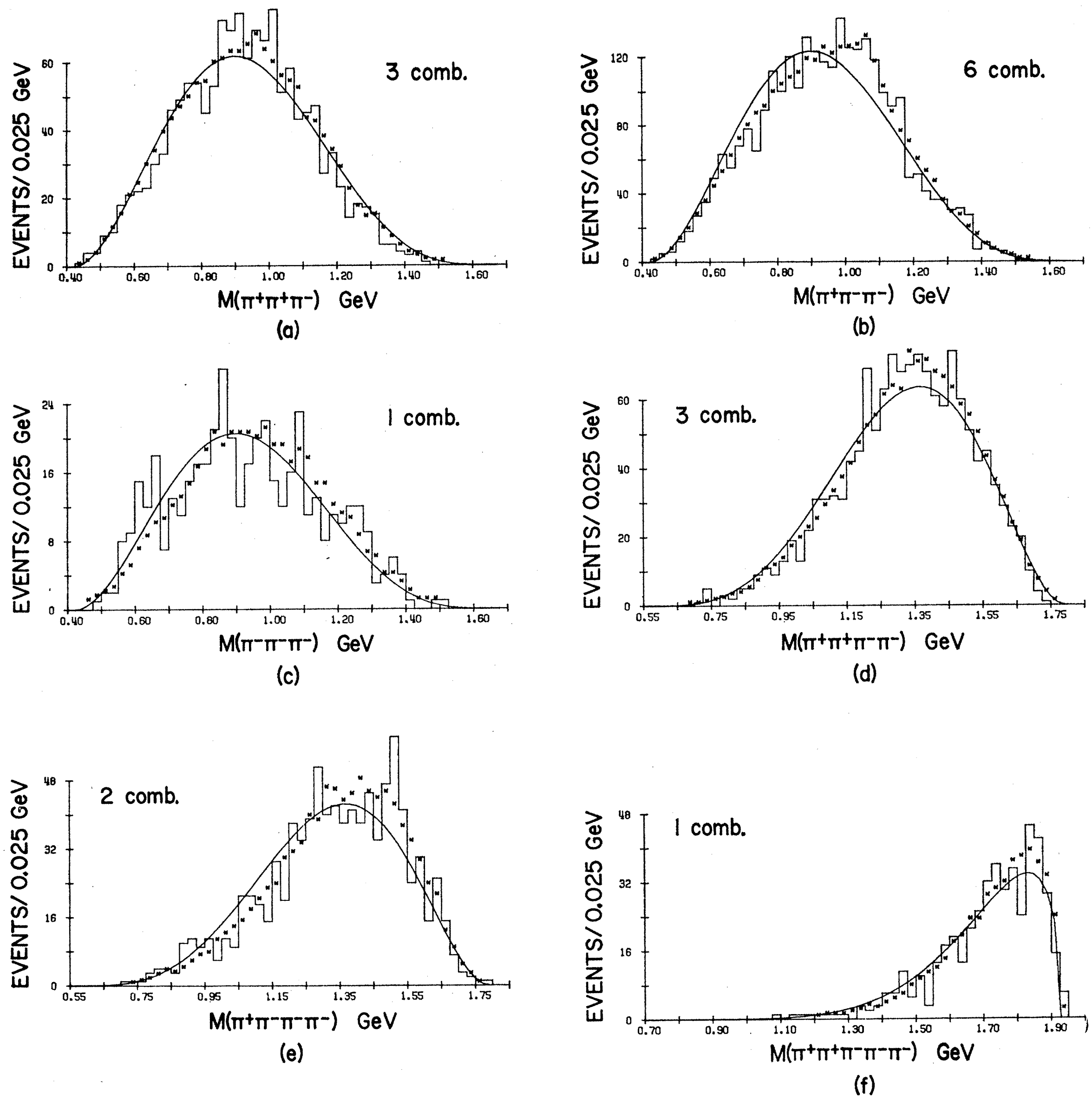

FIG. 2. Invariant-mass distributions. The solid curves are pure phase space. The asterisks are predictions of the resonance-model fit.

Finally, we observe angular correlations between pions. The asymmetry ratios $\gamma = B/F$ are given in Table II, where B is the number of pion pairs with relative angles in the over-all c.m. system greater than 90° and F is the number of pion pairs with the angles less than 90°. It is noted that the ratios for like pions are much different from one for unlike pions. The six-body phase-space predictions, also given in the table, yield equal values for the ratios.

TABLE I. The number of data and the phase-space predictions in the Δ^{++}, ρ^0, and A_1^- regions. The excesses of events in those resonance regions over the phase-space predictions are also given, where the phase spaces are normalized to the total number of events.

Mass	Mass interval (GeV)	Number of events	Phase-space prediction	Excess of events over phase space
$\pi^+ p$	1.125–1.175	454	349	105±28
$\pi^+\pi^-$	0.675–0.825	487	374	113±29
$\pi^+\pi^-\pi^-$	1.0 –1.125	596	497	99±33

IV. MAXIMUM-LIKELIHOOD FIT

In this section we describe the results of an attempt to fit the data to a model of incoherent resonance production. The method used was first introduced by Friedman and Ross[10] and we have utilized their com-

[10] J. Friedman, Alvarez Programming Group Note No. P-156, 1966, Lawrence Radiation Laboratory, University of California, Berkeley (unpublished).

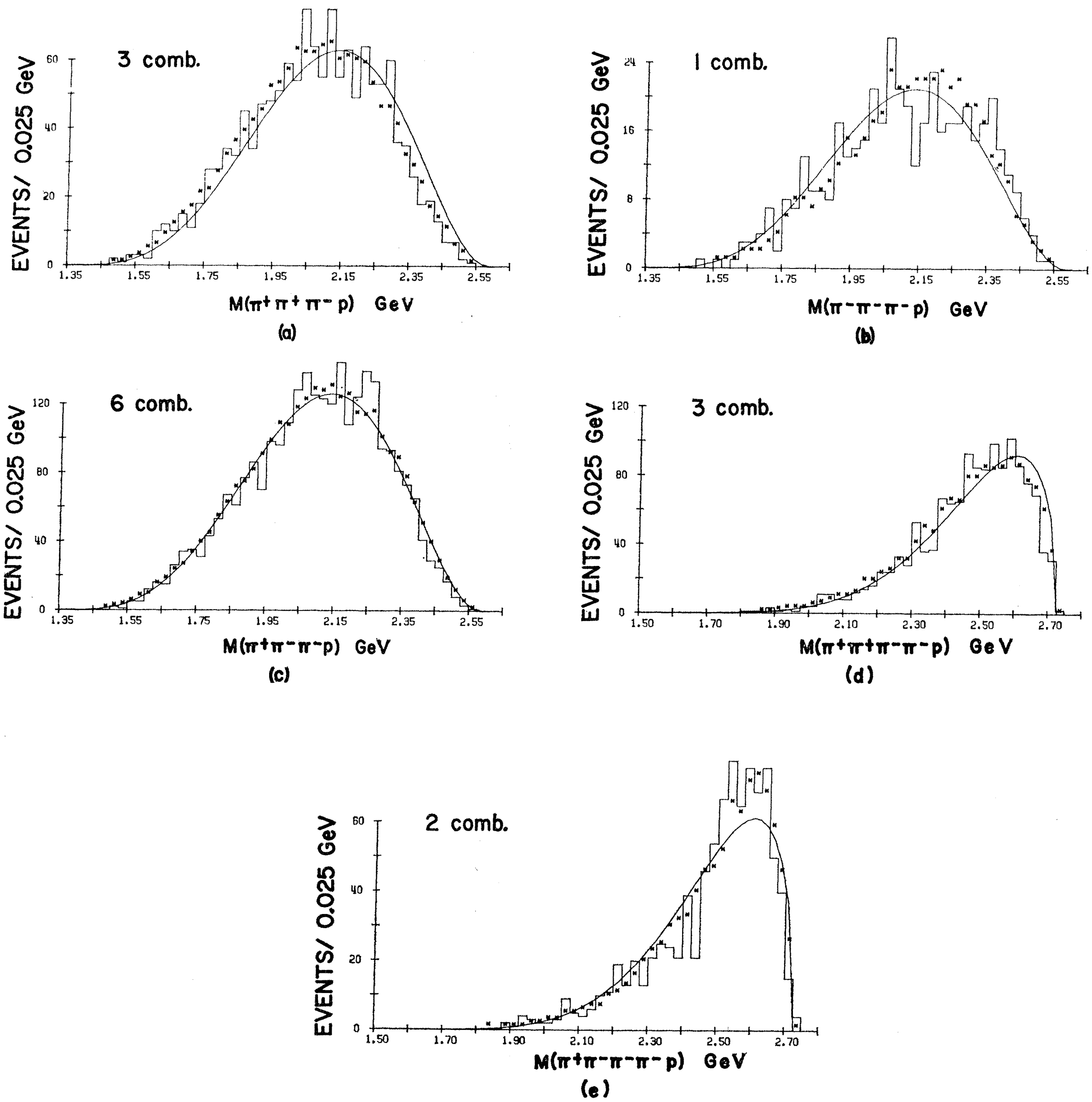

FIG. 3. Invariant-mass distributions. The solid curves are pure phase space. The asterisks are predictions of the resonance-model fit.

puter program MURTLEBERT. This program was modified to handle more than four bodies in the final state and cascade decay processes. An application of the method to four-body final states with peripheral processes has also been made.[11]

We have considered a model in which the following processes are assumed to account for the major portion of the data:

$$\begin{array}{l} \pi^-p \longrightarrow \Delta^{++}\pi^+\pi^-\pi^-\pi^- \\ \qquad\quad \searrow \\ \qquad\qquad \pi^+p \end{array} \qquad , \qquad (2)$$

$$\begin{array}{l} \pi^-p \longrightarrow \rho^0\pi^+\pi^-p \\ \qquad\quad \searrow \\ \qquad\qquad \pi^+\pi^- \end{array} \qquad , \qquad (3)$$

$$\begin{array}{l} \pi^-p \longrightarrow A_1{}^-\pi^+\pi^-p \\ \qquad\quad \searrow \\ \qquad\qquad \rho^0\pi^- \\ \qquad\qquad\quad \searrow \\ \qquad\qquad\qquad \pi^+\pi^- \end{array} \qquad , \qquad (4)$$

$$\begin{array}{l} \qquad\qquad\quad \pi^+p \\ \qquad\qquad \nearrow \\ \pi^-p \longrightarrow \rho^0\Delta^{++}\pi^-\pi^- \\ \qquad\quad \searrow \\ \qquad\qquad \pi^+\pi^- . \end{array} \qquad (5)$$

[11] P. G. Wohlmut, K. Abe, A. D. Johnson, and V. J. Stenger, Nucl. Phys. (to be published).

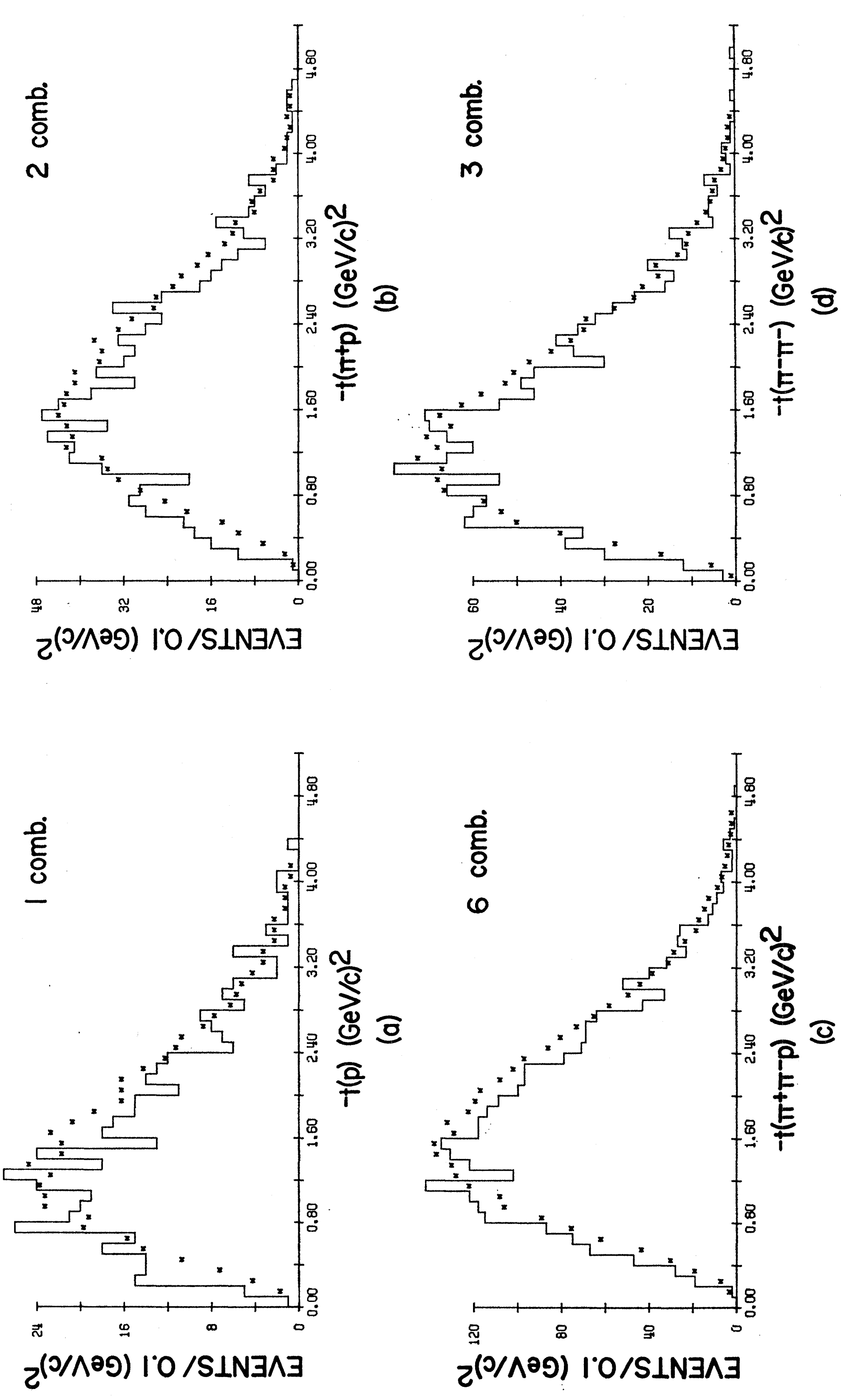

FIG. 4. Invariant-momentum-transfer-squared distributions. The asterisks are predictions of the resonance-model fit—essentially the same as pure phase space.

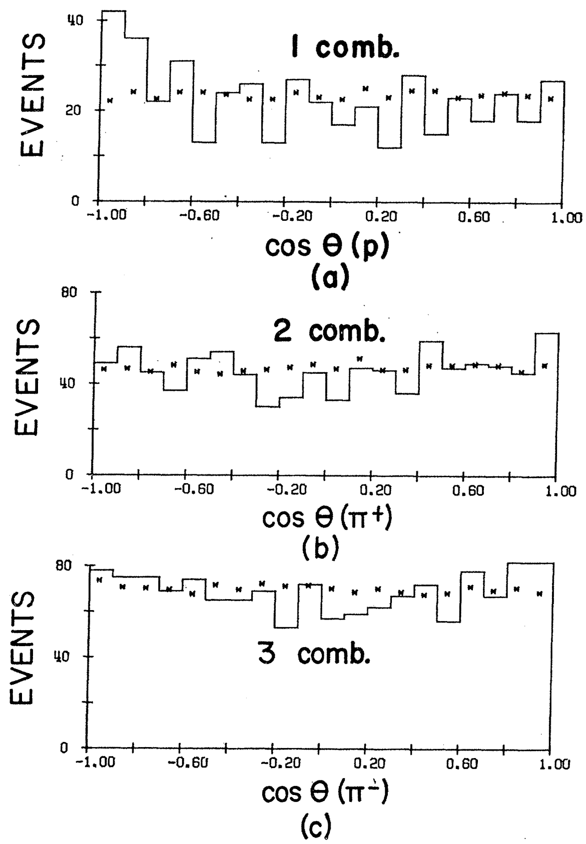

FIG. 5. Angular distributions of outgoing particles in the over-all c.m. system. θ is the angle with respect to the incident π^-. The asterisks are predictions of the resonance-model fit—isotropic.

The masses and widths of the resonant states, given in Table III, are fixed in the fitting procedure and the relative amounts of the four processes above are allowed to vary. The processes are assumed to add incoherently.

The procedure is to compute for each event i and each process j a phase-space density function $\rho_j(\mathbf{x}_i)$, where $\mathbf{x}_i$ is the point in phase space defined by the four-vectors of the outgoing particles in the event. In the model used, $\rho_j(\mathbf{x}_i)$ is a Breit-Wigner function or a product of such functions (resonance model). Constant production amplitudes are thus assumed. From this we form, for each process j, a probability density function

$$P_j(\mathbf{x}_i)=\rho_j(\mathbf{x}_i)\Big/\int\rho_j(\mathbf{x})d\mathbf{x}\,, \tag{6}$$

where the integral is over the allowed phase-space volume. The probability density for the ith event is taken to be

$$P(\mathbf{x}_i)=\sum_j \beta_j P_j(\mathbf{x}_i)\,. \tag{7}$$

The β_j parameters are the relative amounts of each process and are determined by the fit. One process in addition to those given by (2)–(5) is included in which $\rho_j(\mathbf{x}_i)$ is taken to be unity. This is assumed to represent, on the average, the background of the many other processes which are undoubtedly present in small amounts but which we cannot hope to resolve because of the limited statistics. The denominator in (6) in this case is just the six-body uniform phase-space volume. The β_j's are constrained to sum to unity.

A likelihood function for the experiment of N events is then formed as the product of the $P(\mathbf{x}_i)$ for $i=1, \ldots, N$,

$$\mathcal{L}=\prod_{i=1}^{N}\sum_j \beta_j P_j(\mathbf{x}_i)\,. \tag{8}$$

The best values of the β_j's are found as those which maximize $\mathcal{L}$.

The results of our fit are given in Table III. Preceding this result, an attempt was made to fit the data for the processes (2) and (3) and the background which will be described by the uniform density function. The mass of Δ^{++} was allowed to vary in the fitting procedure in addition to the relative amounts of the processes. A mass of 1.195 ± 0.005 GeV was obtained and was used in the four-parameter resonance-model fit mentioned above in order to fit the experimental peak better. We have tried variable-width Breit-Wigner forms in an attempt to account for the downward shift of the Δ^{++} mass in our data but these do not produce enough of a displacement to explain the effect.

TABLE II. The asymmetry ratio γ's for the present data, the pure phase space, and the resonance model, where γ is defined as the ratio of the number of pion pairs with relative angles greater than 90° in the over-all c.m. system to the number of pion pairs with the angles less than 90°.

Combination	Present data	Phase-space prediction	Resonance model
γ^{++}	1.10 ± 0.10	1.41	1.37
γ^{--}	1.12 ± 0.06	1.41	1.38
γ^{+-}	1.77 ± 0.07	1.41	1.53

TABLE III. Results of the four-parameter resonance-model fit. The fractions of each process determined by the fit and the corresponding cross sections are given. The errors are purely statistical. The resonance masses and widths used are shown. These were held constant in the fitting but the values were manually adjusted to give the best fit to the mass plot.

Process	Fraction	Cross section (mb)	Resonance mass (GeV)	Resonance width (GeV)
$\pi^-p\to\Delta^{++}\pi^+\pi^-\pi^-\pi^-$	0.35 ± 0.05	0.040 ± 0.006	1.195	0.100
$\pi^-p\to\rho^0 p\pi^+\pi^-\pi^-$	0.03 ± 0.08	0.003 ± 0.009	0.765	0.120
$\pi^-p\to A_1^-p\pi^+\pi^-$	0.22 ± 0.07	0.025 ± 0.008	1.070	0.080
$\searrow \rho^0\pi^-$			0.765	0.120
$\pi^-p\to\rho^0\Delta^{++}\pi^-\pi^-$	0.20 ± 0.10	0.023 ± 0.012	1.195	0.100
			0.765	0.120

Our results indicate that the bulk of our data can be explained as the production of Δ^{++}, ρ^0, and A_1^- resonances. The indication is that ρ^0 in our data is not produced alone but results either from A_1^- decay or production in association with Δ^{++}. Certainly one would not conclude this from the mass plots by themselves, but it must be remembered that there are many particle combinations on several plots, e.g., six combinations on the $M(\pi^+\pi^-)$ distribution, which wash out the resonance peaks and make the distributions resemble those predicted by pure phase space. The maximum-likelihood analysis fits the density of events in the multidimensional phase space which contains considerably more information about particle correlations than the individual mass plots.

We have made other attempts to fit other resonances but can find no statistically significant evidence for Δ^0, $N^*(1688)$, or other states.

Monte Carlo events have been generated and weighted according to the resonance model. The distributions of these events are indicated by the asterisks in Figs. 1–5. Some fluctuation of these points results from limited Monte Carlo event statistics.

The "fake" data are of major value since they enable one to study carefully the kinematic reflections of the model on the various distributions. We see that in several cases, notably $M(\pi^+\pi^-\pi^-)$ in Fig. 2(b), $M(\pi^+\pi^+\pi^-\pi^-)$ in Fig. 2(d), $M(\pi^+\pi^+\pi^-p)$ in Fig. 3(a), and $M(\pi^+\pi^+\pi^-\pi^-p)$ in Fig. 3(d), the data are not quite consistent with uniform phase space and the Monte Carlo events reproduce the differences. The χ^2 and the confidence levels for the several invariant-mass distributions are given in Table IV on both the phase space and the resonance model.

For the $\pi^+\pi^+p$ system there is a clear downward shifting of the distribution as seen in Fig. 1(f), presumably resulting from the strong Δ^{++} production. The X^2 of the fit which is given in Table IV is considerably improved as compared to phase space. Although the shape is not fitted perfectly, the effect is clearly in the right direction.

For the nonresonant systems $\pi^+\pi^+$ and $\pi^-\pi^-$, the resonance model yields the distributions which resemble those given by the pure six-body phase space as seen in Figs. 1(c) and 1(e). However, the fake events do not reproduce perfectly the experimental data as shown in Table IV. For $M(\pi^-\pi^-)$ in Fig. 1(e), the fake events give a poorer fit than the pure phase space due to the discrepancy at the lower end of the mass distributions. For $M(\pi^+\pi^+)$ in Fig. 1(c), the resonance model again deviates from the data in the low-mass region.

It has been indicated in Sec. III that the asymmetry ratios of the opening angles of like pions are considerably different from one of unlike pions. We also give the values of the ratios predicted by the present resonance model in Table II. The value for γ^{+-} is rather improved, but the ones for γ^{++} and γ^{--} are very little different from the phase-space predictions. This fact seems to be related to the excess of the data in the lower ends of the $\pi^+\pi^+$ and $\pi^-\pi^-$ mass distributions and does not contradict the Bose-statistical low-$\pi\pi$-mass enhancement and angular correlations indicated by Goldhaber *et al.* in their study of $\bar{p}p$ annihilations.[12,13] Under Bose-statistical symmetrization, like pions have a tendency to go to the same direction, resulting in a low-mass enhancement of like pions.

TABLE IV. Given are the χ^2 values and confidence levels in the several invariant-mass distributions for both the phase space and the resonance model. In the estimation of the χ^2 values, 0.05-GeV bins are taken.

	Phase space			Resonance model		
Mass	Degrees of freedom	χ^2	Confidence level	Degrees of freedom	χ^2	Confidence level
$\pi^+\pi^+\pi^-$	18	29.0	0.05	14	16.2	0.37
$\pi^+\pi^-\pi^-$	19	50.9	<0.001	15	14.1	0.51
$\pi^+\pi^+\pi^-\pi^-$	17	19.9	0.28	13	9.3	0.75
$\pi^+\pi^+\pi^-p$	19	39.0	~0.005	15	8.5	0.90
$\pi^+\pi^-\pi^-p$	19	25.6	0.14	15	22.3	0.10
$\pi^+\pi^+\pi^-\pi^-p$	15	39.3	<0.001	11	14.5	0.20
$\pi^+\pi^+$	11	24.5	~0.01	7	19.1	~0.01
$\pi^+\pi^-$	16	70.3	<0.001	12	28.3	~0.005
$\pi^-\pi^-$	15	19.5	0.19	11	25.5	~0.01
$\pi^+\pi^+p$	15	43.5	<0.001	11	17.5	0.10
π^+p	14	96.5	<0.001	10	25.1	~0.006

The momentum transfer and angular distributions in Figs. 4 and 5 are fairly well reproduced by the fake events, but there is clear evidence for some peripheralism not accounted for by the resonance model.

ACKNOWLEDGMENTS

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