Comments and Addenda

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Comment on "Threshold Kinematic Constraints and Correlations of Helicity Amplitudes"

JERROLD FRANKLIN
Department of Physics, Temple University, Philadelphia, Pennsylvania 19122

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Previously derived threshold constraints on helicity amplitudes are simplified for high-spin cases, using a suggestion made by King and Kuo.

IN a recent paper,[1] we used partial-wave threshold behavior[2] to derive threshold (and pseudothreshold) constraints on helicity amplitudes. These relations appeared to require derivative (with respect to t) constraints for some cases when the total spin (S) was ≥ 2. On the other hand, the results, for similar cases, of Cohen-Tannoudji *et al.*[3] and of King and Kuo[4] do not involve derivative constraints. The purpose of this comment is to reconcile this apparent difference by noting that the presence of the derivatives in our original derivation was unnecessary. The derivatives appeared to be required because the helicity (λ) dependence could not be factored out of the partial-wave sum (on J) for some cases with $S\geq 2$. This was because the J, λ dependence in Eq. (40) of I appeared in a nonfactorizable polynomial $P_N(S,\lambda;J)$. However, it turns out that only the leading power of λ in $P_N(S,\lambda;J)$ need be considered for the constraint equation. This follows immediately from the simple observation of King and Kuo[4] that lower-order terms in λ would be removed by the lower-order constraints. The remaining J, λ dependence is obviously factorizable. The result is that derivatives are unnecessary in any of the constraint relations, and the complicated operator $P_N(S,\lambda;J_{\text{op}})$ can be replaced by simply λ^N in the general constraint equations (45)–(48) of I. These equations can then be written[1]

$$\lim_{W\to m+\mu} q^{-S+N}\sum_{\lambda}(-1)^{-\lambda/2}[(S+\lambda)!(S-\lambda)!]^{-1/2}\times\lambda^N A_{S'\lambda';S\lambda}(W,t)=\alpha_{S'\lambda',S}{}^{(N)}(t),\quad N\leq S \tag{45'}$$

$$\lim_{W\to m'+\mu'} q'^{-S'+N'}\sum_{\lambda'}(-1)^{\lambda'/2}[(S'+\lambda')!(S'-\lambda')!]^{-1/2}\times\lambda'^{N'} A_{S'\lambda';S\lambda}(W,t)=\alpha'_{S',S\lambda}{}^{(N')}(t),\quad N'\leq S' \tag{47'}$$

for inelastic scattering, and

$$\lim_{W\to m+\mu} q^{-S-S'+N+N'}\times\sum_{\lambda,\lambda'}\frac{(-1)^{(\lambda'-\lambda)/2}\lambda^N\lambda'^{N'}}{[(S+\lambda)!(S-\lambda)!(S'+\lambda')!(S'-\lambda'!)]^{1/2}}\times A_{S'\lambda';S\lambda}(W,t)=\alpha_{S'S}{}^{(N,N')}(t),\quad N\leq S,\ N'\leq S'$$

$$(-1)^{S-S'-N+N'}=\eta\ \text{(relative intrinsic parity)} \tag{48'}$$

for elastic scattering. These agree with the equivalent results of King and Kuo.[4]

[1] J. Franklin, Phys. Rev. **170**, 1606 (1968).
[2] J. Franklin, Phys. Rev. **152**, 1437 (1966).
[3] G. Cohen-Tannoudji, A. Morel, and H. Navelet, Ann. Phys. (N. Y.) **46**, 239 (1968).
[4] M. J. King and P. K. Kuo, Phys. Rev. D **1**, 442 (1970).