Pion-Nucleon Scattering from 50 to 500 GeV/c

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(Received 10 April 1970)

We show the general features to be expected in scattering at energies high enough that the scattering amplitude consists of only the Pomeranchuk Regge pole and the two-Pomeranchuk-cut terms. The predictions of the Reggeized multiple-scattering quark model for pion-nucleon scattering at lab momenta up to 500 GeV/c are given as a specific example.

THE unexpected constancy of meson-nucleon total cross sections from[1] 30 to 65 GeV/c has presented high-energy phenomenology with an unpleasant choice. We either (a) accept the observed constant values as asymptotes and discard the Pomeranchuk theorem, or (b) continue to expect asymptotic equality of particle and antiparticle amplitudes, but admit that asymptotic energies are as yet far away.

That either choice requires a revision of the Regge-pole treatment of these amplitudes is clear, since extensions of the best pre-Serpukhov fits[2] are in disagreement with the new data. Choosing (a) necessitates some terms effectively breaking line reversal. The second alternative maintains the traditional Pomeranchuk pole, but assumes that the remainder of the Regge description is more complicated than previously thought; either a proliferation of poles or a cut is required.

In this paper we take the latter viewpoint and assume that a Regge cut corresponding to double Pomeranchuk exchange is the culprit. The feasibility of such a model has been shown by our previous work[3] using a Reggeized multiple-scattering quark model, which correctly predicted the Serpukhov data, as well as by more recent fits by Barger and Phillips.[4] In both of these calculations, it is found that the cut contribution to the total cross section is negative and of the order of several mb. Since this contribution will shrink proportionally to $(\ln s)^{-1}$, the principal new result of its inclusion is the prediction that total cross sections will ***rise*** by several mb before approaching their asymptotes. The logarithmic rise is so slow, however, that it seems impossible ever to attain even roughly asymptotic energies.

In other words, it appears that we can never get to the asymptotic region. It may nonetheless be possible, however, to reach energies at which scattering amplitudes become simple, although not asymptotic; that is, at which all inelastic effects have disappeared, and only the vacuum pole and its associated cuts are observed. We propose to call this energy region "the Pomeranchuk region."

The general features of scattering in the Pomeranchuk region can be seen by writing a simple form for the scattering amplitude:

$$F(s,t)=-C\left[\left(\frac{s}{is_0}\right)^{\alpha+\beta t}-\frac{\phi}{\ln(s/is_0)}\left(\frac{s}{is_0}\right)^{2\alpha-1+\frac{1}{2}\beta t}\right]. \quad (1)$$

In (1) we have assumed a linear vacuum trajectory, $\alpha_P(t)=\alpha+\beta t$. The phase of the amplitude is correctly given by the dependence on (s/i) only; C, ϕ, and s_0 are real constants. We have assumed that the t dependence of pole and cut residues can be effectively parametrized exponentially, leading to the normalization constant s_0. In order that the slope of the diffraction peak produced by (1) be consistent with observed values (~ 10 GeV^{-2}), s_0 must be of the order of 10^{-2} or 10^{-3} GeV^2. The negative sign of the cut term is chosen in accordance with observation; we feel that (1) reproduces satisfactorily the characteristics of the Pomeranchuk region.

The Pomeranchuk trajectory has not been assumed in (1) to have $\alpha_P(0)=1$ so that vanishing asymptotic amplitudes may also be considered. Taking $s\gg s_0$ so that $\ln(s/is_0)\approx\ln(s/s_0)$, we find total cross sections of the form

$$\sigma_t(s)=\sigma_\infty\left[\left(\frac{s}{s_0}\right)^{\alpha-1}-\frac{\lambda}{\ln(s/s_0)}\left(\frac{s}{s_0}\right)^{\alpha-2}\right], \quad (2)$$

where

$$\lambda=\phi\,\sin\tfrac{1}{2}\pi\alpha/\cos\tfrac{1}{4}\pi(\alpha-1).$$

By substituting $x=(s/s_0)^{\alpha-1}$, it is easily shown that the function defined by (2) has no minima if $\alpha<1$ and a single minimum if $\alpha\geq 1$. An increase in the total cross section therefore guarantees that $\sigma_t(\infty)$ does not vanish.

Let us now assume that $\alpha=1$ and examine the differential cross section. It can be written

$$\frac{d\sigma}{dt}=C\left[\left(\frac{s}{s_0}\right)^{2\beta t}-2\frac{\phi}{\ln(s/s_0)}\cos(\tfrac{1}{4}\pi\beta t)\left(\frac{s}{s_0}\right)^{\frac{3}{2}\beta t}+\frac{\phi^2}{\ln^2(s/s_0)}\left(\frac{s}{s_0}\right)^{\beta t}\right]. \quad (3)$$

Near $t=0$, (3) may be equated to the usual diffraction peak parametrization $d\sigma/dt\propto\exp[b(s)t+c(s)t^2]$ by identifying powers of t. The results are

$$b(s)=\frac{[2\ln(s/s_0)-\phi]}{[\ln(s/s_0)-\phi]}\beta\ln\left(\frac{s}{s_0}\right), \quad (4a)$$

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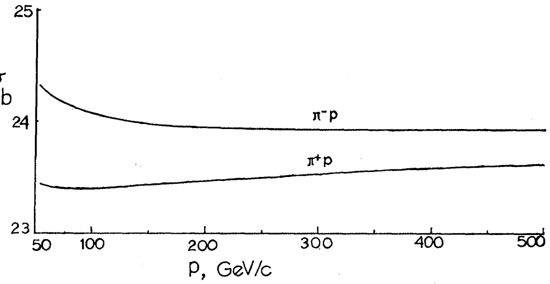

FIG. 1. Predictions for pion-nucleon total cross sections from 50 to 500 GeV/c lab momentum.

$$c(s)=\frac{[\frac{1}{16}\pi^2-\frac{1}{4}\ln^2(s/s_0)]}{[\ln(s/s_0)-\phi]^2}\beta^2\phi\ln\left(\frac{s}{s_0}\right), \tag{4b}$$

which, for sufficiently large s, yield

$$\begin{aligned} b(s)&\approx 2\beta\ln(s/s_0)\,,\\ c(s)&\approx -\tfrac{1}{4}\beta^2\phi\ln(s/s_0)\,. \end{aligned} \tag{5}$$

The value of $b(s)$ thus predicts shrinkage of the diffraction peak, in agreement with the result obtained by neglecting the cut. The curvature $c(s)$, being proportional to ϕ, is a consequence of the cut; its negative sign indicates that the differential cross section drops more rapidly with t than for a purely pole-dominated amplitude.

The variation of (3) with t is also of interest for larger momentum transfers, where interference between pole and cut terms produces a structure in the differential cross section. To investigate this structure, we differentiate (3) with respect to t and assume that the structure occurs for momentum transfers small enough that the variation of $\cos\frac{1}{4}\pi\beta t$ is negligible. Then it follows that

$$\frac{d}{dt}\left(\frac{d\sigma}{dt}\right)=0$$

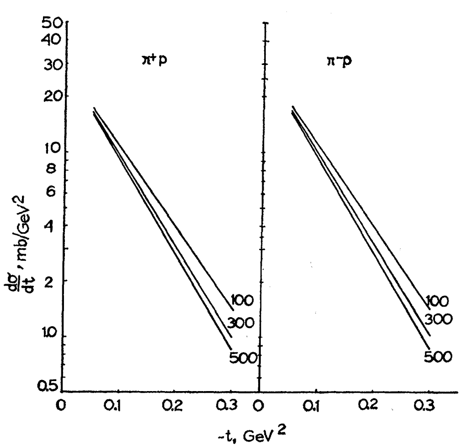

FIG. 2. Differential cross sections for elastic $\pi^{\pm}p$ scattering at 100, 300, and 500 GeV/c lab momentum.

when

$$\left(\frac{s}{s_0}\right)^{\frac{1}{2}\beta t}\approx\frac{1}{4}\frac{\phi}{\ln(s/s_0)}\{3\cos(\tfrac{1}{4}\pi\beta t)\pm[1-9\sin^2(\tfrac{1}{4}\pi\beta t)]^{1/2}\} \tag{6a}$$

$$\approx\frac{1}{4}\frac{\phi}{\ln(s/s_0)}(3\pm1)\,. \tag{6b}$$

That is, we expect that a minimum will occur when

$$\left(\frac{s}{s_0}\right)^{\frac{1}{2}\beta t_1}\approx\frac{\phi}{\ln(s/s_0)},$$

and a maximum when

$$\left(\frac{s}{s_0}\right)^{\frac{1}{2}\beta t_2}\approx\frac{1}{2}\frac{\phi}{\ln(s/s_0)},$$

provided that both of these t values are such that $\cos\frac{1}{4}\pi\beta t_{1,2}\approx 1$. Including the variations of $\cos\frac{1}{4}\pi\beta t$ should not alter the locations of these points appreciably; therefore we expect the structure to occur near

$$t_s\approx\frac{2\ln[\phi/\ln(s/s_0)]}{\beta\ln(s/s_0)}\,. \tag{7}$$

It follows from (7) that t_s approaches the origin, but only loglogarithmically, i.e., extremely slowly.

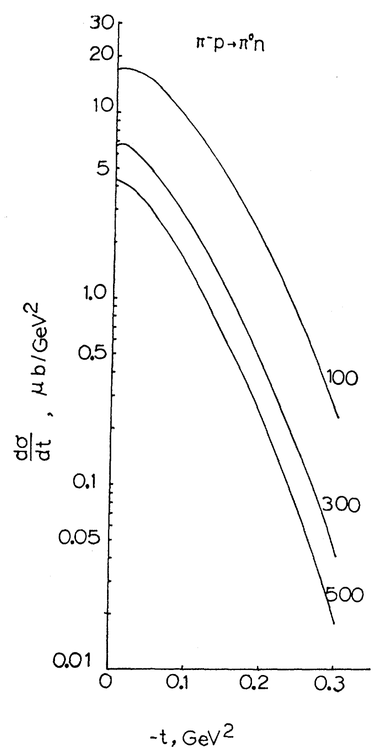

FIG. 3. Differential cross section for ($\pi^-p\rightarrow\pi^0n$) at 100, 300, and 500 GeV/c lab momentum.

The location of this structure can be related to the ratio of cut to pole contributions in (2). For $\alpha=1$, we have

$$\frac{\sigma_{\text{cut}}}{\sigma_{\text{pole}}}=\frac{\phi}{\ln(s/s_0)};$$

thus,

$$\left(\frac{s}{s_0}\right)^{\frac{1}{2}\beta t_s}\approx\frac{\sigma_{\text{cut}}}{\sigma_{\text{pole}}}.$$

If the differential cross section near $t=0$ is taken as $d\sigma/dt(s,t)\approx[d\sigma/dt(s,0)](s/s_0)^{2\beta t}$, then the structure occurs when

$$\frac{d\sigma}{dt}(s,t)\approx\left(\frac{\sigma_{\text{cut}}}{\sigma_{\text{pole}}}\right)^4\frac{d\sigma}{dt}(s,0). \tag{8}$$

At experimentally available energies, structure is generally seen at $t\approx-0.6$ GeV^2, which for the observed diffraction peaks corresponds to a cut/pole ratio of 0.2–0.3. This figure is in accord with the cut contribution found by fitting the data.[3,4]

Since the spin-flip amplitude for Pomeranchuk exchange seems to be small compared to the nonflip term (as shown by the absence of a dip in the forward differential cross section), it is reasonable to assume that its contributions may be neglected entirely in the Pomeranchuk region. We mention, however, that if it is parametrized similarly to (1), a nonvanishing polarization may result from pole-cut interference. Its form is

$$P\frac{d\sigma}{dt}\propto\frac{(-t)^{1/2}}{\ln(s/s_0)}\left(\frac{s}{s_0}\right)^{\frac{3}{2}\beta t}\sin\tfrac{1}{4}\pi\beta t. \tag{9}$$

In summary, then, the principal features to be expected when the new accelerators allow us to reach the Pomeranchuk region are (a) shrinkage of the diffraction peak, (b) slowly rising total cross sections (if $\alpha=1$), and (c) structure in the differential cross section around $t\sim-0.6$, moving extremely slowly toward the origin. In order to put quantitative flesh on these qualitative bones, we give in Figs. 1–5 the predictions obtained by

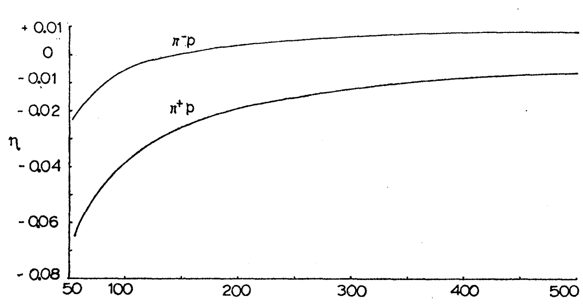

FIG. 4. Predictions for the phase of the forward elastic $\pi^{\pm}p$ amplitudes from 50 to 500 GeV/c lab momentum.

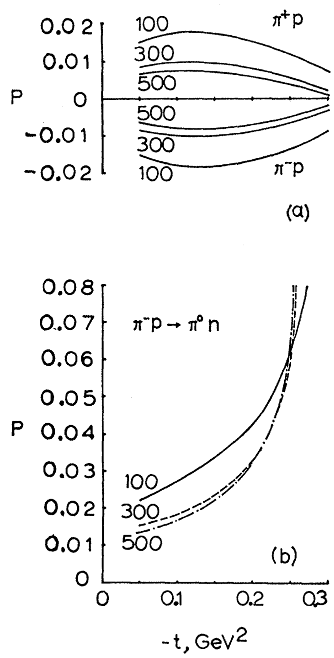

FIG. 5. Polarizations predicted for (a) elastic and (b) charge-exchange pion-nucleon scattering at lab momenta 100, 300, and 500 GeV/c.

extending our earlier pion-nucleon fit up to 500 GeV/c. Since these curves go far beyond the region we fitted to obtain them, they may disagree with experiment; their general features, however, should be qualitatively correct if the amplitude is approaching the form (1).

The $\pi^{\pm}p$ total cross sections we obtain are shown in Fig. 1. For $\sigma_t(\pi^-p)$, the P' and ρ poles and the cut terms in our model cancel each other quite well, leading to a nearly constant total cross section. For $\sigma_t(\pi^+p)$, however, a minimum is reached around 90 GeV/c lab momentum, and thereafter a slow [$\sim$0.05 mb/(100 GeV/c)] increase is observed. The differential cross sections for elastic scattering shown in Fig. 2 show substantial shrinkage of the diffraction peak; the slope $b(s)$ increases from 9.2 GeV^{-2} at 50 GeV/c to 11.5 GeV^{-2} at 500 GeV/c. Our fits did not, unfortunately, include points with $-t>0.3$ GeV^2, so the $t=-0.6$ structure cannot be shown. The charge-exchange differential cross section shown in Fig. 3 has fallen to the order of a few μb/GeV^2. For completeness, we give in Figs. 4 and 5 the predicted phases and polarizations. The Pomeranchuk contributions are becoming essentially imaginary in this region, and the variations of the real-to-imaginary ratio from zero result from the small ρ terms. The predicted elastic polarizations are very small; the sharp rise in the charge-exchange polarization is extremely parameter dependent and may not be reliable for that reason.