Current Commutators and the 2^+ Nonet in Electroproduction and Neutrino-Induced Production*

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Commutators of the type $[\dot{J}_i,J_j]$, where J_μ is a vector or an axial-vector current, are expressed in terms of canonical 2^+ meson fields. (More generally, a commutator involving the nth time derivative is expressed in terms of fields of spin $n+1$.) One consequence is that electromagnetic mass differences are finite. Predictions are made for electroproduction and neutrino-induced production; where comparisons are possible, the agreement with experiment is excellent.

I. INTRODUCTION

OFTEN, it is relatively easy to extract physical information from, and to make models of, commutators which involve at least one time component of a current:

$$\delta(x_0)[J_0(x),J_\mu{}^b(0)],$$
$$\delta(x_0)[\partial_t J_0{}^a(x),J_\mu{}^b(0)], \qquad (1)$$

where the J_μ are the usual vector or axial-vector currents. With the help of conservation of vector current (CVC) and partial conservation of axial-vector current (PCAC), it often turns out that simple tree graphs (i.e., phenomenological Lagrangians) give very good answers,[1] usually in the form of low-energy theorems. This is not the case for commutators of the type

$$\delta(x_0)[(\partial_t)^n J_i{}^a(x),J_j{}^b(0)], \quad n=0,\,1,\,\ldots \qquad (2)$$

for two reasons: (a) There are no associated low-energy sum rules, but instead sum rules at asymptotic energies,[2] and (b) tree graphs or Born approximations fail miserably (by a factor of 6 in electroproduction).

It is therefore urgent to produce well-defined, testable hypotheses for commutators of the type (2). The following ideas are relevant: (a) field algebra[3]; (b) the Sugawara model[4]; (c) Feynman's parton theory, as developed by Bjorken and Paschos,[5] and by Smith.[6] Of these, (a) and (b) yield ill-defined products of currents at the same space-time point for the commutator (2) with $n=1$. In general, no meaningful predictions can be made in such a case. (Nonetheless, the Sugawara model contains one very important testable feature, which was pointed out by Gross and Callan,[7] and which we retain in slightly modified form.) The parton model[5,6] is more successful, but sufficiently loose so that no really definitive predictions can be made; the typical result is in the form of an inequality.

The basic idea of the present paper is that the symmetric part [under the exchange $(a,i)\leftrightarrow(b,j)$] of (2) is expressible in terms of canonical meson fields of spin $n+1$.[8] The case of greatest interest is $n=1$, where $[\dot{J}_i,J_j]$ is expressed in terms of the fields which describe the nonet of spin-2 mesons.[9] This concept, when combined with several other known ideas (Bjorken's[10] scale invariance, nonet symmetry[11] for 2^+ mesons,[9,12] 2^+ meson dominance of the traceless part of the stress-energy tensor,[13] a modified form of the Gross-Callan[7] commutator) leads to several definite predictions for electroproduction and neutrino-induced production which are in excellent agreement with experiment.

Additionally, electromagnetic mass differences are finite, even though $[\dot{J}_i,J_j]\neq 0$. Bjorken[14] pointed out that such mass differences are necessarily finite (in the absence of operator Schwinger terms) if this commutator vanishes, and Cornwall and Norton[15] gave the precise connection between the mass difference and matrix elements of the commutator. From Ref. 15, it is clear that there can be cancellations of these matrix elements, leading to a finite mass difference. That is precisely what happens in the present case.

There are at least two drawbacks from a theoretical point of view. The first is that certain matrix elements of (2) get contributions from the Pomeranchon (P) and P'. It is not clear any more whether the P is an ordinary Regge trajectory, or if it is, what 2^+ meson lies on it. This, of course, makes it hard to see how the P con-

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[1] For references to the literature, see S. L. Adler and R. F. Dashen, *Current Algebra* (Benjamin, New York, 1968).

[2] J. M. Cornwall and R. E. Norton, Phys. Rev. **177**, 2584 (1969).

[3] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[4] H. Sugawara, Phys. Rev. **170**, 1659 (1968).

[5] J. D. Bjorken and E. A. Paschos, Phys. Rev. **185**, 1975 (1969).

[6] C. H. Llewellyn Smith, CERN Report No. TH. 1097, 1969 (unpublished).

[7] D. J. Gross, Phys. Rev. Letters **21**, 308 (1968); D. J. Gross and C. G. Callan, *ibid.* **21**, 311 (1968). Note the misprint of sign in Eq. (4) of the first article; replace $3i$ by $-3i$.

[8] The antisymmetric part of these commutators is known in terms of commutators with fewer time derivatives; this is a consequence of Lorentz invariance. For a demonstration of the relation between $[\dot{J}_i,J_j]$ and $[J_i,J_j]$, see the Appendix of Ref. 15.

[9] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).

[10] J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).

[11] S. Okubo, Phys. Letters **5**, 165 (1963). For a generalization, see H. Sugawara and F. von Hippel, Phys. Rev. **141**, 1331 (1966).

[12] V. Barger, M. Olsson, and K. V. L. Sarma, Phys. Rev. **147**, 1115 (1966).

[13] P. G. O. Freund, Phys. Letters **2**, 136 (1962); **16**, 291 (1966); M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[14] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

[15] J. M. Cornwall and R. E. Norton, Phys. Rev. **173**, 1637 (1968).

tributions can be related to 2^+ fields. [The P' may lie on the $f(1260)$ trajectory.]

The second drawback is that nonet symmetry for 2^+ mesons, which looked so good in 1965,[9] is partially upset by A_2 splitting.[16] There are a number of more or less *ad hoc* ways of evading these objections, none of which will be discussed here. The model gives such good agreement with experiment that one hopes that these drawbacks can be explained away. Note that, even if the Pomeranchon is not correctly included in the model, the conclusions about mass differences would still be valid.

Section II states the basic commutators; Sec. III contains motivations and models. Section IV gives the predictions of the theory and a comparison with experiment, while Sec. V contains a summary.

II. PROPOSED COMMUTATORS

In this section, we write down the symmetric part of the commutator (2) for $n=1$, which is the case of greatest interest theoretically. Let $J_\mu{}^a(x)$ be the usual vector current, $J_{5\mu}{}^a(x)$ the axial-vector current, and $T_{\mu\nu}{}^a(x)$ a set of traceless conserved tensors describing 2^+ mesons. The index a runs from 0 to 8. Define symmetric and antisymmetric[8] commutators by

$$[\dot{A}_i{}^a,B_j{}^b]^{(\pm)}=\tfrac{1}{2}[\dot{A}_i{}^a,B_j{}^b]\pm\tfrac{1}{2}[\dot{A}_j{}^b,B_i{}^a],\qquad(3)$$

where A_μ and B_μ are any currents. The proposed commutators are

$$\begin{aligned}\delta(x_0)[\dot{J}_i{}^a(x),J_j{}^b(0)]^{(+)}&=\delta(x_0)[\dot{J}_{5i}{}^a(x),J_{5j}{}^b(0)]^{(+)}\\&=-i\alpha d_{abc}[T_{ij}{}^c(x)+\beta\delta_{ij}T_{00}{}^c(x)]\delta_4(x)+\cdots,\qquad(4)\end{aligned}$$

$$\delta(x_0)[\dot{J}_i{}^a(x),J_{5j}{}^b(0)]^{(+)}=-i\lambda f_{abc}\epsilon_{ijk}T_{k0}{}^c(x)\delta_4(x)+\cdots,\qquad(5)$$

where the omitted terms involve one or more powers of the gradient operator. In the next section, it is argued that

$$\alpha=\lambda,\quad\beta=-1.\qquad(6)$$

In addition to (4) and (5), it is assumed that a modified form of the Gross-Callan[7] commutator in the Sugawara[4] model holds:

$$\begin{aligned}\delta(x_0)\sum_{a=1}^{8}[\dot{J}_i{}^a(x),J_j{}^a(0)]&=\delta(x_0)\sum_{a=1}^{8}[\dot{J}_{5i}{}^a(x),J_{5j}{}^a(0)]\\&=-Si[\bar{\theta}_{ik}(x)+\beta\delta_{ij}\bar{\theta}_{00}(x)]\delta_4(x)+\cdots,\qquad(7)\end{aligned}$$

where again the omitted terms contain gradients. In (7), $\bar{\theta}_{\mu\nu}$ is the traceless part of the stress-energy tensor $\theta_{\mu\nu}$:

$$\bar{\theta}_{\mu\nu}=\theta_{\mu\nu}-\tfrac{1}{4}g_{\mu\nu}\theta,\quad\theta=\theta_\mu{}^\mu.\qquad(8)$$

This should be compared to the Gross-Callan[7] result, which would replace the right-hand side of (7) by

$$-3i\theta_{ik}(x)\delta_4(x)+\text{terms in }\delta_{ij},\qquad(9)$$

where the terms in δ_{ij} are not all well defined. A comparison of (7) and (9) suggests that $S=3$, a value which fits very well with experiment (see Sec. IV).

Obviously, it would be possible to loosen the scheme of Eqs. (4), (5), and (7) by adding scalar operators multiplying δ_{ij} to the right-hand side of any appropriate $SU(3)$ representation (1,8,27), allowing separate α's and β's for the vector and axial-vector commutators, splitting off the 2^+ octet from the singlet in (4), etc. But the idea is not to write down the most general commutator allowed by $SU(3)$ and Lorentz invariance. Rather, it is to provide a *specific* scheme with as few arbitrary constants as possible. The specific scheme compared with experiment in Sec. IV consists of Eqs. (4), (5), and (7), with $\alpha=\lambda$, $\beta=-1$, and $S=3$. The only unknown is therefore α, which cancels out of all predictions for electroproduction and neutrino-induced production, when 2^+ dominance of $\theta_{\mu\nu}$ is assumed.

The next section motivates this scheme with various models.

III. MOTIVATIONS AND MODELS

Equations (4) and (5) identify certain equal-time commutators with canonical meson fields. This is hardly new; in field algebra,[3] one makes the identification

$$J_\mu{}^{(a)}(x)=(m_V{}^2/2\gamma)V_\mu{}^a(x),\qquad(10)$$

where m_V is a vector-meson mass, and γ the vector coupling constant ($\gamma^2/4\pi\simeq\frac{1}{2}$). The usual charge algebra then provides that (modulo c-number Schwinger terms)

$$\delta(x_0)[J_0{}^a(x),J_\mu{}^b(0)]=if_{abc}(m_V{}^2/2\gamma)V_\mu{}^c(x)\delta_4(x),\qquad(11)$$

thereby identifying the commutator with a meson field.

Equations (4) and (5) contain a time derivative of one current, which means that α (or λ) in these equations has dimensions of (mass)^3. In (11) the dimensional constant is $m_V{}^2/2\gamma$; it is perhaps reasonable that $\alpha\simeq m_V{}^3/2\gamma$, with m_V as a suitable scale for the time derivative. In fact, it will be shown later that $\alpha\simeq1.3m_\rho{}^3/2\gamma$.

Consider now the time-ordered product of two currents of either parity, momenta q_1 and q_2, taken between spin-zero hadron states (covariantly normalized):

$$i\int d^4x\,e^{iQ\cdot x}\langle p_2|(J_\mu{}^a(\tfrac{1}{2}x)J_\nu{}^b(-\tfrac{1}{2}x))_+|p_1\rangle=P_\mu P_\nu T_2-g_{\mu\nu}T_1-\tfrac{1}{2}i\epsilon_{\mu\nu\alpha\beta}P^\alpha Q^\beta T_3+\cdots.\qquad(12)$$

The omitted terms have Q or Δ as kinematical factors.

[16] For references, see Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970).

If the currents J_μ each have definite parity, then either T_1 and T_2, or T_3 occur, but not both. The notation is

$$P=\tfrac{1}{2}(p_1+p_2),\quad Q=\tfrac{1}{2}(q_1+q_2),\quad \Delta=q_1-q_2=p_2-p_1. \tag{13}$$

For simplicity, we assume $p_1^2=p_2^2$, so $P\cdot\Delta=0$.

Define

$$2\pi W_i=\mathrm{Im}T_i. \tag{14}$$

Bjorken[10] has argued that, in the scale limit

$$Q^2\to\infty,\quad P\cdot Q\equiv\nu\to\infty,\quad \text{with}\quad \omega\equiv-Q^2/2\nu,$$
$$\Delta^2\equiv t,\quad \text{and}\quad Q\cdot\Delta\quad \text{fixed},$$

the imaginary parts exhibit scale invariance[17]:

$$W_1\to F_1(\omega,t),\quad \nu W_2\to F_2(\omega,t),\quad \nu W_3\to F_3(\omega,t). \tag{15}$$

It is quite plausible that all Regge poles survive in the scale limit[18] (however, see Harari[19] for a contrary opinion); in such a case, for a trajectory $\alpha(t)$,

$$F_1\sim\beta_1(t)\omega^{-\alpha(t)},\quad F_2\sim\beta_2(t)\omega^{1-\alpha(t)},\quad F_3\sim\beta_3(t)\omega^{-\alpha(t)}. \tag{16}$$

With the help of the Bjorken limit[14] (taken for $\mathbf{Q}=0$), it follows that[2,10]

$$\begin{aligned}\int d^3x\langle p_2|(\partial_t)^n[J_i{}^a(\tfrac{1}{2}x),J_j{}^b(-\tfrac{1}{2}x)]|_{t=0}|p_1\rangle \\ =8i^n\bigg[\delta_{ij}\int_0^1 d\omega\,(2\omega)^nF_1(\omega,t)P_0{}^{n+1} \\ -P_iP_j\int_0^1 d\omega\,(2\omega)^{n-1}F_2(\omega,t)P_0{}^{n-1} \\ -\tfrac{1}{2}i\epsilon_{ijk}P_kP_0{}^n\int_0^1 d\omega\,(2\omega)^nF_3(\omega,t)\bigg]+\cdots.\end{aligned} \tag{17}$$

The omitted terms in (17) are of power less than $n+1$ in the components of P. Substitute (16) into (17) to find that all terms behave like $[n+1-\alpha(t)]^{-1}$. This fact, plus the kinematic structure in (17), allows one to identify the commutators in (17) as spin-$(n+1)$ meson fields (or at least to say that they are approximately dominated by such fields). (There is, of course, the problem of the Pomeranchon trajectory alluded to in Sec. I, but this will be ignored.) The simplest and most elegant ansatz based on (17) is that the commutator is composed solely of spin-$(n+1)$ meson fields—that is, nth-rank completely symmetric tensors which are traceless and conserved. Note that this has implications for the terms omitted in (17), which are of lesser rank in the components of P. This will be quite important in the electromagnetic mass-difference problem.

Further insight can be gained from free quark models. Consider the time-ordered product (12) in first Born approximation for quarks taken in the forward direction ($\Delta=0$) and spin averaged. Define a matrix element $t_{\mu\nu}{}^c$ by

$$t_{\mu\nu}{}^c=\tfrac{1}{2}\lambda_c(2P_\mu P_\nu), \tag{18}$$

where λ_c ($c=0, \ldots, 8$) are the usual $SU(3)$ matrices. One finds that the coefficient of $Q_0{}^{-2}$ in (12), in the Bjorken limit, has the following values, depending on the currents (V or A) used in (12);

$$\begin{aligned}(VV)&\quad -2d_{abc}(t_{ij}{}^c-\delta_{ij}t_{00}{}^c),\\ (AA)&\quad -2d_{abc}(t_{ij}{}^c-\delta_{ij}t_{00}{}^c),\\ (VA)&\quad -2f_{abc}\epsilon_{ijk}t_{0k}{}^c.\end{aligned} \tag{19}$$

These numbers multiplied by $+i$ give the commutators specified in (17) for $n=1$. The tensor $t_{\mu\nu}{}^c$ is, of course, not traceless unless the quark mass vanishes, so that the free quark model does not completely justify the ansatz. Nevertheless, it suggests that $\beta=-1$, $\alpha=\lambda$ [see (9) and (5)]. One may also compare with the modified Gross-Callan commutator in (7); comparing coefficients of P_iP_j, we would conclude that $S=16/3$. The Gross-Callan value is $S=3$, which seems to fit much better with experiment (see Sec. IV).

It is also possible to recover commutators of the type (4) and (5) from a field-algebra model. Let there be a coupling of the type $F_{\mu\alpha}{}^aF_b{}^{\nu\alpha}T_\nu{}^{\mu c}d_{abc}$, where $F_{\mu\nu}$ is the usual antisymmetric tensor $\partial_\mu V_\nu-\partial_\nu V_\mu+\cdots$. If the commutator (4) is sandwiched between the vacuum and a 2^+ meson state, the result should be the 2^+ wave function (polarization tensor). This matrix element can be gotten from the time-ordered product

$$T_{\mu\nu}=i\int d^4x\, e^{iq\cdot x}\langle 0|(J_\mu{}^a(x)J_\nu{}^b(0))_+|p_c\rangle, \tag{20}$$

where $|p_c\rangle$ is a 2^+ meson state. With the field-current relation (10), one finds (up to an irrelevant numerical constant)

$$\begin{aligned}T_{\mu\nu}=\frac{\epsilon_{\alpha\beta}(p)d_{abc}}{(q^2-m_V{}^2)[(p-q)^2-m_V{}^2]}\{q\cdot(p-q)g_{\mu\alpha}g_{\nu\beta} \\ -q_\nu(p-q)_\alpha g_{\mu\beta}-q_\alpha(p-q)_\mu g_{\nu\beta}+g_{\mu\nu}q_\alpha(p-q)_\beta\}.\end{aligned} \tag{21}$$

Apply the Bjorken limit to T_{ij}:

$$T_{ij}\to(d_{abc}/q_0{}^2)[-\epsilon_{ij}(p)+\delta_{ij}\epsilon_{00}(p)], \tag{22}$$

which is consistent with the commutator relation (4), if $\beta=-1$. Note that, when μ or $\nu=0$, the coefficient of $q_0{}^{-2}$ vanishes, as it should.

We conclude this section by using 2^+ dominance of the stress-energy tensor[13] to evaluate the constant α

[17] Observe that the dependence on $Q\cdot\Delta$ drops out in the scale limit. That this must happen follows from a generalization of the work in Ref. 2 to the nonforward direction, which will be discussed in a separate publication.

[18] H. D. I. Abarbanel, M. Goldberger, and S. B. Treiman, Phys. Rev. Letters 22, 500 (1969); R. A. Brandt, *ibid.* 22, 1149 (1969); J. M. Cornwall and R. E. Norton (unpublished).

[19] H. Harari, Phys. Rev. Letters 22, 1078 (1969).

(although its value will not be needed in the rest of the paper). In the commutator, set $a=b$ and sum over a from 1 to 8, then compare with the modified Gross-Callan commutator (7) to conclude

$$T_{ij}^0(x)=(\sqrt{\tfrac{3}{2}})(S/8\alpha)\bar{\theta}_{ij}(x)\,. \tag{23}$$

Glashow and Socolow[9] have argued that Okubo's[11] nonet symmetry holds for the 2^+ mesons, which means that $T_{\mu\nu}^0$ can be expressed in terms of physical meson fields as

$$T^0=(\sqrt{\tfrac{2}{3}})f(1260)-(\sqrt{\tfrac{1}{3}})f'(1515)\,. \tag{24}$$

Write the phenomenological Lagrangian coupling 2^+ mesons to the pseudoscalars as (m_T is a mean 2^+ mass)

$$\mathcal{L}=f_T m_T^2 d_{abc} T_{\mu\nu}{}^a \partial_\mu \pi^b \partial_\nu \pi^c\,. \tag{25}$$

One easily calculates the decay rate $f\to\pi\pi$:

$$\Gamma(f\to 2\pi)=(P^5 f_T^2 m_T^2)/10\pi \tag{26}$$

from which $f_T\sim 5$ GeV^{-3}. Sandwich (23) between two single-pion states and use (25); at zero momentum transfer, one finds

$$f_T=3S/16\alpha\,. \tag{27}$$

For $S=3$, $\alpha=0.11$ GeV$^3\simeq 1.3\ m_\rho{}^3/2\gamma$.

IV. EXPERIMENTAL PREDICTIONS

The commutators (4), (5), and (7) cannot be directly tested without some additional assumptions. The most important of these is nonet symmetry of 2^+ coupling to hadrons in the form given by Barger, Olsson, and Sarma,[12] which forbids coupling of $f'(1515)$ to nonstrange hadrons. For purposes of numerical predictions, we assume $S=3$, $\alpha=\lambda$, and $\beta=-1$, in view of the results of Sec. III, although S, α, and λ will be displayed in the formulas. $\beta=-1$ has the following implications: (a) $F_2(\omega)=2\omega F_1(\omega)$ in electroproduction or neutrino-induced production experiments [this follows from (17) and the fact that $F_2-2\omega F_1\geq 0$ in the forward direction]; (b) there are no operator Schwinger terms in the $[J_0,J_i]$ commutator,[20] and hence there is no quadratic divergence in electromagnetic mass differences.[15]

In principle, there are still logarithmic divergences in electromagnetic mass differences, but these are removed by the requirement that the fields $T_{\mu\nu}$ be traceless. To see this, note that the matrix elements of $T_{\mu\nu}$ in the forward direction between states of mass M must have the form

$$\langle p|T_{\mu\nu}(0)|p\rangle\sim P_\mu P_\nu-\tfrac{1}{4}M^2 g_{\mu\nu}\,. \tag{28}$$

Then, using (4) and (28), the matrix elements of the equal-time commutator

$$\int d^3x\langle p|[\dot{J}_i,J_j]|p\rangle\,, \tag{29}$$

where J_μ is the electromagnetic current, have the form

$$A[P_iP_j+\beta P_0{}^2\delta_{ij}(\tfrac{1}{4}M^2)(1-\beta)]\,. \tag{30}$$

In Ref. 15, the connection between the coefficients of kinematic tensors in (30) and the logarithmically divergent electromagnetic mass shift were given. It is easy to see, by comparing Eqs. (26), (27), (33), and (34) of Ref. 15 with Eq. (30) (and noting that scale invariance requires K_1' of Ref. 15 to vanish), that the coefficient of the logarithmic divergence vanishes, for any β. For $\beta=-1$, there are no Schwinger terms or quadratically divergent mass differences, so the commutator (4) implies finiteness of electromagnetic mass differences.

We now turn to electroproduction and neutrino-induced production. Specific numerical predictions will be made on the basis of nonet symmetry for the coupling of 2^+ mesons to baryons, as elucidated by Barger, Olsson, and Sarma.[12] Up to an over-all constant, the phenomenological Lagrangian of these authors is written as

$$\mathcal{L}=\mathrm{Tr}T\{B,\bar{B}\}+(F/D)\,\mathrm{Tr}T[B,\bar{B}]+(F/D-1)(\mathrm{Tr}T)(\mathrm{Tr}B\bar{B})\,, \tag{31}$$

where T is the 3×3 matrix describing the nine 2^+ mesons. The specific Lagrangian (31) forbids the coupling of $f'(1515)$ to nonstrange particles, which is in reasonable agreement with experiment.[9,12] When J_μ is the electromagnetic current, the commutator (4) sandwiched between protons reads

$$\int d^3x\langle p|[\dot{J}_i(\mathbf{x},0),J_j(0)]|p\rangle=-i\alpha\tfrac{2}{3}\langle p|\tfrac{1}{3}\sqrt{3}T_{ij}{}^8+T_{ij}{}^3+2(\sqrt{\tfrac{2}{3}})T_{ij}{}^0|p\rangle+\cdots. \tag{32}$$

Recall that

$$T_{ij}^0(x)=(S/8\alpha)(\sqrt{\tfrac{3}{2}})\bar{\theta}_{ij}(x) \tag{33}$$

and, of course,

$$\langle p|\bar{\theta}_{ij}(0)|p\rangle=2p_ip_j+\cdots. \tag{34}$$

If one uses the phenomenological Lagrangian (31) in conjunction with (32)–(34) and (17), it is easy to find

$$\int_0^1 d\omega\,F_2{}^{\gamma p}(\omega)=\frac{S}{48}\left(\frac{3F/D-\frac{1}{3}}{F/D-\frac{1}{3}}\right). \tag{35}$$

In Ref. 12, it is found that $-\infty<F/D<-1$, with a preferred value $F/D\simeq -2$. With the Gross-Callan value of $S=3$, the right-hand side of (35) is 0.17; the experimental value is 0.16.[21] Born approximation would give 1, and the parton-model prediction is $\gtrsim 0.22$.[5,6] In the same way,

$$\int_0^1 d\omega\,F_2{}^{\gamma n}(\omega)=\frac{S}{48}\left(\frac{2F/D-\frac{4}{3}}{F/D-\frac{1}{3}}\right)\simeq 0.14. \tag{36}$$

As mentioned above, $\beta=-1$ implies $F_2(\omega)=2\omega F_1(\omega)$.

[20] J. M. Cornwall, D. Corrigan, and R. E. Norton, Phys. Rev. Letters **24**, 1141 (1970).

[21] E. D. Bloom *et al.*, Phys. Rev. Letters **23**, 930 (1969); M. Breidenbach *et al.*, *ibid.* **23**, 935 (1969).

For the neutrino experiments, can we only make predictions about the sum of ν and $\bar{\nu}$ cross sections, since the current commutators used in this paper give no information about the differences of ν and $\bar{\nu}$ cross sections. We find (transitions to strange final states ignored)

$$\int_0^1 d\omega\, \tfrac{1}{2}[F_2^{\nu p}(\omega)+F_2^{\bar{\nu} p}(\omega)] = \int_0^1 d\omega\, \tfrac{1}{2}[F_2^{\nu n}(\omega)+F_2^{\bar{\nu} n}(\omega)] = \tfrac{3}{16}S = 0.56. \quad (37)$$

The second equality follows from isospin symmetry, which says that $F_i^{\nu p}=F_i^{\bar{\nu} n}$ and $F_i^{\bar{\nu} p}=F_i^{\nu n}$. Also, we find

$$\int_0^1 d\omega\, 2\omega \tfrac{1}{2}[F_3^{\nu p}(\omega)-F_3^{\bar{\nu} p}(\omega)] = \frac{\alpha S}{8\lambda}\left(\frac{F/D+1}{F/D-\frac{1}{3}}\right) \simeq 0.16 \quad (\alpha=\lambda). \quad (38)$$

This prediction is rather more sensitive to a negative F/D ratio than (35) or (36). We combine (35)–(38) to find, with $\alpha=\lambda$ and $F_2=2\omega F_1$,

$$12\int_0^1 d\omega\, 2\omega[F_1^{\gamma p}(\omega)-F_1^{\gamma n}(\omega)] = \int_0^1 d\omega\, 2\omega[F_3^{\nu p}(\omega)-F_3^{\bar{\nu} p}(\omega)] = \int_0^1 d\omega\, 2\omega[F_3^{\nu p}(\omega)-F_3^{\nu n}(\omega)], \quad (39)$$

$$\int_0^1 d\omega[F_2^{\gamma p}(\omega)+F_2^{\gamma n}(\omega)] = \frac{5}{18}\int_0^1 d\omega[F_2^{\nu p}(\omega)+F_2^{\bar{\nu} p}(\omega)] = \frac{5}{18}\int_0^1 d\omega[F_2^{\nu p}(\omega)+F_2^{\nu n}(\omega)]. \quad (40)$$

These should be compared with the parton-model sum rules of Smith,[6] which assert the equality of the integrands in (39), and which would remove the integral signs from (40), and replace the = sign by $\geq$.

If one is willing to extend the scheme to its logical conclusion, and to assume an infinite sequence of spin-$(n+1)$ nonets, all coupled to baryons as in (31) (but with different F/D ratio), then the integral signs can be removed from (39) and (40). The reason is that one would generate a series of equations like (39), with 2ω replaced by $(2\omega)^n$.

At present it is not possible to make a precise comparison with the CERN neutrino experiment[22] in a heavy-liquid bubble chamber. If the difference between neutron number and proton number in the target is ignored, the CERN experiment yields

$$\tfrac{1}{2}(\sigma^{\nu p}+\sigma^{\nu n}) \simeq (0.8\pm0.2)\times 10^{-38}E \quad (\sigma \text{ in cm}^2,\ E \text{ in GeV}). \quad (41)$$

We can, however, predict another cross section[6,10]:

$$\tfrac{1}{4}(\sigma^{\nu p}+\sigma^{\bar{\nu} p}+\sigma^{\nu n}+\sigma^{\bar{\nu} n}) \to \frac{G^2ME}{\pi}\int_0^1 d\omega\, \tfrac{2}{3}F_2(\omega) - \tfrac{1}{6}(2\omega)F_3(\omega), \quad (42)$$

where

$$F_2(\omega)=\tfrac{1}{4}(F_2^{\nu p}+F_2^{\bar{\nu} p}+F_2^{\nu n}+F_2^{\bar{\nu} n})=\tfrac{1}{2}(F_2^{\nu p}+F_2^{\nu n}), \quad (43)$$

$$F_3(\omega)=\tfrac{1}{4}(F_3^{\nu p}-F_3^{\bar{\nu} p}+F_3^{\nu n}-F_3^{\bar{\nu} n})\equiv 0. \quad (44)$$

$F_3(\omega)$ vanishes by isospin symmetry. The right-hand side of (42) is valid when E is so big that scaling has set in, but not so big that effects from massive W bosons are important. From (37) and (43), the right-hand side of (42) is predicted to be $0.5\times10^{-38}E$. There is some indication[22] from the CERN data that the contribution of the F_3's to (41) is small; to the extent that it is negligible, the second equality in (43) allows us to compare the predicted value of 0.5 directly with the 0.8 ± 0.2 in (41). The possibilities for agreement of theory and experiment are not wholly discouraging.

V. SUMMARY

The excellent agreement with experiment found in Sec. IV lends some credence to the basic hypothesis, expressed in the commutators (4), (5), and (7). How the Pomeranchon is to be theoretically incorporated into the scheme remains obscure at present, but to the extent that diffractive scattering is present in the experimental data, our theory accounts for it numerically at least. Another feature which remains unexplained is why the value $S=3$, taken from the Sugawara model, should work so well, when the literal Sugawara model is not an acceptable fundamental theory.[23] It may well be that there is an underlying Lagrangian framework in which the commutators (4), (5), and (7) are true; we hope to report on this in the future.

These commutators also have consequences for divergences in weak interactions. It is known[24] that, if the $SU(3)\times SU(3)$ symmetry-breaking Hamiltonian is in the representation $(3,\bar{3})+(\bar{3},3)$, to $O(G)$, there are no quadratic divergences in off-diagonal weak processes. The remaining logarithmic divergences are governed by commutators like (4), plus the commutator

$$[\dot{D}^\dagger(\mathbf{x},0),D(0)]^{(+)} \quad (45)$$

[22] I. Budagov *et al.*, Phys. Letters **30B**, 364 (1969).

[23] See, e.g., R. F. Dashen and Y. Frishman, Phys. Rev. Letters **22**, 572 (1969).

[24] C. Bouchiat, J. Iliopoulos, and J. Prentki, Nuovo Cimento **56A**, 1150 (1969).

[where $D(x)$ is the divergence of the Cabibbo current], as one readily sees from the Bjorken limit. The commutators (4) do not contribute to the divergence, as in Sec. IV. If one takes PCAC literally, so that $D(x)$ is related to canonical spin-zero fields, then (44) is a harmless c number, and first-order weak processes would be finite.

It is intriguing that, in the quark model of Sec. III, the commutators are not expressible in terms of traceless tensors unless the quark mass vanishes. This may have something to do with the idea that part of the Lagrangian of the model is scale invariant.[25] One would hope that whatever breaks scale invariance does not affect the commutators (4), (5), and (7), in much the same way that mass terms in Yang-Mills theories do not affect the usual current-algebra commutators.

Note added in manuscript. After this manuscript was finished, it was brought to my attention that a paper by R. A. Brandt, Phys. Rev. D (to be published), deals with some of the topics of the present paper in somewhat different form.

[25] See, e.g., G. Mack and A. Salam, Ann. Phys. (N. Y.) **53**, 174 (1969); D. J. Gross and J. E. Wess, Phys. Rev. D (to be published); P. Carruthers and M. Gell-Mann (unpublished).

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Absorption Model Applied to $\pi-\pi$ Scattering

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A study of the possibility of using an absorption model to obtain a unique s-wave $I=0$ $\pi\pi$ phase shift from the reaction $\pi^-p\rightarrow\pi^-\pi^+n$ was made. Applying the model to combined data from several latoratories, we obtained a set of resaonable (but twofold ambiguous) values for the isospin-zero s-wave phase shift $\delta_0{}^0$, using the *isotropic* moment of the dipion decay distribution. Values for $\delta_0{}^0$ deduced by using the s-p wave *interference* term in the dipion decay cross section agreed well with previous (twofold ambiguous) results obtained by extrapolation procedures. Comparing the two sets of twofold ambiguous solutions for $\delta_0{}^0$, the lower or "down" branch was preferred marginally in the dipion mass region below the ρ meson. No unique conclusion could be drawn in the higher-mass region. We emphasize the model dependence of these results, but we believe that similar studies with improved statistics for $-t>\mu^2$ would yield similar results. Good data for $-t<\mu^2$ are required to do definitive model-independent analyses.

I. INTRODUCTION

SEVERAL methods have been used recently to deduce $\pi\pi$ phase shifts $\delta_l{}^I$ in the dipion energy range 500–900 MeV from the reactions[1,2]

$$\pi^-p\rightarrow\pi^-\pi^0p \tag{1}$$

and

$$\pi^-p\rightarrow\pi^-\pi^+n. \tag{2}$$

These methods generally use "evasive" Chew-Low[3] extrapolations of polynomials representing $(\mu^2-t)^2d\sigma/dt$, and various moments thereof, which are constrained to vanish at $t=0$, where μ is the charged pion mass and t is the square of the four-momentum transfer to the nucleon. In particular, using the $I=2$ s-wave phase shifts from Baton and Laurens,[2] Marateck *et al.*[1] obtained sets of $I=0$ s-wave $\pi\pi$ phase shifts $\delta_0{}^0$ from the extrapolated moments A_2 and A_1 in the decay distribution of the dipion in reaction (2) in the $\pi\pi$ c.m. frame,

$$W(\theta)=A_0+A_1\cos\theta+A_2\cos^2\theta, \tag{3}$$

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[1] S. Marateck *et al.*, Phys. Rev. Letters **21**, 1613 (1968); for many details of the collaboration data see also V. Hagopian, in Proceedings of the Conference on $\pi\pi$ and $K\pi$ Scattering, Argonne National Laboratory, 1969, p. 149 (unpublished). The data of the Florida State University group at 2.26 GeV/c were reported by B. Reynolds *et al.*, Phys. Rev. **184**, 1424 (1969). The nonevasive production mechanism is indicated in a recent study by J. Scharenguivel *et al.*, Phys. Rev. Letters **24**, 332 (1970).
[2] J. Banton and G. Laurens, Phys. Rev. **176**, 1574 (1968).
[3] G. Chew and F. Low, Phys. Rev. **113**, 1640 (1959).