all operators acting on a state of momentum $\mathbf{p}$ and mass $m=(\omega^2-\mathbf{p}^2)^{1/2}=i\mu$, μ being real. The Hermitian matrices s_1', s_2', and s_3' generate the group $O(2,1)$ which is the little group appropriate to a spacelike vector. We have now only to take the matrix element of Eq. (6) between, for example, two spacelike states and take the limit $\kappa\to\infty$ exactly as before. The result corresponding to Eq. (12) is

$$\begin{aligned}\lim\langle\mathbf{p}'_\kappa,n'|[Z(p'_\kappa,p_\kappa),A]|\mathbf{p}_\kappa,n\rangle &= (m'^2-m^2)\lim\langle\mathbf{p}'_\kappa,n'|[S_3',A]|\mathbf{p}_\kappa,n\rangle \\ &\quad -2k_\perp\cdot\lim\langle\mathbf{p}'_\kappa,n'|\mu'S_\perp'A-\mu AS_\perp'|\mathbf{p}_\kappa,n\rangle \\ &\quad -k_\perp^2\lim\langle\mathbf{p}'_\kappa,n'|[S_3',A]_+|\mathbf{p}_\kappa,n\rangle,\end{aligned}\tag{16}$$

where $S_\perp'=(-S_2',S_1',0)$. It follows that the angular condition can still be written in the form of Eq. (1), with $J_k(\theta)$ given by Eq. (3), but $I_k(\theta)$ replaced by

$$I_k'(\theta)=[M^2,[\mathcal{J}_3',\theta]]-2k_\perp\cdot[M\mathcal{J}_\perp',\theta]-k_\perp^2[\mathcal{J}_3',\theta]_+,\tag{17}$$

where the correspondence between the spin operators S_i' and the operators $\mathcal{J}_i'$ is given by $S_3'\to\mathcal{J}_3'$, $-S_2'\equiv\exp(-\frac{1}{2}i\pi S_3')S_1'\exp(\frac{1}{2}i\pi S_3')\to i\mathcal{J}_1'$, and

$$S_1'\equiv\exp(-\tfrac{1}{2}i\pi S_3')S_2'\exp(\tfrac{1}{2}i\pi S_3')\to i\mathcal{J}_2'.$$

In other words, $i\mathcal{J}_\perp'$ are two *Hermitian* operators which, with $\mathcal{J}_3'$, give rise to a unitary representation of $O(2,1)$. Therefore, $M\mathcal{J}_\perp'$ are also *Hermitian* operators satisfying the commutation relations

$$[M\mathcal{J}_+',M\mathcal{J}_-']=2M^2\mathcal{J}_3',$$

the same as in the case of timelike states.

It is obvious that the case of one timelike state and one spacelike state is dealt with in the same way, leading to the conclusion that in the general case, the angular condition is indeed given by Eqs. (1)–(3).

Apart from providing a very simple derivation of the angular conditions, our work shows precisely what conditions *are* obeyed by the explicit models containing spacelike states presented in the literature.[2]

PHYSICAL REVIEW D VOLUME 2, NUMBER 2 15 JULY 1970

Schwinger Terms, Hard Pions, and the Structure of the Deuteron

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The npd vertex, Ward and Jacobi identities are used to evaluate the equal-time commutator (ETC) of the deuteron axial vector with the nucleon field. Schwinger terms are shown to be related to the deuteron wave function. They are important for nuclear short-range correlations. Ward identities yield the pole-term amplitudes for $pp\to d\pi^+$ and $pn\to d\gamma$ at threshold. Pion-induced nucleon-nucleon scattering is traced to the deuteron ETC with the weak axial-vector current. The Bjorken limit with respect to the deuteron yields the nonrelativistic limit of the reaction amplitudes.

RECENTLY, it was found[1] that *soft-pion techniques*[2] *fail* for the reaction $pp\to d\pi^+$ at threshold. The zero-mass pion result gives three times the experimental cross section and decreases by a factor of about 20 in going to the physical pion mass, at fixed pion momentum $\mathbf{k}\neq 0$. The reaction mechanism of the inverse process, slow-pion absorption (π^-,NN) on nuclei—the pion in flight or in an atomic orbit—sheds some light on the rapid variation of the amplitude. At the physical pion mass, two nucleons are emitted predominantly back to back.[3] In fact, they can, more easily than a single nucleon, share the *high energy* M_π and *little momentum* $\mathbf{k}$ transferred to the nucleus by the pion and yet satisfy energy-momentum conservation. Hence the soft-pion limit is not applicable because

$$M_\pi^2\gg\mathbf{k}^2,\quad \mathbf{k}_N^2=2M_\pi M\gg\mathbf{p}_F^2\gg\mathbf{k}^2\tag{1}$$

is essential for suppressing single-nucleon emission. Here $\mathbf{p}_F$ denotes the Fermi momentum of the nucleus and $\mathbf{k}_N$ the single-nucleon momentum. This suggests using *hard-pion* techniques[4] for those slow-pion reactions in which the absorbed pion mass induces nuclear rearrangement or disintegration. In contrast, the absorbed pion of radiative pion capture on nuclei[5] may be treated as soft including the application of current algebra[6] because its mass is largely carried away by the

* Supported in part by a University of Virginia NSF Institutional Grant.

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kinematically similar photon. Now the low-energy pionic disintegration of the deuteron involves nuclear structure effects in terms of rapidly varying high-momentum components of the deuteron wave function, i.e., essentially nuclear short-range correlations. Hence, the deuteron's off-shell Ward identity may be expected to generate the threshold pole diagrams of the amplitude. Furthermore, we have to allow for Schwinger terms in the deuteron-nucleon equal-time commutator (ETC) to take care of the deuteron's form factors at high momentum. To this end, it is convenient to use a description of the deuteron in terms of an axial field vector[7] $D^{5\mu}(x)$ subject to

$$\partial_\mu D^{5\mu}(x)=0\,, \tag{2}$$

in view of the absence of a bound 0^- two-nucleon state, and[8]

$$\langle \mathbf{d},j|D^5{}_\mu(x)|0\rangle=(2\pi)^{-3/2}(2d_0)^{-1/2}e^{+idx}U_\mu{}^{*(j)}(\mathbf{d})\,,$$

where $U_\mu{}^{(j)}(\mathbf{d})$ denotes the deuteron's polarization pseudovector. Near threshold, it is safe to assume that all ETC are free of time derivatives of δ functions.[9] Thus we write

$$\delta(x^0-y^0)[D^{50}(x),\psi(y)]=S(-i\boldsymbol{\partial}_x,-i\boldsymbol{\partial}_y)\delta(x-y)\bar\psi(x)\mathcal{C}\,, \tag{3}$$

where $\mathcal{C}=i\tau_2C$ characterizes the deuteron's triplet S, isospin-0 state at rest,[7,10] and $C=i\gamma^2\gamma^0$ is the charge conjugation matrix. By parity, S depends bilinearly on the momentum vectors.

The next step is to use the spatial Fourier transform of the Jacobi identity

$$[a\{bc\}]+\{c[ba]\}+\{b[ca]\}=0\,, \tag{4}$$

with $a=D^{50}(t,\mathbf{x})$, $b=\psi(t,\mathbf{y})$, and $c=\psi(t,\mathbf{z})$. Upon partial integration, using the canonical nucleon anticommutator, and $\{a,bc\}=b\{ac\}+[a,b]c=\{a,b\}c+b[ca]$, $\mathcal{C}^T=\mathcal{C}$, we find $S(\mathbf{p},\mathbf{q})=S(\mathbf{p},\mathbf{r})$ for $\mathbf{p}+\mathbf{q}+\mathbf{r}=0$. This implies, in view of $(\mathbf{p}+2\mathbf{q})^2=(\mathbf{p}+2\mathbf{r})^2$, that

$$S(\mathbf{p},\mathbf{q})=F((\mathbf{p}+2\mathbf{q})^2)\,, \tag{5}$$

with some scalar function F which we evaluate next. Factoring out the kinematical poles of the npd vertex $\Gamma_{npd}{}^\mu(p_1,p_2)$, we write

$$\begin{aligned}\int d^4x_1d^4x_2d^4x_3\,&\exp(ip_1x_1+ip_2x_2-idx_3)\\&\times\langle T(\psi_p(x_1)\psi_n(x_2)D_\mu{}^5(x_3))\rangle_0\\=S_F(p_1)&\Gamma_{npd}{}^\nu(p_1,p_2)\mathcal{C}S_F{}^T(p_2)\Delta_{\nu\mu}(d)\\&\times(2\pi)^4\delta(p_1+p_2-d)\,,\end{aligned} \tag{6}$$

with the propagators

$$\begin{aligned}\Delta_{\mu\nu}(d)&=\frac{-g_{\mu\nu}+d_\mu d_\nu/M_D{}^2}{d^2-M_D{}^2}\\iS_F(p)&=\int d^4x\,e^{ipx}\langle T(\psi(x)\bar\psi(0))\rangle_0=i(\gamma\cdot p-M)^{-1}.\end{aligned} \tag{7}$$

The deuteron Ward identity reads

$$\begin{aligned}\partial_\mu T(\psi(x_1)\psi(x_2)D^{5\mu}(x_3))&=T(\psi(x_1)\psi(x_2)\partial_\mu D^{5\mu}(x_3))\\&+T(\delta(x_3{}^0-x_1{}^0)[D^{50}(x_3),\psi(x_1)]\psi(x_2))\\&+T(\psi(x_1)\delta(x_3{}^0-x_2{}^0)[D^{50}(x_3),\psi(x_2)])\end{aligned} \tag{8}$$

and yields, together with (2), (3), and (6),

$$\begin{aligned}\Gamma_{npd}{}^\nu d_\nu=-M_D{}^2(\gamma\cdot p_1-M)[&-F((-\mathbf{d}+2\mathbf{p}_1)^2)S_F(-p_2)\\&+F((-\mathbf{d}+2\mathbf{p}_2)^2)S_F(p_1)](\gamma\cdot p_2+M)\,.\end{aligned} \tag{9}$$

In the deuteron rest frame $\mathbf{p}_1=-\mathbf{p}_2=\mathbf{p}$, $\mathbf{d}=0$, $\Gamma_{npd}{}^\mu$ reduces to the nonrelativistic wave function[11]

$$\Gamma_{npd}{}^\mu=A(\mathbf{p}^2)\gamma^\mu-\tfrac{1}{2}(p_1-p_2)^\mu B(\mathbf{p}^2)\,. \tag{10}$$

Herein

$$\begin{aligned}A(\mathbf{p}^2)=(8\pi/M)^{1/2}&(\mathbf{p}^2+BM)\\&\times[\tilde u(|\mathbf{p}|)+\tilde w(|\mathbf{p}|)/\sqrt{2}]\,,\end{aligned} \tag{11}$$

with the S- and D-wave functions in momentum space $\tilde u$ and $\tilde w$ of the deuteron, and its binding energy B. Comparison of (9) and (10) gives[12]

$$F(4\mathbf{p}^2)=-M_D{}^{-2}A(\mathbf{p}^2)\,, \tag{12}$$

while the derivative coupling term $B(\mathbf{p}^2)$ drops out of the Ward identity (9) because $d\cdot(p_1-p_2)=0$ at rest.

To justify (2), we transform the npd three-point function to its *nonrelativistic* ($c\to\infty$) *limit*, $d_0\to\infty$ while $\mathbf{d}$ is kept fixed, using *Bjorken's technique*.[13] The result is[9]

$$\begin{aligned}\int d^4x_1d^4x_2d^4x_3\,&\exp(-idx_3+ip_1x_1+ip_2x_2)\\&\times\{id_\mu\langle T(\psi(x_1)\psi(x_2)D^{5\mu}(x_3))\rangle_0\\&-\langle T(\delta(x_3{}^0-x_1{}^0)[D^{50}(x_3),\psi(x_1)]\psi(x_2))\rangle_0\\&-\langle T(\psi(x_1)\delta(x_3{}^0-x_2{}^0)[D^{50}(x_3),\psi(x_2)])\rangle_0\}\to0\,.\end{aligned} \tag{13}$$

In the nonrelativistic region, therefore, the deuteron structure is determined by pole diagrams alone while a nontrivial $\partial_\mu D^{5\mu}$, as a two-nucleon operator, would describe nucleon-nucleon rescattering effects.

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$$S(\mathbf{p},\mathbf{q})=F((\mathbf{p}+\mathbf{q})^2)\quad\text{and}\quad A(\mathbf{p}^2)=-M_D{}^2F(0).$$
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We may also note that additive nucleon resonance terms in (3) from the nucleon Regge trajectory produce nucleon-resonance poles and will take care of mesonic effects in view of duality.[14] It is now a simple matter to make use of the off-shell Ward identities for the $np \to \gamma d$ amplitude

$$\int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \exp(ip_1x_1+ip_2x_2-idx_3-ikx_4) \times\langle T(\psi_n(x_1)\psi_p(x_2)D_\nu^5(x_3)j_\mu(x_4))\rangle_0$$
$$=-i(2\pi)^4\delta(p_1+p_2-d-k)S_F(p_1) \times\Gamma_{npd\gamma}{}^{\lambda\rho}\mathcal{C}S_F{}^T(p_2)\Delta_{\lambda\mu}{}^{(\gamma)}(k)\Delta_{\rho\nu}(d). \quad (14)$$

It splits up into pole and nonpole terms of which the latter may be discarded at threshold because of the foregoing nonrelativistic theorem established by means of Bjorken's limit. The deuteron-pole diagram [Fig. 1(a)] is generated by the ETC

$$\delta(x^0-y^0)[D^{50}(x),j_8{}^\mu(y)]=eD^{5\mu}(y)\delta(x-y), \quad (15)$$

while the antisymmetrized nucleon-pole graph [Fig. 1(b)] is due to the ETC of (3).

Instead of (15) and Fig. 1(a), the pionic deuteron disintegration involves the ETC of the deuteron with the weak axial-vector current upon inserting partial conservation of axial-vector current (PCAC) for the pion field. Let us study this ETC.

To establish the threshold result

$$\delta(x^0-y^0)[A_\alpha{}^0(x),D^{50}(y)] =g_1\bar\psi(x)F((-i\boldsymbol{\partial}_y-2i\boldsymbol{\partial}_x)^2) \times\tfrac{1}{2}\tau^\alpha\gamma^5\gamma^0\mathcal{C}\bar\psi^T(y)\delta(x-y), \quad (16)$$

we use the Fourier transform of the Jacobi identity $[a[bc]]+[b[ca]]+[c[ab]]=0$ with $a=A_\alpha{}^0(t,\mathbf{x})$, $b=D^{50}(t,\mathbf{y})$, $c=\psi(t,\mathbf{z})$, insert the ETC

$$\delta(x^0-y^0)[A_\alpha{}^0(x),\psi(y)]=-\tfrac{1}{2}g_1\tau^\alpha\gamma^5\psi(x)\delta(x-y),$$

to get

$$\int d^3x d^3y d^3z \exp(i\mathbf{p}\cdot\mathbf{x}+i\mathbf{q}\cdot\mathbf{y}+i\mathbf{r}\cdot\mathbf{z}) \times[\psi(t,\mathbf{z})[A_\alpha{}^0(t,\mathbf{x}),D^{50}(t,\mathbf{y})]]$$
$$=-\tfrac{1}{2}g_1\tau^\alpha\gamma^5\mathcal{C}[F((\mathbf{q}+2\mathbf{r})^2)+F((\mathbf{q}+2\mathbf{p}+2\mathbf{r})^2)] \times\int d^3x \exp[(i\mathbf{p}+i\mathbf{q}+i\mathbf{r})\cdot\mathbf{x}]\bar\psi(t,\mathbf{x}). \quad (17)$$

Next we substitute (16), note that $[\psi,\bar\psi\Gamma]=\{\psi,\bar\psi\}\Gamma-\bar\psi\{\Gamma,\psi\}$, $\{\psi,\bar\psi F\}=\{\psi,\bar\psi\}F+\bar\psi[F,\psi]$, and then insert the Fourier expansion for $\bar\psi$ to verify (17).

It is tempting to generalize (16) by replacing $\gamma^0\to\gamma^\mu$ and $A_\alpha{}^0\to A_\alpha{}^\mu$ or $D^{50}\to D^{5\mu}$, this way constructing a deuteron vector operator.

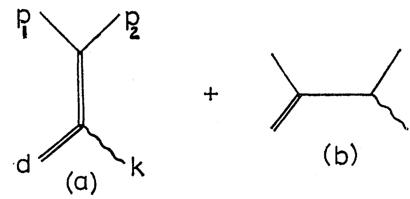

FIG. 1. Pole diagrams.

If we now integrate partially and substitute the Fourier expansion of the δ function involved in the foregoing ETC, the relevant pole term of the integrated Ward identity becomes

$$[F((\mathbf{d}+2\mathbf{k})^2)-F((2\mathbf{d}+2\mathbf{k})^2)]\int d^4p d^4x_1 d^4x_2 d^4x_3 d^4x_4 \times\exp[ip_1x_1+ip_2x_2-i(k+p)x_3-i(d-p)x_4] \times\langle T(\psi(x_1)\psi(x_2)\bar\psi_n(x_3)\bar\psi_m(x_4))\rangle_0(\tfrac{1}{2}\tau^\alpha\gamma^5\gamma^\mu\mathcal{C})_{nm}.$$

It obviously involves nucleon-nucleon scattering amplitudes. Hence it describes pion-induced nucleon-nucleon scattering which vanishes at threshold, however. Pion rescattering[15] and nucleon resonance admixtures to (3) and (16) will be studied in detail elsewhere.

Summarizing, we may state that Ward identities in conjunction with Bjorken's nonrelativistic limit establish for the deuteron the Low and Adler-Dothan theorems[16] in the sense that a finite sum of pole diagrams dominate the amplitude at *threshold* provided the relevant ETC are linear in the field operators. The pionic deuteron disintegration near threshold involves in addition pion-induced NN scattering because of the nonlinear nature of the ETC in (16).

Finally we note that, because pion-induced NN scattering seems to be intimately connected with the pseudovector nature of the deuteron, pion absorption on heavier nuclei may or may not reflect these features depending on the absorbing cluster. In fact, it seems[17] that the central potential plays a major role which, because of the one-body (impulse) approximation used for the pion absorption operator, produces vanishing rates for two uncorrelated absorbing nucleons and thus requires highly accurate final-state wave functions over the nuclear region to reproduce this zero overlap.

The advantage of hard-pion techniques for nuclear reactions including pickup (p,d) and stripping (d,p) seems at least twofold: They avoid perturbation theory as well as phenomenological interactions.

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