and

$$\langle iT(\Phi_a(x)\Phi_b(x'))\rangle_{\text{new}}=\delta_{ab}\Delta(x-x')[1+F_\pi^{-2}i\Delta(0)]. \quad (21b)$$

This alteration brings Eqs. (19a) and (19b) into agreement and verifies the invariance to order F_π^{-4}.

This work was begun at the Stanford Linear Accelerator Center and completed at the Aspen Center for Physics. I have enjoyed conversations with Y. P. Yao, S. Weinberg, and D. G. Boulware. I have had particularly fruitful discussions with S. Gasiorowicz.

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Reply to "On Summing Soft Pions"*

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IN his paper Brown[1] is correct in pointing out that my use[2] of the tree approximation in evaluating the effects of soft virtual pions introduces a dependence on the definition of the pion field. My statement to the contrary was an error, arising from a misuse of the theorem that the S matrix in the tree approximation is independent of the definition of the fields.

This error does not in my view affect the physical significance of the results. As emphasized in my original paper, there is no known current-algebra justification for using a chiral-invariant Lagrangian to deal with *virtual* pions. These calculations were offered only as a mathematical model, interesting chiefly because a large number of complicated Feynman diagrams could be explicitly summed. What Brown has shown is that this model is somewhat more arbitrary than I had thought.

However, Brown's comments raise some interesting questions. The S matrix for multi-soft-pion processes (like pion-pion scattering) can be calculated with confidence by using a chiral-invariant Lagrangian in the tree approximation, because current-algebra tells us that these S-matrix elements are uniquely determined by the chiral invariance of the Lagrangian, and determined, moreover, to be the lowest order in the "coupling constant" $1/F_\pi$. Is it possible that current algebra could also be used to validate the tree approximation for Feynman amplitudes involving soft pions off the mass shell? It now appears[3] that this may be possible, but only for one particular definition of the pion field, namely, that used in the "nonlinear σ model." If this is so, then it would be natural to calculate the effects of soft virtual pions by using the tree approximation, but only for the nonlinear σ-model definition of the pion field, rather than the Callan, Coleman, Wess, and Zumino[4] (CCWZ) definition. A nonlinear differential equation would then need to be solved in order to take account of the pion-pion interactions, but pionic loops would affect the final result only through a renormalization of F_π. Detailed calculations will have to be performed to see whether these ideas will prove useful.

There is just one point where my paper went beyond mathematical models to make a prediction about the real world. I noted that in the eikonal approximation, the matrix element for the emission of any number of soft real pions in a given process would vanish if the S matrix for that process commutes with the chiral matrix $\mathbf{X}$. The emission of pions by the terms in the S matrix which do not commute with $\mathbf{X}$ might be expected, via unitarity, to suppress those terms, so that the S matrix for any hard-hadron process would approximately commute with $\mathbf{X}$. It is important to note that this argument dealt with *real* soft pions, and is therefore unaffected by Brown's comments. In addition, the model for soft-virtual-pion effects worked out in my paper shows, *even without using the tree approximation*, that soft virtual pions have no effect on the terms in the hard-hadron S matrix which commute with $\mathbf{X}$. Any suppression of the S matrix by virtual-pion exchange can affect only those terms of the S matrix which do not commute with $\mathbf{X}$, and would thus yield an approximately chiral-invariant S matrix. Only the explicit formulas for the suppression factors given in my paper do depend on the use of the tree approximation with the CCWZ pion field.

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[1] Lowell S. Brown, preceding paper, Phys. Rev. D 2, 3083 (1970).

[2] S. Weinberg, Phys. Rev. D 2, 674 (1970).

[3] I. Gerstein, R. Jackiw, B. W. Lee, and S. Weinberg (unpublished).

[4] C. G. Callan, S. Coleman, J. W. Wess, and B. Zumino, Phys. Rev. 177, 2247 (1968).