(as in the discussion of $\Pi^{(1)}$ and $\Pi^{(2)}$). If this part of the single logarithm vanishes for a special value of α, all higher powers of the logarithm will also cancel for the same value of α. For the higher powers are generated by inserting the original self-energies back into the original graphs. The vanishing of the single logarithmic terms is like the condition $F_1(\alpha')=0$, but the set of graphs considered is different. The exact connection between the two conditions remains to be discovered, but presumably the two functions are related in such a way that they vanish at the same point.

ACKNOWLEDGMENTS

The author wishes to express his thankfulness to Professor D. R. Yennie for suggesting the problem and many helpful suggestions, and to Dr. P. K. Kuo for many illuminating discussions.

PHYSICAL REVIEW D VOLUME 2, NUMBER 12 15 DECEMBER 1970

Energy-Momentum Tensors and Scale Invariance in the Thirring Model

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(Received 18 June 1970)

It is shown that in the Thirring model the Sugawara form of the energy-momentum tensor, $T_S^{\mu\nu}$, is a more natural choice than the usual form in terms of canonical fields, $T_C^{\mu\nu}$, in the sense that $\int x_\mu T_S^{\mu 0}(x)dx^1$ is the generator of scale transformations which leave the solution of the model invariant.

I. INTRODUCTION

THE Thirring model[1] is an explicitly solvable field theory in two-dimensional space-time. The fact that a solution is available in closed form makes it a convenient testing ground for ideas which may be relevant in more complicated theories. In this paper, we will investigate the commutation relations of the energy-momentum tensor with the fields in the theory.

Any expression for an energy-momentum tensor involves products of local fields at the same point, which are not well-defined quantities. So our first task must be to define precisely what we mean by these products. Then we can compute the commutation relations using the Bjorken-Johnson-Low (BJL) technique.[2]

In Sec. II, we will define the model, write down a solution, and comment on its behavior under scale transformations. In Sec. III we will define the two forms of the energy-momentum tensor and write down some commutators. The detailed calculations will be given in the Appendix. Section IV will contain conclusions and comments.

II. DEFINITION OF MODEL

The equation of motion in the Thirring model is[3]

$$-i\gamma^\mu\partial_\mu\psi(x)=\sigma\gamma^\mu j_\mu(x)\psi(x)\,,$$

with the product defined in the following sense:

$$j_\mu(x)\psi(x)=\lim_{\eta_\mu\to 0}[j_\mu(x+\eta)\psi(x)+j_\mu(x-\eta)\psi(x)]\,,$$

where η_μ is an arbitrary two-vector. Here j^μ is the current associated with gauge transformations:

$$\begin{aligned}j^0(x)&=\lim_{\eta_\mu\to 0}(1/2)[\bar\psi(x+\eta)\gamma^0\psi(x-\eta)\\&\qquad+\bar\psi(x-\eta)\gamma^0\psi(x+\eta)]\,,\\ j^1(x)&=\lim_{\eta_\mu\to 0}(1/2\lambda)[\bar\psi(x+\eta)\gamma^1\psi(x-\eta)\\&\qquad+\bar\psi(x-\eta)\gamma^1\psi(x+\eta)]\,,\end{aligned}\tag{1}$$

where in performing the limit we keep $\eta^0=0$, and where $\lambda=1+\sigma/\pi$. This defines the model completely, and we can write down the solution in the form of the n-fermion Green's functions[1]:

$$\begin{aligned}&G(x_1\cdots x_n,y_1\cdots y_n)\\&=i^n\langle 0|T(\psi(x_1)\cdots\psi(x_n)\bar\psi(y_n)\cdots\bar\psi(y_1))|0\rangle\\&=\exp\{i\sigma[n(1-\lambda^{-1})\Delta_F(0)\\&\qquad+\sum_{1\le i<j\le n}(1-\lambda^{-1}\gamma^5{}_i\gamma^5{}_j)\Delta_F(x_i-x_j)\\&\qquad+\sum_{1\le i<j\le n}(1-\lambda^{-1}\gamma^5{}_{i'}\gamma^5{}_{j'})\Delta_F(y_i-y_j)\\&\qquad-\sum_{i,j=1}^{n}(1-\lambda^{-1}\gamma^5{}_i\gamma^5{}_{j'})\Delta_F(x_i-y_j)]\}\\&\qquad\qquad\times G_0(x_1\cdots x_n,y_1\cdots y_n)\,,\end{aligned}\tag{2}$$

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[1] C. M. Sommerfield, Ann. Phys. (N.Y.) **26**, 1 (1963), and references cited therein.

[2] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966); K. Johnson and F. Low, Progr. Theoret. Phys. (Kyoto) Suppl. **37**, 74 (1966).

[3] We use $-g^{00}=g^{11}=1$, $\gamma^0=\sigma_2$, $\gamma^1=i\sigma_1$, and $\gamma^5=\gamma^0\gamma^1=\sigma_3$; $\epsilon^{\mu\nu}$ is completely antisymmetric; $\epsilon^{01}=1$.

where

$$\Delta_F(x)=\frac{1}{(2\pi)^2}\int\frac{e^{ipx}d^2p}{p^2-i\epsilon}$$

and

$$G_0(x_1\cdots x_n,y_1\cdots y_n)=\det\begin{bmatrix}G_0(x_1-y_1) & \cdots & G_0(x_1-y_n)\\ \vdots & \vdots & \vdots\\ G_0(x_n-y_1) & \cdots & G_0(x_n-y_n)\end{bmatrix}$$

with

$$G_0(x)=i\gamma^\mu\partial_\mu\Delta_F(x).$$

The subscripts on the γ^5's specify the spinor index on which the matrix multiplication acts. In each term of the expansion of the determinant, $\gamma^5{}_i$ acts on the G_0 whose argument contains x_i, and $\gamma^5{}_{i'}$ acts on the G_0 whose argument contains y_i.

We can now derive vacuum expectation values for time-ordered products involving currents by performing the limits indicated in (1) using the explicit forms

$$\begin{aligned}G_0(x)&=(1/2\pi)\gamma^\mu x_\mu/x^2,\\ \Delta_F(x)&=-(i/4\pi)\ln(x^2)+C+i\epsilon,\end{aligned}\qquad(3)$$

where C is a constant. The results which we will need are[4]

$$\begin{aligned}\{j^\mu\}_{x,y}&=i\langle 0|T(j^\mu(\xi)\psi(x)\bar\psi(y))|0\rangle\\ &=(\partial^\mu+\epsilon^\mu{}_\nu\lambda^{-1}\gamma^5\partial^\nu)\\ &\quad\times[\Delta_F(\xi-x)-\Delta_F(\xi-y)]G(x-y),\end{aligned}\qquad(4)$$

$$\begin{aligned}D^{\mu\nu}(\xi,\xi')&=i\langle 0|T(j^\mu(\xi)j^\nu(\xi'))|0\rangle,\\ D^{00}(\xi,\xi')&=D^{11}(\xi,\xi')=-(1/\pi\lambda)\partial^1\partial^1\Delta_F(\xi-\xi'),\\ D^{01}(\xi,\xi')&=D^{10}(\xi,\xi')=-(1/\pi\lambda)\partial^1\partial^0\Delta_F(\xi-\xi'),\end{aligned}\qquad(5)$$

$$\begin{aligned}&i\langle 0|T(j^\mu(\xi_1)j^\nu(\xi_2)\psi(x)\bar\psi(y))|0\rangle\\ &=(\partial^\mu+\lambda^{-1}\gamma^5\epsilon^\mu{}_\kappa\partial^\kappa)[\Delta_F(\xi_1-x)-\Delta_F(\xi_1-y)]\\ &\quad\times(\partial^\nu+\lambda^{-1}\gamma^5\epsilon^\nu{}_\kappa\partial^\kappa)[\Delta_F(\xi_2-x)-\Delta_F(\xi_2-y)]\\ &\quad\times G(x-y)-iD^{\mu\nu}(\xi_1,\xi_2)G(x-y).\end{aligned}\qquad(6)$$

As we might have suspected from the equation of motion, since σ is dimensionless, the solution is scale invariant. All the expressions for vacuum expectation values will be unchanged if we multiply each coordinate by a real number s and at the same time multiply the entire expression by $s^{2n\beta+m}$, where n is the number of ψ's (and hence $\bar\psi$'s) and m the number of j's in the vacuum expectation value, with $\beta=(1+\lambda^2)/4\lambda$. Notice that the interacting fermion field does not behave in the same way as the free fermion field.[5] A theory of free massless fermions in two dimensions would be scale invariant with $\beta=1/2$. In what follows, we will refer to β as the dimension of the field.

[4] We use the convention that the partial-derivative operator ∂^μ acts on the first variable occurring after it. Thus $\partial^\mu\equiv\partial/\partial\xi_\mu$ in Eqs. (4) and (5); $\partial^\mu\equiv\partial/\partial(\xi_1)_\mu$ in the first term on the right of Eq. (6); etc.

[5] See also K. G. Wilson, Phys. Rev. D 2, 1473 (1970).

III. TWO FORMS OF ENERGY-MOMENTUM TENSOR

The canonical form for the energy-momentum tensor is given by[1]

$$\begin{aligned}T_C{}^{00}(x)&=T_C{}^{11}(x)\\ &=-\tfrac12 i\bar\psi(x)\gamma^1\partial^1\psi(x)+\tfrac12 i\partial^1\bar\psi(x)\gamma^1\psi(x)\\ &\quad+\tfrac12\sigma[j^0(x)j^0(x)-\lambda j^1(x)j^1(x)],\\ T_C{}^{01}(x)&=T_C{}^{10}(x)\\ &=-\tfrac12 i\bar\psi(x)\gamma^0\partial^1\psi(x)+\tfrac12 i\partial^1\bar\psi(x)\gamma^0\psi(x).\end{aligned}\qquad(7)$$

We assign meaning to the products involved in the following way. To compute a matrix element of this tensor, separate the points in the product arbitrarily and throw out all terms which may be singular or ill-defined in the limit as the points come together. What remains will have a unique limit, which is the desired matrix element. When we compute commutation relations using this prescription, we find that it gives the same results as formal use of the canonical commutation relations. The results are as follows (the calculation will be given in the Appendix):

$$\begin{aligned}&[T_C{}^{00}(x^1,0),\psi(0)]\\ &=i\gamma^5\partial^1\psi(0)\delta(x^1)-\sigma j^0(0)\psi(0)\delta(x^1)\\ &\quad+\sigma j^1(0)\gamma^5\psi(0)\delta(x^1)-\tfrac12 i\gamma^5\psi(0)\partial^1\delta(x^1)\\ &=i\partial^0\psi(0)\delta(x^1)-\tfrac12 i\gamma^5\psi(0)\partial^1\delta(x^1),\\ &[T_C{}^{01}(x^1,0),\psi(0)]=i\partial^1\psi(0)\delta(x^1)-\tfrac12 i\psi(0)\partial^1\delta(x^1).\end{aligned}\qquad(8)$$

There are two features of the Thirring model which allow us to write a Sugawara form for the energy-momentum tensor. First,

$$[j^0(x^1,0),j^1(0)]=-(i/\pi\lambda)\partial^1\delta(x^1),$$

that is, the Schwinger term is a c number, and second, we can write the equation of motion in the following form[6]:

$$\partial_\mu\psi(x)=i\lambda\pi j_\mu(x)\psi(x)+i\pi\gamma^5\tilde j_\mu(x)\psi(x),\qquad(9)$$

where $\tilde j_\mu=\epsilon_\mu{}^\nu j_\nu$. The Sugawara form is the following[7]:

$$T_S{}^{\mu\nu}(\xi)=\pi\lambda[j^\mu(\xi)j^\nu(\xi)-\tfrac12 g^{\mu\nu}j^k(\xi)j_k(\xi)],\qquad(10)$$

with the following prescription for handling the products: To find a commutator, look at matrix elements involving the Sugawara form with the points split *in the space component only*. Then use the BJL technique and afterwards let the points come together symmetrically.[8] The results are

$$\begin{aligned}&[T_S{}^{00}(\xi^1,0),\psi(0)]=j\partial^0\psi(0)\delta(\xi^1)-\tfrac12 i\gamma^5\psi(0)\partial^1\delta(\xi^1),\\ &[T_S{}^{01}(\xi^1,0),\psi(0)]=j\partial^1\psi(0)\delta(\xi^1)-i\frac{1+\lambda^2}{4\lambda}\psi(0)\partial^1\delta(\xi^1).\end{aligned}\qquad(11)$$

[6] C. M. Sommerfield, Phys. Rev. 176, 2019 (1968).

[7] H. Sugawara, Phys. Rev. 170, 1659 (1968). See also Ref. 6 above and C. G. Callan, R. F. Dashen, and D. H. Sharp, Phys. Rev. 165, 1883 (1968).

[8] This is the analog of the procedure suggested by Schwinger for defining the electromagnetic current, in J. Schwinger, Phys. Rev. Letters 3, 296 (1959).

IV. CONCLUSIONS AND COMMENTS

Looking at (7) and (11), we see that both $D_S=\int x_\mu T_S{}^{\mu 0}(x)dx^1$ and $D_C=\int x_\mu T_C{}^{\mu 0}(x)dx^1$ generate scale transformations, but the transformations generated by D_S have $\beta=(1+\lambda^2)/4\lambda$, while those generated by D_C have $\beta=1/2$. Both D_S and D_C commute with the Hamiltonian; they are formed from conserved currents, since both $T_S{}^{\mu\nu}$ and $T_C{}^{\mu\nu}$ are traceless, at least formally. But while D_S generates an invariance of the theory, D_C does not. What is wrong? One possibility, since there are massless particles in the theory, is that the vacuum is not invariant under $\exp(i\alpha D_C)$.[9] It is also possible that the form we wrote down for $T_C{}^{\mu\nu}$ is not quite consistent. In deriving it, Sommerfield[1] eliminated time derivatives from $T_C{}^{11}$ by making use of the field equations and by combining products of fields into currents without using the same limiting procedures necessary in the rest of his development of the theory. This would not affect what we have written down, but might mean that $T_C{}^{\mu\nu}$ is not really traceless, allowing us to shift the blame for noninvariance from the vacuum to the Hamiltonian.

We have looked without success for some other way of handling the singular products in $T_C{}^{\mu\nu}$ which would give both the right Lorentz generators, and the correct dimension for the interacting field. Certainly no procedure for splitting points will do it.

Our interpretation of the singular products in $T_S{}^{\mu\nu}$ contrasts sharply with that of Coleman, Gross, and Jackiw.[10] They show that in a theory of free, massless fermions in two dimensions, $T_S{}^{\mu\nu}$ reduces to $T_C{}^{\mu\nu}$ when one lets the points approach each other from purely spatial separation and from purely time separation and averages the results. Our procedure, which can also be carried out in the free theory, differs in two respects. We allow only spatial separation (and hence $T_S{}^{\mu\nu}$ does not reduce to exactly $T_C{}^{\mu\nu}$), and we let the points come together only after we have computed commutation relations.

Our results indicate that, in two dimensions at least, the Sugawara form may yield significant information about the dimension of the interacting fields.

ACKNOWLEDGMENTS

The author wishes to thank Professor Sommerfield for suggesting this interesting line of investigation.

APPENDIX

Here we give the calculations of

$$[T_C{}^{01}(\xi^1,0),\psi(0)] \quad \text{and} \quad [T_S{}^{01}(\xi^1,0),\psi(0)].$$

From (2) we find

$$\begin{aligned}
&\langle 0|\,T(\bar\psi(\xi)\gamma^0\psi(\bar\xi)\psi(0)\bar\psi(x))\,|0\rangle \\
&\quad= -\exp\{i\sigma(1-\lambda^{-1})[2\Delta_F(0)-\Delta_F(\xi-\bar\xi)-\Delta_F(\bar\xi-x)+\Delta_F(\xi-x)+\Delta_F(\bar\xi)-\Delta_F(\xi)-\Delta_F(x)]\}G_0(-\xi)\gamma^0G_0(\bar\xi-x) \\
&\qquad -2\exp\{i(\sigma/\lambda)[\Delta_F(\bar\xi)-\Delta_F(\xi)-\Delta_F(\bar\xi-x)+\Delta_F(\xi-x)]\}\sin\{(\sigma/\lambda)[\Delta_F(\bar\xi)-\Delta_F(\xi)-\Delta_F(\bar\xi-x)+\Delta_F(\xi-x)]\} \\
&\qquad\qquad \times\exp\{i\sigma(1-\lambda^{-1})[2\Delta_F(0)-\Delta_F(\bar\xi-\xi)-\Delta_F(x)]\}\bar\partial^1\Delta_F(\bar\xi-\xi)\gamma^5G_0(-x)\,, \qquad \text{(A1)}
\end{aligned}$$

where we have assumed for convenience that $\xi^0=\bar\xi^0$, and we have done the matrix multiplication. To find $\langle 0|\,T(T_C{}^{01}(\xi)\psi(0)\bar\psi(x))\,|0\rangle$, we take $-\frac{1}{2}i\bar\partial^1$ and $\frac{1}{2}i\partial^1$ acting on the above and let $\bar\xi-\xi\to 0$, throwing away terms like $\bar\partial^1\Delta_F(\bar\xi-\xi)$ which are singular in the limit. We also use the following identity (true in two dimensions):

$$\begin{aligned}
&G_0(-\xi)\gamma^0G_0(\bar\xi-x) \\
&\qquad= -[G_0(\xi)-G_0(\bar\xi-x)]\gamma^0G_0(\bar\xi-\xi-x)\,. \qquad \text{(A2)}
\end{aligned}$$

The result is[4]

$$\begin{aligned}
&\langle 0|\,T(T_C{}^{01}(\xi)\psi(0)\bar\psi(x))\,|0\rangle = i[G_0(\xi)-G_0(\xi-x)] \\
&\quad\times\gamma^0\exp\{i\sigma(1-\lambda^{-1})[\Delta_F(0)-\Delta_F(x)]\}\partial^1G_0(-x) \\
&\quad+\tfrac{1}{2}i\partial^1[G_0(\xi)+G_0(\xi-x)]\gamma^0G(-x)+\sigma(1-\lambda^{-1}) \\
&\quad\times\{\partial^1[\Delta_F(\xi)-\Delta_F(\xi-x)]\} \\
&\qquad\qquad\times[G_0(\xi)-G_0(\xi-x)]\gamma^0G(-x)\,. \qquad \text{(A3)}
\end{aligned}$$

Now to use BJL, we define

$$\begin{aligned}
&M_C{}^{01}(p,k) \\
&\quad=\int\langle 0|\,T(T_C{}^{01}(\xi)\psi(0)\bar\psi(x))\,|0\rangle e^{-ip\xi}e^{ikx}d^2\xi d^2x \\
&\quad=iG_0(p)\gamma^0[K(k)-K(k-p)] \\
&\qquad+i\sigma(1-\lambda^{-1})\int G_0(p-q)q^1\Delta_F(q)\gamma^0 \\
&\qquad\quad\times[G(k)-G(k-q)-G(k-p+q)+G(k-p)]\frac{d^2q}{(2\pi)^2} \\
&\qquad\qquad-\tfrac{1}{2}p^1G_0(p)\gamma^0[G(k)+G(k-p)]\,, \qquad \text{(A4)}
\end{aligned}$$

where

$$\begin{aligned}
K(k)=\int&\exp\{i\sigma(1-\lambda^{-1})[\Delta_F(0)-\Delta_F(x)]\} \\
&\qquad\times[\partial^1{}_xG_0(-x)]e^{ikx}d^2x\,. \qquad \text{(A5)}
\end{aligned}$$

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[9] J. Goldstone, Nuovo Cimento **19**, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962).

[10] S. Coleman, D. Gross, and R. Jackiw, Phys. Rev. **180**, 1359 (1969).

Now we expand $M_C^{01}(p,k)$ in powers of $1/p^0$, and pick out $N_C^{01}(p^1,k)$, the coefficient of $1/p^0$:

$$N_C^{01}(p^1,k)=-iK(k)-i\sigma(1-\lambda^{-1})\int q^1\Delta_F(q)\times[G(k)-G(k-q)]\frac{d^2q}{(2\pi)^2}+\tfrac{1}{2}p^1G(k). \quad (A6)$$

Now BJL tells us that

$$\langle 0|T([T_C^{01}(\xi^1,0),\psi(0)]\bar\psi(x))|0\rangle =-i\int N_C^{01}(p^1,k)e^{ip^1\xi^1}e^{-ikx}\frac{dp^1}{(2\pi)}\frac{d^2k}{(2\pi)^2}$$
$$=-\partial_x{}^1G(-x)\delta(\xi^1)-\tfrac{1}{2}G(-x)\partial^1\delta(\xi^1) \quad (A7)$$

or

$$[T_C^{01}(\xi^1,0),\psi(0)]=i\partial^1\psi(0)\delta(\xi^1)-\tfrac{1}{2}i\psi(0)\partial^1\delta(\xi^1), \quad (A8)$$

which is the second result in (8).

Using (6), we find, for

$$T_S^{01}(\xi,\epsilon)=\tfrac{1}{2}\pi\lambda[j^0(x+\epsilon)j^1(x-\epsilon)+j^0(x-\epsilon)j^1(x+\epsilon)], \quad (A9)$$

$$\begin{aligned}\langle 0|T(T_S^{01}(\xi,\epsilon)\psi(0)\bar\psi(x))|0\rangle &=-i\tfrac{1}{2}\pi\lambda\{[(\partial^0+\lambda^{-1}\gamma^5\partial^1)(\Delta_F(\xi+\epsilon)-\Delta_F(\xi+\epsilon-x))]\\&\times[(\partial^1+\lambda^{-1}\gamma^5\partial^0)(\Delta_F(\xi-\epsilon)-\Delta_F(\xi-\epsilon-x))]\\&+[(\partial^1+\lambda^{-1}\gamma^5\partial^0)(\Delta_F(\xi+\epsilon)-\Delta_F(\xi+\epsilon-x))]\\&\times[(\partial^0+\lambda^{-1}\gamma^5\partial^1)(\Delta_F(\xi-\epsilon)\\&-\Delta_F(\xi-\epsilon-x))]\}G(-x),\end{aligned} \quad (A10)$$

where $\epsilon^0=0$, $\epsilon^1=\epsilon$. As before, we find the Fourier transform

$$\begin{aligned}M_S^{01}(p,k)&=i\tfrac{1}{2}\pi\lambda\int\{(q^0+\lambda^{-1}\gamma^5q^1)[(p^1-q^1)+\lambda^{-1}\gamma^5(p^0-q^0)]\\&+(q^1+\lambda^{-1}\gamma^5q^0)[(p^0-q^0)+\lambda^{-1}\gamma^5(p^1-q^1)]\}\\&\times\Delta_F(q)\Delta_F(p-q)e^{i(p^1-2q^1)\epsilon}[G(k)-G(k-q)\\&-G(k-p+q)+G(k-p)]\frac{d^2q}{(2\pi)^2}.\end{aligned} \quad (A11)$$

Then

$$\begin{aligned}N_S^{01}(p,k)&=i\pi\lambda\int[(q^1+\lambda^{-1}\gamma^5q^0)+\lambda^{-1}\gamma^5(q^0+\lambda^{-1}\gamma^5q^1)]\\&\times\Delta_F(q)G(k-q)\cos[(p^1-2q^1)\epsilon]\frac{d^2q}{(2\pi)^2}\\&-\tfrac{1}{2}i\pi\lambda\int[(q^1+\lambda^{-1}\gamma^5q^0)+\lambda^{-1}\gamma^5(q^1+\lambda^{-1}\gamma^5q^0)]\\&\times\Delta_F(q)e^{-2iq^1\epsilon}\frac{d^2q}{(2\pi)^2}(e^{ip^1\epsilon}-e^{-ip^1\epsilon})G(k).\end{aligned} \quad (A12)$$

In the first term we can take the $\epsilon\to 0$ limit inside the integral. In the second, the q^0 terms do not contribute, so it is

$$\begin{aligned}&-\tfrac{1}{2}i\pi\lambda(1+\lambda^{-2})\int q^1\Delta_F(q)e^{-iq^12\epsilon}\frac{d^2q}{(2\pi)^2}(e^{ip^1\epsilon}-e^{-ip^1\epsilon})G(k)\\&=\tfrac{1}{2}\pi\lambda(1+\lambda^{-2})\left[\frac{\partial}{\partial\epsilon}\Delta_F(2\epsilon)\right](e^{ip^1\epsilon}-e^{-ip^1\epsilon})G(k)\\&=-\frac{i\lambda}{8\epsilon}(1+\lambda^{-2})(e^{ip^1\epsilon}-e^{-ip^1\epsilon})G(k).\end{aligned} \quad (A13)$$

Now as $\epsilon\to 0$, this goes to

$$\frac{(1+\lambda^2)}{4\lambda}p^1G(k).$$

So

$$\begin{aligned}\langle 0|T([T_S^{01}(\xi^1,0),\psi(0)]\bar\psi(x))|0\rangle&=i\pi\lambda\delta(\xi^1)\{[(\partial^1+\lambda^{-1}\gamma^5\partial^0)+\lambda^{-1}\gamma^5(\partial^0+\lambda^{-1}\gamma^5\partial^0)]\\&\times\Delta_F(-x)\}G(-x)-\frac{(1+\lambda^2)}{4\lambda}\partial^1\delta(\xi^1)G(-x).\end{aligned} \quad (A14)$$

Now using (4) and (9), we find

$$[T_S^{01}(\xi^1,0),\psi(0)]=i\partial^1\psi(0)\delta(\xi^1)-i\frac{1+\lambda^2}{4\lambda}\psi(0)\partial^1\delta(\xi^1). \quad (A15)$$