Comments and Addenda

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Unitarity and the Phase of the Mixing Parameter in Superweak Theories

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The Bell-Steinberger unitarity condition is used to discuss the relation between the phase ϕ_W of the "unitarity sum" appearing in that relation and the Lee-Wolfenstein phase ϕ_ϵ, for which an explicit formula is given in terms of $|\epsilon|$, γ_1, γ_2, and m_2-m_1.

ASSUMING *TCP* invariance, the short- and long-lived neutral kaon states $|K_1^0\rangle$ and $|K_2^0\rangle$ can be written in the form[1]

$$|K_{1,2}^0\rangle=[2(1+|\epsilon|^2)]^{-1/2}\times[(1+\epsilon)|K^0\rangle\pm(1-\epsilon)|\bar{K}^0\rangle]. \quad (1)$$

Lee and Wolfenstein[2] showed that if all transition matrix elements from K^0 and $\bar{K}^0$ states satisfy the requirements of *CP* invariance, all *CP*-nonconserving effects in neutral kaon decay are determined by the parameter ϵ. They also gave an implicit formula for the phase of ϵ in such a theory which reduces, when $|\epsilon|^2$ can be neglected in comparison with unity, to the simple expression

$$\phi_\epsilon\approx\tan^{-1}[2(m_2-m_1)/(\gamma_1-\gamma_2)]. \quad (2)$$

This result was obtained by analyzing the structure of the effective Hamiltonian[1,3] which determines the time evolution of the K^0-$\bar{K}^0$ system in the Weisskopf-Wigner approximation. On the other hand, discussions based on the Bell-Steinberger unitarity relation,[4,5]

$$[\tfrac{1}{2}(\gamma_1+\gamma_2)+i(m_2-m_1)]\langle K_1^0|K_2^0\rangle =\sum_f \langle f|T|K_2^0\rangle\langle f|T|K_1^0\rangle^*\equiv\sum_f \eta_f\gamma_{1f}, \quad (3)$$

naturally involve the phase

$$\phi_\omega=\tan^{-1}[2(m_2-m_1)/(\gamma_1+\gamma_2)]. \quad (4)$$

Some authors even state, erroneously, that the phase of ϵ is given by Eq. (4). The difference between ϕ_ϵ and ϕ_W disappears if γ_2 is negligible in comparison with γ_1, an approximation which is well justified by the measured K_1^0 and K_2^0 lifetimes. Nevertheless, it is interesting to examine the relation between ϕ_ϵ and ϕ_W as a logical question since the physical conditions which determine Eq. (2) already incorporate the requirement of unitarity. Unitarity is assured by the Hermiticity of the primary Hamiltonian from which the effective Hamiltonian is derived; it is well known that the Weisskopf-Wigner approximation preserves this feature of the theory. In fact, Eq. (3) could be derived directly from the effective Hamiltonian.[6,7] Also, there are similar situations, e.g., the ρ-ω system, where (γ_2/γ_1) is not as small and the difference between equations corresponding to (2) and (4) can be significant.

The sum in Eq. (3) extends over any complete set of states. It will be convenient to choose these as self-conjugate states

$$\Theta|\alpha\rangle=\lambda_\alpha|\alpha\rangle$$

(where $\lambda_\alpha=\pm1$ and $\Theta\equiv TCP$ replaces all particles by their corresponding antiparticles with reversed helicity) which are also scattering eigenstates if weak interactions could be neglected. In terms of these states, the re-

[1] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).
[2] T. D. Lee and L. Wolfenstein, Phys. Rev. **138**, B1490 (1965).
[3] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 380 (1964).
[4] J. S. Bell and J. Steinberger, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford Laboratory, Chilton, Didcot, Berks, 1966).
[5] Despite alarms that an *S*-matrix approach might lead to a *different* unitarity relation, it has been shown that Eq. (3) holds exactly in that case also; see G. C. Wick, Phys. Letters **30B**, 126 (1969); and L. Wolfenstein, Phys. Rev. **188**, 2536 (1969).
[6] T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **16**, 519 (1966).
[7] P. K. Kabir, *The CP Puzzle* (Academic, London, 1968), p. 107.

requirement of TCP invariance can be stated in the form

$$\langle\alpha|T|\bar{K}^0\rangle=\lambda_\alpha A_\alpha{}^*e^{i\delta_\alpha} \tag{5}$$

if

$$\langle\alpha|T|K^0\rangle=A_\alpha e^{i\delta_\alpha}. \tag{6}$$

Here δ_α is the scattering eigenphase in the channel α. CP invariance requires $A_\alpha=\mathrm{Re}A_\alpha$. Combining Eqs. (5) and (6) with Eq. (1), the right-hand side of Eq. (3) can be expressed in terms of ϵ and the A_α's and, if $A_\alpha=\mathrm{Re}A_\alpha$ for all channels, assumes the simple form

$$2(1+|\epsilon|^2)^{-1}(\epsilon\sum_{\lambda_\alpha=+1}A_\alpha{}^2+\epsilon^*\sum_{\lambda_\alpha=-1}A_\alpha{}^2).$$

The imaginary part of Eq. (3) can then be written as

$$(m_2-m_1)\,\mathrm{Re}\epsilon=\mathrm{Im}\epsilon\,(\sum_\alpha\lambda_\alpha A_\alpha{}^2). \tag{7}$$

Beside Eq. (3), unitarity also requires

$$\gamma_{1,2}=\sum_k|\langle k|T|K_{1,2}{}^0\rangle|^2. \tag{8}$$

If we again choose the complete set k as the set of self-conjugate channels α, we obtain

$$\gamma_1-\gamma_2=2\left(\frac{1-|\epsilon|^2}{1+|\epsilon|^2}\right)(\sum_\alpha\lambda_\alpha A_\alpha{}^2). \tag{9}$$

Equations (7) and (9) yield

$$\tan\phi_\epsilon=\frac{\mathrm{Im}\epsilon}{\mathrm{Re}\epsilon}=2\left(\frac{1-|\epsilon|^2}{1+|\epsilon|^2}\right)\left(\frac{m_2-m_1}{\gamma_1-\gamma_2}\right). \tag{10}$$

Thus, while unitarity and TCP invariance require the sum on the right-hand side of Eq. (3) to have the phase ϕ_W, Eq. (4), the same conditions require ϵ to have the phase ϕ_ϵ given above,[8] which could also have been deduced directly from Eq. (65) of Ref. 2. Equation (10) admits two solutions for ϕ_ϵ differing by 180°; thus it might be thought that a sign factor, e.g., the sign of $\cos\phi_\epsilon$, is an additional independent parameter.[9] However, this can always be taken positive *by convention*. The redefinitions $|K^0\rangle\leftrightarrow|\bar{K}^0\rangle$, $\epsilon\to-\epsilon$, $|K_1{}^0\rangle\to|K_1{}^0\rangle$, and $|K_2{}^0\rangle\to-|K_2{}^0\rangle$ leave Eq. (1) unchanged; hence it is possible to regard $\mathrm{Re}\epsilon>0$ as the *convention* which defines the distinction between K^0 and $\bar{K}^0$ states.[10]

Consequently, in superweak theories (defined by $A_\alpha=\mathrm{Re}A_\alpha$ for all channels), all CP-noninvariant phenomena in neutral kaon decays are determined by the single real parameter $|\epsilon|$.

[8] If all channels α could be characterized by the same value of λ_α, we would have $\tan\phi_\epsilon=\lambda_\alpha\tan\phi_\omega$.

[9] In this connection, see the discussion of M. Nauenberg, T. D. Lee, C. N. Yang, and L. Wolfstein, in *Proceedings of the Thirteenth International Conference on High Energy Physics* (University of California Press, Berkeley, 1967), p. 81.

[10] By a happy coincidence, this convention agrees with the usua terrestrial definition of K^0 and $\bar{K}^0$ states.

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Veneziano Model for the Reaction $\pi^-p\to\eta n$†

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A Veneziano model is proposed for the reaction $\pi^-p\to\eta n$. This reaction is interesting since only the A_2 trajectory can contribute in the t channel, and only $T=\frac{1}{2}$ trajectories appear in the s and u channels. We limit the terms in our model to those which have leading asymptotic behavior in two channels. We emphasize the intimate connection between resonance parameters and both forward and backward cross sections.

I. INTRODUCTION

In this paper, we consider a Veneziano-type model for the reaction $\pi^-p\to\eta n$. Our main purpose is to emphasize the intimate connection between the low-energy resonance parameters and the residue functions for *both* the forward (small t) and the backward (small u) differential cross sections for large s. With a similar purpose in mind, the Veneziano scheme has been investigated for πN elastic scattering by several authors.[1] It has been found that, in general, there is a satisfactory correlation between the low-energy resonance parameters and the forward differential cross sections. However, the predicted backward cross sections are, on the whole, unsatisfactory. In I, the predicted cross section is too big by several orders of magnitude. Berger and Fox[1] come within a factor of 2 for the Δ width, but they do not consider the fit to be

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[1] S. Fenster and K. C. Wali, Phys. Rev. D 1, 1409 (1970), hereafter called I; see also E. Berger and G. Fox, Phys. Rev. 188, 2120 (1969). References to their related papers are contained in these papers.