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C Invariance and the Reaction $e^- + e^+ \to \eta + \pi^0$

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We show how electron-positron annihilation into η and π^0 may be used to obtain information on C invariance in the electromagnetic interactions of hadrons. This is illustrated by studying the angular distributions in the c.m. system, the analysis of which should give a model-independent check on the validity of C invariance.

SINCE the observation of CP violation in $K_2{}^0$ decay, there have been many discussions on the origin of this effect. In particular, the possibility has been suggested that the effect is caused by violations of time-reversal invariance and of charge-conjugation invariance in the electromagnetic interactions of hadrons.[1] From this viewpoint, the recent report[2] of a breakdown of detailed balance between the reaction $n+p \to d+\gamma$ and its inverse is of particular interest. So far as C invariance in the electromagnetic interactions of hadrons is concerned, electron-positron annihilation processes into hadrons[3] seem to be good electromagnetic processes involving hadrons.

In this paper, we consider the electron-positron annihilation into η and π^0 to check the validity of charge-conjugation invariance in the electromagnetic interactions of hadrons:

$$e^- + e^+ \to \eta + \pi^0. \tag{1}$$

Since the cross section of process (1) may be very small, the validity of C invariance must be checked model independently by experiment with relatively poor statistics. The purpose of this paper is to point out that the angular distribution of η in the c.m. system could give a model-independent check for C invariance in reaction (1), even if the observed number of events is not large.

We shall now give the general form of the matrix element for reaction (1). As for the decay process

$$\eta \to \pi^0 + e^- + e^+, \tag{2}$$

Hiida, Lee, and the present author[4] gave the most general expression for the matrix element to make a model-independent check for C invariance. The reaction process (1) can be obtained by just the same diagram as the decay process (2) except for transposing only the neutral pion, so we refer to HLS for notation.

Since the process (1) involves the exchange of any number of photons, the matrix element is invariant (not invariant) under charge conjugation if the process includes an even (odd) number of photons. We shall assume vanishing electron mass throughout this work, since the electron mass is very small compared with any finite quantity involved.

Assuming the space-reflection invariance, we can write the general matrix element in the form

$$\bar{v}(p_-)\gamma \cdot p_\eta u(p_+)[th_1(s,t^2)+h_2(s,t^2)]. \tag{3}$$

Here p_η, p_+, and p_- denote the momenta of the η, electron, and positron, respectively. The two invariant variables are

$$s=-(p_+ + p_-)^2, \quad t=-p_\eta \cdot (p_+ - p_-). \tag{4}$$

h_1 and h_2 are the invariant structure functions. As is easily seen, the first term gives a C-invariant matrix element which comes mainly from the two-photon-exchange diagram, and the second term gives a C-violating one coming mainly from the one-photon-exchange diagram.

If C invariance is violated in the electromagnetic interactions of hadrons, process (1) may occur through the electromagnetic current $\langle \eta\pi^0 | j_\mu | 0 \rangle$. This current vanishes in the $SU(3)$-symmetry limit,[5] provided it transforms in the same way as the usual C-conserving electromagnetic current does. Thus, even if C invariance is badly broken in the electromagnetic interactions of hadrons, there is still a possibility that the magnitude of the C-violating matrix element is comparable with that of the C-conserving matrix element. Hence, it is necessary to know whether the process comes from C-conserving or C-violating electromagnetic interactions or from both, when we observe reaction (1). Therefore, we must know not only the magnitude of the reaction cross section but also the angular distribution to check the validity of C invariance.

The relevant matrix element, in the limit $m_e=0$, is given by

$$M=\frac{\delta^4(p_\eta+p_\pi-p_+-p_-)}{(2\pi)^2}\frac{m_e}{(4\omega_\eta\omega_\pi E_+E_-)^{1/2}} \times \bar{v}(p_-)\gamma \cdot p_\eta u(p_+)[th_1(s,t^2)+h_2(s,t^2)], \tag{5}$$

where E_+, E_-, ω_η, and ω_π are the energies of the electron, positron, η, and π^0, respectively. In the c.m. system, $t=-2\bar{p}_\eta\bar{p}_+ \cos\theta$, where the bar means the magnitude of the three-momentum and θ is the angle between the

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[2] D. F. Bartlett, C. E. Friedberg, K. Goulianos, I. S. Hammerman, and D. P. Hutchinson, Phys. Rev. Letters **23**, 893 (1969).
[3] General discussions on electron-positron collisions and charge-conjugation invariance have been given by A. Pais and S. B. Treiman, Phys. Letters **29B**, 308 (1969).
[4] K. Hiida, Y-Y. Lee, and T. Sakuma, Phys. Rev. **161**, 1428 (1967); hereafter referred to as HLS.
[5] T. D. Lee, Phys. Rev. **140**, B967 (1965).

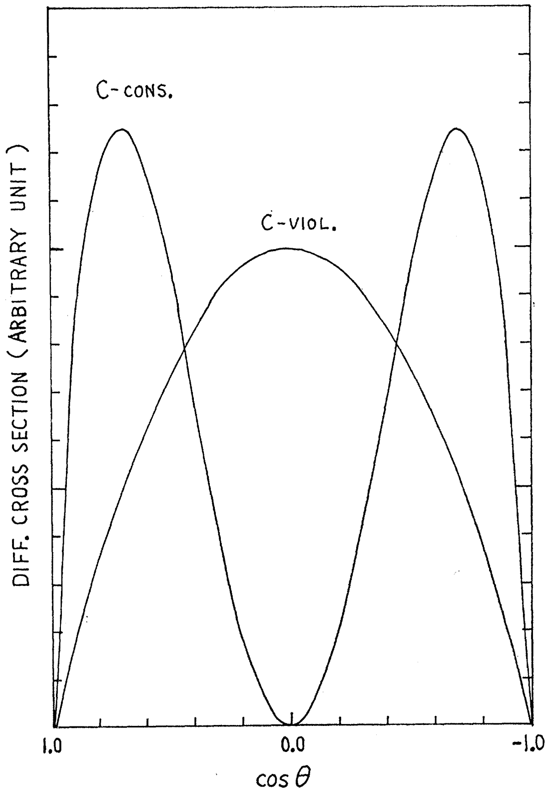

FIG. 1. Angular distributions for the reaction $e^-+e^+\to\eta+\pi^0$ for the pure C-conserving and pure C-violating cases.

electron and η. Hence if the h_1 and h_2 terms exist together, and they interfere, then the angular distribution of η should be asymmetric with respect to the $\cos\theta=0$ axis. If C is violated and the C-conserving term th_1 is negligible compared with the C-violating term h_2, it will be shown that the angular distribution has a maximum at $\cos\theta=0$ due to the phase-space factor. Thus the angular distribution will determine whether C is conserved or not in the electromagnetic reactions.

In order to calculate the angular distribution, we shall neglect s and t^2 dependences of the structure functions h_1 and h_2. Therefore, the part $[th_1+h_2]$ in (5) can be written, except for a common phase factor, in the form

$$\frac{1}{m_\eta^2}\left[a-2be^{i\varphi}\frac{\bar{p}_\eta\bar{p}_+}{m_\eta^2}\cos\theta\right], \tag{6}$$

where a and b are real and dimensionless constants coming from h_2 and h_1, respectively. φ is the relative phase angle between h_1 and h_2. Thus the angular distribution of η is now given by

$$\frac{d\sigma}{d\Omega}=\frac{1}{32(2\pi)^2}\frac{\bar{p}_\eta^3}{m_\eta^4E_+}\sin^2\theta$$
$$\times\left[a^2+4b^2\frac{\bar{p}_\eta^2\bar{p}_+^2}{m_\eta^4}\cos^2\theta-4ab\cos\varphi\frac{\bar{p}_\eta\bar{p}_+}{m_\eta^2}\cos\theta\right]. \tag{7}$$

Figure 1 shows the angular distribution of η in the c.m. system for both cases $a=0$, $b=b'$ and $a=a'$, $b=0$ $[a'/b'=\bar{p}_\eta\bar{p}_+/(\sqrt{5})m_\eta^2]$. In the former case the angular distribution has two maxima at $\cos\theta=\pm1/\sqrt{2}$ and one minimum at $\cos\theta=0$, whereas in the latter case the distribution has only one maximum at $\cos\theta=0$. This makes these two cases unambiguously distinguishable.

We show in Fig. 2 the cases where both a^2 and b^2 terms have the same magnitude $[a'/b'=\bar{p}_\eta\bar{p}_+/(\sqrt{5})m_\eta^2]$. In the cases $\varphi=0$ and $\varphi=\pi$, the interference between the C-conserving and C-violating terms is maximum, and the angular distributions are very asymmetric with respect to the $\cos\theta=0$ axis. If the angle φ is equal to $\frac{1}{2}\pi$, then there is no interference and the angular distribution is symmetric with two maxima at about $\cos\theta=\pm0.69$ and one minimum at $\cos\theta=0$ as in the C-conserving case. In this case, however, the distribution at $\cos\theta=0$ does not vanish, which shows the C violation.

Finally, we estimate the rough order of magnitude of the total cross section, under the assumption of separate conservation of C and P. As stated above, the main contributor to the C-conserving matrix element is the two-photon-exchange diagram, shown in Fig. 3. The $(\gamma+\gamma)\to\eta+\pi^0$ part of this diagram, where the parentheses mean that the two photons may be virtual, has been given in HLS. The matrix element of this part

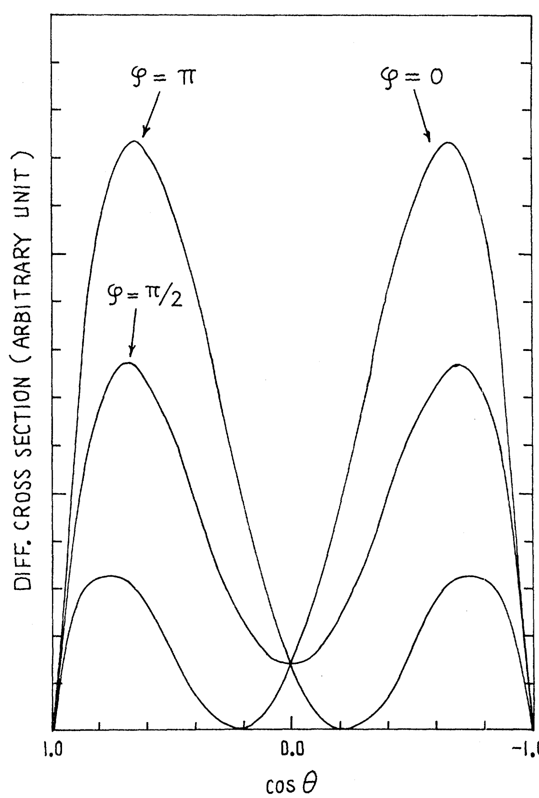

FIG. 2. Angular distributions for the reaction $e^-+e^+\to\eta+\pi^0$ for the mixed case.

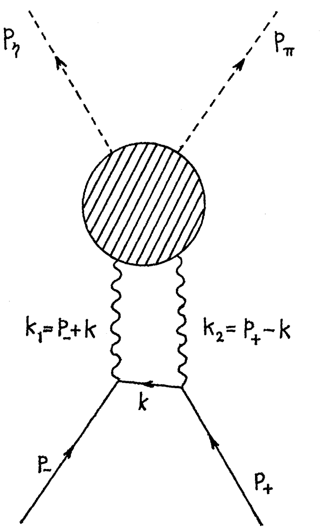

FIG. 3. Two-photon-exchange contribution for the reaction $e^-+e^+ \to \eta+\pi^0$.

consists of following five independent amplitudes.[4]

$$\{(k_1\cdot k_2)[\epsilon(k_1)\cdot\epsilon(k_2)]-[k_2\cdot\epsilon(k_1)][k_1\cdot\epsilon(k_2)]\}f_1, \tag{8}$$

$$\begin{aligned}\{(p_\eta\cdot k_1)(p_\eta\cdot k_2)[\epsilon(k_1)\cdot\epsilon(k_2)]&\\ +(k_1\cdot k_2)[p_\eta\cdot\epsilon(k_1)][p_\eta\cdot\epsilon(k_2)]&\\ -(p_\eta\cdot k_2)[p_\eta\cdot\epsilon(k_1)][k_1\cdot\epsilon(k_2)]&\\ -(p_\eta\cdot k_1)[k_2\cdot\epsilon(k_1)][p_\eta\cdot\epsilon(k_2)]\}f_2,&\end{aligned} \tag{9}$$

$$k_1^2k_2^2[\epsilon(k_1)\cdot\epsilon(k_2)]f_3, \tag{10}$$

$$\begin{aligned}\{(k_1^2+k_2^2)(k_1\cdot k_2)[p_\eta\cdot\epsilon(k_1)][p_\eta\cdot\epsilon(k_2)]&\\ -(p_\eta\cdot k_1)k_2^2[k_2\cdot\epsilon(k_1)][p_\eta\cdot\epsilon(k_2)]&\\ -(p_\eta\cdot k_2)k_1^2[k_1\cdot\epsilon(k_2)][p_\eta\cdot\epsilon(k_1)]\}f_4,&\end{aligned} \tag{11}$$

$$k_1^2k_2^2[p_\eta\cdot\epsilon(k_1)][p_\eta\cdot\epsilon(k_2)]f_5. \tag{12}$$

The structure functions f_1–f_5 depend on the four invariant variables $(k_1+k_2)^2$, $k_1^2+k_2^2$, $k_1^2k_2^2$, and $(p_\eta\cdot k_1)(p_\eta\cdot k_2)$. Of these five amplitudes, (8)–(12), to describe the $(\gamma+\gamma)\to\eta+\pi^0$ part of the diagram, only two amplitudes, (8) and (9), survive, if the two photons are both real.

Since we are to estimate the rough order of magnitude of the cross section, we shall neglect the other three amplitudes, (10)–(12). Then we have following matrix element for Fig. 3:

$$\begin{aligned}&\bar v(p_-)\gamma\cdot p_\eta u(p_+)th_1(s,t^2)\\ &=\frac{-ie}{(2\pi)^4}\int d^4k\,\frac{1}{[k_1^2-i\epsilon][k_2^2-i\epsilon][k^2+m_e^2-i\epsilon]}\\ &\times\bar v(p_-)\{[(k_1\cdot k_2)\gamma_\mu\gamma\cdot k\gamma_\mu-\gamma\cdot k_2\gamma\cdot k\gamma\cdot k_1]f_1\\ &+[(p_\eta\cdot k_1)(p_\eta\cdot k_2)\gamma_\mu\gamma\cdot k\gamma_\mu+(k_1\cdot k_2)\gamma\cdot p_\eta\gamma\cdot k\gamma\cdot p_\eta\\ &-(p_\eta\cdot k_2)\gamma\cdot p_\eta\gamma\cdot k\gamma\cdot k_1\\ &\qquad-(p_\eta\cdot k_1)\gamma\cdot k_2\gamma\cdot k\gamma\cdot p_\eta]f_2\}u(p_+),\end{aligned} \tag{13}$$

where $k_1=p_-+k$ and $k_2=p_+-k$. Here we shall assume the functions f_1 and f_2 to be constant in order to perform the k integration. Then we find that the f_1 term does not contribute to h_1 in the limit $m_e=0$.

Performing the k integration and keeping only the divergent part, we find

$$h_1(s,t^2)\cong\frac{\alpha}{4\pi}f_2\ln\left[\frac{\Lambda^2-s}{s-(m_\eta+m_\pi)^2}\right], \tag{14}$$

where Λ is the cutoff parameter. Now the rough order of magnitude of f_2 can be estimated by the partial decay rate of the process $\eta\to\pi^0+\gamma+\gamma$, whose expression has been given in (5.5) of HLS. If we take the branching ratio of $\eta\to\pi^0+\gamma+\gamma$ to be 2.5%,[6] we obtain

$$m_\eta^8|f_2|^2\simeq 0.22. \tag{15}$$

Using the expressions (13)–(15), and assuming that the cutoff is around twice the nucleon mass, we estimate the cross section of reaction (1):

$$\sigma_{e^-+e^+\to\eta+\pi^0}\simeq 0.5\times 10^{-10}\text{ mb}\quad\text{at } s=m_N^2, \tag{16}$$

where m_N is the nucleon mass. The well-known process $e^-+e^+\to\pi^++\pi^-$ gives the following order of magnitude of the cross section at the same energy, $s=m_N^2$, if we assume the ρ-dominance model:

$$\sigma_{e^-+e^+\to\pi^++\pi^-}\simeq 0.89\times 10^{-4}\text{ mb}. \tag{17}$$

Then expressions (16) and (17) give the ratio of those cross sections to be about 10^{-6}, which is nearly 10^{-2} of the usual expected ratio α^2.

An alternative way to estimate the order of magnitude of the total cross section for $e^-+e^+\to\eta+\pi^0$ is to make use of the experimental decay rate of the process $\eta\to\pi^0+e^-+e^+$.

Assuming the separate conservation of C and P, and hence $h_1=0$ and h_2 constant, we can obtain the maximum cross section allowed by the present experimental upper limit on $\eta\to\pi^0+e^-+e^+$ whose branching ratio is less than 0.01%.[6] We find

$$\sigma_{e^-+e^+\to\eta+\pi^0}<0.5\times 10^{-6}\text{ mb}\quad\text{at } s=m_N^2. \tag{18}$$

Although this upper limit (18) is much larger than the value (16) estimated from the two-photon-exchange diagram, we can expect that this value would be much reduced because the upper limit of the branching ratio on $\eta\to\pi^0+e^-+e^+$ is being reduced in the present experimental situation.

To conclude, if our approximation is valid and if we actually observe some number of events of reaction (1), we may conclude that charge-conjugation invariance would be seriously violated in the electromagnetic interactions of hadrons.

[6] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).