Finally, we relate our covariants[9] $\tilde{A}_i$ to the sets of Fox and Freedman,[7] B_i, and of Hearn and Leader,[11] A_i:

$$B_1=2(P^2\tilde{A}_2+2m\tilde{A}_3)\,,\quad B_2=2\tilde{A}_4\,,\quad B_3=4\tilde{A}_3\,,$$
$$B_4=A_3=-2(\nu\tilde{A}_{45}+mt\tilde{A}_6)=-A_P\,,$$
$$B_5=-4[2P^2(\tilde{A}_1+m\tilde{A}_3)+\nu\tilde{A}_{45}]\,,$$
$$B_6=A_4+A_5=4m\tilde{A}_{45}=4m(m\tilde{A}_4+4\tilde{A}_5)\,,$$
$$A_1+A_2=-2[t(\tilde{A}_1+m\tilde{A}_3)-2\nu\tilde{A}_{45}]\,,$$
$$A_1-A_2=-(tP^2+4\nu^2)\tilde{A}_2+4\nu\tilde{A}_{45}\,,$$
$$A_4-A_5=4(2\nu\tilde{A}_3+P^2\tilde{A}_4)\,,$$
$$A_6=t\tilde{A}_3-2\nu\tilde{A}_4\,.$$

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Low-t Theorems for Charged-Pion Photoproduction*

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Low-t theorems for charged-pion photoproduction are proven on the basis of gauge-invariance restrictions. They can be simply formulated by means of the minimal gauge-invariant extension of pion exchange (MPE). It is shown that whereas MPE leads in $\gamma N\to\pi^\pm N$ only to weak restrictions for $d\sigma/dt$ near the forward direction ($|t|\lesssim\mu^2$), it does actually account for all the structure observed in $\gamma p\to\pi^-\Delta^{++}$ in this t domain. Results for these reactions as well as the other charge modes of $\gamma N\to\pi^\pm\Delta$ are discussed and compared with experiment.

I. INTRODUCTION

THE properties of the photon lead to unique consequences for any process in which a photon participates. We know that in any such four-point function, one finds low-energy theorems in all channels in which a Born term exists[1] at the position relevant to this particular exchange. In high-energy photoproduction, one may reach very low t values; therefore, one can look for a manifestation of the low-t theorems. Alas, the low-t theorems affect only some of the amplitudes, and therefore we are not guaranteed *a priori* that their effect will be visible in the shape of the differential cross sections.

The low-t theorem is enforced by gauge invariance. Whereas, in general, any individual exchange can be written in a manifestly gauge-invariant form, that is not the case when a Born term is involved. (We mean here by Born term the exchange of a particle that appears also among the external ones.) It turns out that the sum of all possible Born terms is a gauge-invariant combination. As a matter of fact, it is usually built out of several pieces that are separately gauge invariant. The conditions of gauge invariance[1–3] may force one amplitude to have a specific value at the point where another amplitude has a pole. This behavior is exhibited in Sec. II, in which we review the situation in $\gamma N\to\pi^\pm N$. Thus the form of a certain set of invariant amplitudes is determined at the position of the Born term. In our case, this corresponds to the pion exchange in the t channel.

Since we cannot determine all possible amplitudes at $t=\mu^2$ on these grounds, we cannot predict in general the shape of the cross section near $t=0$. Nevertheless, we may take the following attitude: Compute $d\sigma/dt$ as if only the minimal gauge-invariant combination corresponding to a π exchange existed (the exact meaning of this minimal combination, to be denoted MPE, will be clarified in the text). Compare the result at $t=0$ with experiment. If you find an agreement in magnitude, it means that the other contributions are small and you should therefore predict the right shape near the forward direction. If, however, the result at $t=0$ does not agree with the experimental number, then all you can hope for are bounds on the rate of the variation of $d\sigma/dt$ near $t=0$. This procedure should work in the range $t\gtrsim-\mu^2$ and is simply based on the assumption that

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[1] D. Horn and M. Jacob, Nuovo Cimento **56A**, 83 (1968).

[2] J. S. Ball, Phys. Rev. **124**, 2014 (1961).

[3] F. S. Henyey, Phys. Rev. **170**, 1619 (1968); T. Ebata and K. E. Lassila, Phys. Rev. Letters **21**, 250 (1968); W. A. Bardeen and W.-K. Tung, Phys. Rev. **173**, 1423 (1968).

every rapid variation is determined by a $t-\mu^2$ pole whose residue is given by the low-t theorem.

In the case of $\gamma N\to\pi^\pm N$, we are indeed left with only bounds on the variation as shown in Sec. II. However, in $\gamma N\to\pi^\pm\Delta$ there are charge states for which we can actually predict (after knowing $d\sigma/dt|_{t=0}$) the right shape of the experimental $d\sigma/dt$. This is discussed in Secs. III–V.

The importance of the Born terms in high-energy π photoproduction is well known.[4,5] Nevertheless, it is often regarded as a puzzle. We try to point out the part of the story which is determined by general principles and explore it in the cases mentioned above.

II. $\gamma N\to\pi^\pm N$

This reaction is particularly simple because there are only two kinds of hadrons involved. Following Ball,[2] we write the matrix element in terms of eight invariant amplitudes (B_i):

$$T^\mu\epsilon_\mu(k)=\sum_{i=1}^{8}B_i(s,t)N_i\,,\tag{1}$$

where

$$N_i=\bar{u}(p_f)I_i{}^\mu u(p_i)\epsilon_\mu(k)\,,$$

and

$$\begin{aligned}I_1{}^\mu&=i\gamma_5\gamma^\mu\boldsymbol{k}\,, & I_5{}^\mu&=-i\gamma_5\gamma^\mu\,,\\ I_2{}^\mu&=2i\gamma_5P^\mu\,, & I_6{}^\mu&=i\gamma_5P^\mu\boldsymbol{k}\,,\\ I_3{}^\mu&=2i\gamma_5q^\mu\,, & I_7{}^\mu&=i\gamma_5k^\mu\boldsymbol{k}\,,\\ I_4{}^\mu&=2i\gamma_5k^\mu\,, & I_8{}^\mu&=i\gamma_5q^\mu\boldsymbol{k}\,,\end{aligned}$$

where we have used the BD metric conventions[6] for the vectors and Dirac matrices; k, q, p_i, and p_f represent the s-channel momenta of the photon, pion, initial nucleon, and final nucleon, respectively, and $P=\frac{1}{2}(p_i+p_f)$. Because of conventional notation, we included here I_4 and I_7 although they do not contribute to photoproduction. The gauge-invariance condition,

$$T^\mu k_\mu=0\,,\tag{2a}$$

implies that

$$B_2[s-M^2+\tfrac{1}{2}(t-\mu^2)]-(t-\mu^2)B_3=0\,.\tag{2b}$$

The pion Born term contributes only to B_3 and B_4. Hence, condition (2b) implies that at $t=\mu^2$, B_2 will be given by a simple pole in $(s-M^2)$ whose residue is just the residue of the pion contribution to B_3. That is the low-t theorem[1] and it holds for every s at $t=\mu^2$. This pole is obviously the exact contribution of the s-channel nucleon Born term; Eq. (2b) assures us that this is the only term that exists in B_2 at $t=\mu^2$. There is another gauge-invariance constraint equation; however, it does not lead to a low-t theorem. Let us now look at $d\sigma/dt$. The leading terms (viz., the highest in s and lowest in t) are represented by amplitudes that are regular at $t=\mu^2$ and contribute the following:

$$\frac{d\sigma}{dt}\approx\frac{1}{32\pi}\left(\left|B_1-MB_6+t\frac{2B_2}{t-\mu^2}\right|^2+|B_1-MB_6|^2-t|B_8|^2-\tfrac{1}{4}t|B_6|^2\right).\tag{3}$$

The only amplitude for which we have a low-t theorem is B_2; however, its contribution to $d\sigma/dt$ vanishes in the forward direction. Since, experimentally, $d\sigma/dt$ is finite there, it is obvious that other amplitudes must play an important role. Nevertheless, we may make the very plausible assumption that the only rapid variation at low t values ($|t|\lesssim\mu^2$) will be determined by the $t-\mu^2$ pole. Then we can approximate (3) in the form

$$\frac{d\sigma}{dt}\approx\frac{1}{32\pi}\left(\left|A+\frac{t2B_2(t=\mu^2)}{t-\mu^2}\right|^2+|A|^2\right)\quad\text{for }|t|\lesssim\mu^2.\tag{4}$$

This determines limits on the behavior of $d\sigma/dt$ (we still have one free parameter—the phase between A and B_2). We predict B_2 to behave like s^{-1} and, experimentally, A behaves the same way. Hence we can compare experiment with prediction for $s^2d\sigma/dt$ for several values of s. This is done in Fig. 1, where we show the allowed range of variation that depends on ϕ, the phase between A and B_2. Since A behaves like s^{-1}, we can expect the phase to be 0° or 180°. Experimentally, the phase seems to be zero.

The nucleon Born term does contribute to B_1 and, therefore, to A. As a matter of fact, A seems to be dominated by the nucleon Born terms. Although that is known to be consistent with finite-energy sum-rule (FESR) considerations,[7] nonetheless it does not rely on a low-t theorem. Therefore, when one states that $\gamma N\to\pi^\pm N$ is determined for $|t|\lesssim\mu^2$ by the sum of all electric Born terms, it is only partially a result of the low-t theorem.

We stated already that the sum of all the Born terms is a gauge-invariant quantity. However, this sum can be broken down to subsets that are gauge invariant by themselves. Thus in $\gamma p\to\pi^+n$ we find the following sum of electric Born terms (i.e., those terms in which no anomalous magnetic moment appears):

$$eg\bar{u}(p_f)i\gamma_5u(p_i)\frac{2\epsilon\cdot q}{t-\mu^2}+eg\bar{u}(p_f)i\gamma_5\frac{\boldsymbol{p}_i+\boldsymbol{k}+M}{s-M^2}\gamma\cdot\epsilon u(p_i)\tag{5}$$

coming from π and N exchange, respectively. Here, e

[4] B. Richter, in *Proceedings of the Third International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, 1967* (Clearing House of Federal Scientific and Technical Information, Washington, D. C., 1968), p. 313.

[5] A. M. Boyarski, F. Bulos, W. Busza, R. Diebold, S. D. Ecklund, G. E. Fischer, J. R. Rees, and B. Richter, Phys. Rev. Letters **20**, 300 (1968).

[6] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[7] For a recent review, see H. Harari, rapporteur talk, in *Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies* (Daresbury Nuclear Physics Laboratory, Daresbury, England, 1969), p. 107.

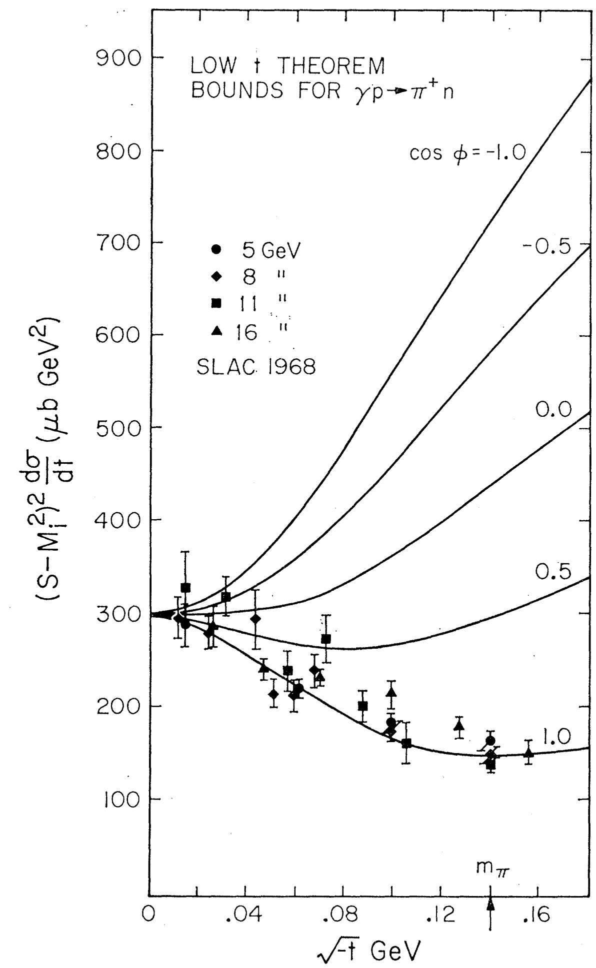


FIG. 1. $d\sigma/dt$ predictions that follow from Eq. (4). Normalization at $t=0$ was chosen so as to fit the data of Ref. 5.

designates the $\gamma\pi^+\pi^-$ coupling constant and g the $pn\pi^+$ coupling constant. Note, however, that the nucleon contribution has a term proportional to $\bar{u}i\gamma_5\boldsymbol{k}\gamma\cdot\epsilon u$ which is gauge invariant by itself. Only the $\bar{u}i\gamma_5(\boldsymbol{p}_i+M)\gamma\cdot\epsilon u$ term has to be there in order to compensate the lack of gauge invariance of the pion term. We will define as the minimal gauge-invariant extension of the pion exchange (MPE) that part of the sum of the Born terms which is necessary and sufficient to complete the pion exchange to a gauge-invariant expression. In $\gamma p \to \pi^+ n$ the MPE is given by expression (5) without the $\bar{u}i\gamma_5\boldsymbol{k}\gamma\cdot\epsilon u$ term. We note, of course, that the MPE is exactly that part which is required to satisfy the condition (2a). Had we just used the MPE, we would have found the same results to which the low-t theorem led us. The $\bar{u}i\gamma_5\boldsymbol{k}\gamma\cdot\epsilon u$ term in (5) is the one that contributes to A and happens to be the right expression experimentally. However, whereas there is a theoretical explanation for why the MPE term exists near $t=\mu^2$, it is not *a priori* clear why the whole expression (5) should dominate there. As a matter of fact, we will see in the following sections that whereas the MPE is present also in $\gamma N \to \pi^\pm\Delta$, the terms analogous to $\bar{u}i\gamma_5\boldsymbol{k}\gamma\cdot\epsilon u$ do not play an important role.

III. LOW-t THEOREMS FOR $\gamma N \to \pi^\pm\Delta$

We can begin, as in (1), by writing the matrix element for Δ photoproduction in terms of a set of independent invariant amplitudes (B_i), where i runs from 1 to 12. A set which agrees with Hearn's criteria[8] for freedom from kinematical singularities is defined by

$$T^\mu\epsilon_\mu(k)=\sum_{i=1}^{12} B_i(s,t)N_i\,, \tag{6}$$

where

$$N_i=\bar{u}_\nu(p_f)I_i^{\nu\mu}u(p_i)\epsilon_\mu(k)$$

and

$$\begin{aligned}
I_1^{\nu\mu}&=q^\nu q^\mu, & I_7^{\nu\mu}&=k^\nu P^\mu\boldsymbol{k},\\
I_2^{\nu\mu}&=q^\nu P^\mu, & I_8^{\nu\mu}&=g^{\nu\mu}\boldsymbol{k},\\
I_3^{\nu\mu}&=k^\nu q^\mu, & I_9^{\nu\mu}&=q^\nu\gamma^\mu\boldsymbol{k},\\
I_4^{\nu\mu}&=k^\nu P^\mu, & I_{10}^{\nu\mu}&=k^\nu\gamma^\mu\boldsymbol{k},\\
I_5^{\nu\mu}&=g^{\nu\mu}, & I_{11}^{\nu\mu}&=q^\nu\gamma^\mu,\\
I_6^{\nu\mu}&=q^\nu P^\mu\boldsymbol{k}, & I_{12}^{\nu\mu}&=k^\nu\gamma^\mu.
\end{aligned} \tag{7}$$

The Dirac subsidiary condition for spinors allows us to write the remaining two apparently independent amplitudes $q^\nu q^\mu\boldsymbol{k}$ and $k^\nu q^\mu\boldsymbol{k}$ in terms of the set in (7) without kinematical singularities.[9]

Out of the set (B_i) used in (6), we expect only eight members to be independent. The gauge-invariance condition $T^\mu k_\mu=0$ ensures this result by providing four equations of constraint:

$$0=B_1(s,t)k\cdot q+B_2(s,t)k\cdot P\,, \tag{8a}$$

$$0=B_3(s,t)k\cdot q+B_4(s,t)k\cdot P+B_5(s,t)\,, \tag{8b}$$

$$0=B_6(s,t)k\cdot P+B_{11}(s,t)\,, \tag{8c}$$

$$0=B_7(s,t)k\cdot P+B_8(s,t)+B_{12}(s,t)\,. \tag{8d}$$

Two of these equations, (8a) and (8b), will be particularly useful in our investigation of low-t theorems.

These gauge-invariance constraint equations permit us to reexpress the transition matrix in a manifestly gauge-invariant form:

$$\begin{aligned}
\sum_{i=1}^{12} B_i(s,t)I_i^{\nu\mu}={}&\frac{2B_2(s,t)}{t-\mu^2}q^\nu(q^\mu k\cdot P-P^\mu k\cdot q)\\
&-\frac{2[B_4(s,t)k\cdot P+B_5(s,t)]}{t-\mu^2}(g^{\nu\mu}k\cdot q-k^\nu q^\mu)\\
&-B_4(s,t)(g^{\nu\mu}k\cdot P-k^\nu P^\mu)+B_6(s,t)q^\nu(P^\mu\boldsymbol{k}-k\cdot P\gamma^\mu)\\
&+B_7(s,t)k^\nu(P^\mu\boldsymbol{k}-k\cdot P\gamma^\mu)+B_8(s,t)(g^{\nu\mu}\boldsymbol{k}-k^\nu\gamma^\mu)\\
&+B_9(s,t)q^\nu\gamma^\mu\boldsymbol{k}+B_{10}(s,t)k^\nu\gamma^\mu\boldsymbol{k}
\end{aligned} \tag{9}$$

[8] A. C. Hearn, Nuovo Cimento **21**, 333 (1961).

[9] See Eqs. (B1) and (B2) in M. Scadron and H. F. Jones, Phys. Rev. **173**, 1734 (1968).

Any singularities in the amplitudes (B_i) must have a dynamical origin. Equation (9) has been written in terms of amplitudes that have no contribution from the pion pole. Pion exchange affects only B_1 and B_3 in the form

$$B_1(s,t) \underset{t\to\mu^2}{\longrightarrow} 2efF_\pi/(t-\mu^2)\,, \tag{10a}$$

$$B_3(s,t) \underset{t\to\mu^2}{\longrightarrow} -2efF_\pi/(t-\mu^2)\,, \tag{10b}$$

where eF_π is the charge of the pion and f is the $N\pi\Delta$ coupling constant. From the substitution of (10a) and (10b) into (8a) and (8b), and the replacement $k\cdot q = -\frac{1}{2}(t-\mu^2)$, it follows that the combinations $B_2(s,t)k\cdot P$ and $B_4(s,t)k\cdot P+B_5(s,t)$ are determined at $t=\mu^2$:

$$\lim_{t\to\mu^2}(B_4(s,t)k\cdot P+B_5(s,t)) = -\lim_{t\to\mu^2} B_2(s,t)k\cdot P = -efF_\pi\,. \tag{11a}$$

The s dependence of $B_2(s,t)$ is therefore determined exactly at $t=\mu^2$:

$$B_2(s,\mu^2) = +2efF_\pi/(s-M_i^2)\,. \tag{11b}$$

Thus we find that the residues of the $(t-\mu^2)$ poles in Eq. (9) are determined by the relations (11). We expect all the B_i amplitudes in (9) to be functions of both s and t, with only a gentle variation with t for $|t|\lesssim\mu^2$ since they should contain only distant singularities in t.

The derivation of $d\sigma/dt$ requires the simplification of the absolute square of an expression which contains (9). We have used Hearn's REDUCE system[10] for algebraic computation to obtain an exact expression for $d\sigma/dt$ in terms of the amplitudes (B_i), the variables s and t, and the external masses. We have also checked separately some of the simpler parts with another program for Dirac matrix algebra[11] which is now a component of the ELP system for noncommutative algebra. We have written numerical programs utilizing these general expressions and the results of the calculations are discussed below for MPE and low-t theorem forms of the amplitudes B_i.

First let us consider the qualitative description of $d\sigma/dt$ for $s\to\infty$ and $|t|\lesssim\mu^2$ which we may obtain by investigating the approximate expression for $d\sigma/dt$ [analogous to (5)] containing terms of the highest order in s and lowest in t. We also approximate the amplitudes B_i by their values at the pion pole $B_i(s)=\lim_{t\to\mu^2}B_i(s,t)\approx B_i(s,0)$. This approximation allows us to replace the $B_4k\cdot P+B_5$ combination in Eq. (9) with $B_2k\cdot P$ given by Eq. (11). The resulting expression is of the form

$$\frac{d\sigma}{dt}\approx\frac{1}{192\pi}\Bigg\{(M_i+M_f)^2\frac{[2\mu^4-t(M_f^2-M_i^2)^2/M_f^2]}{(t-\mu^2)^2} \times|B_2(s)|^2+2\sum_{i=4}^{10}s^{n(i)}\alpha_{2i}\frac{(\beta_{2i}-t)}{t-\mu^2}\,\mathrm{Re}[B_2(s)B_i^*(s)] +2\sum_{j=4}^{10}\sum_{i=4}^{10}s^{[n(i)+n(j)]}\alpha_{ij}\,\mathrm{Re}[B_i(s)B_j^*(s)]\Bigg\}\,, \tag{12}$$

where 5 is excluded from the sums and the α's and β's are functions of the external masses. The extra factors $s^{n(i)}$ appearing in (12) originate in the kinematics and their implications will be discussed below. The values of the powers are $n(i)=0$ for $i=4$ and 8 and $n(i)=1$ for $i=6$, 7, 9, and 10.

It is not possible to reduce (12) to a simpler form such as (4), but we can still utilize it to obtain qualitative predictions for the shape of $d\sigma/dt$ in the region of interest. In particular, (12) predicts that if B_2 were the dominant amplitude, then $d\sigma/dt$ would be finite at $t=0$, rise to a peak, and then drop off.

In Sec. V we use the exact form of $d\sigma/dt$ to obtain numerical comparisons with the experimental data along the lines discussed in Sec. I.

IV. ELECTRIC BORN TERMS AND MPE FOR $\gamma N\to\pi^\pm\Delta$

In this section we consider the electric Born terms and show that the MPE amplitude satisfies the low-t theorem at the pion pole. We represent the $\Delta\pi N$ vertex by the expression $\mathcal{L}_{\Delta\pi N}=f\bar{u}_\nu(p_f)q^\nu u(p_i)$, where f is the coupling constant with dimensions GeV^{-1}, whose magnitude is determined directly from the experimental decay width[12] ($\Gamma=120$ MeV) of the $\Delta^{++}\to p\pi^+$, giving $f^2/4\pi=18.9$ GeV^{-2}.

The electric Born terms for $\gamma N\to\pi^\pm\Delta$ are given by

$$I_{\mathrm{EB}}{}^{\nu\mu}=efq^\nu\left(\frac{2q^\mu F_\pi}{t-\mu^2}+\frac{2p_i{}^\mu F_i}{s-M_i^2}+\frac{2p_f{}^\mu F_f}{u-M_\sigma{}^2}\right) -efF_\pi\left(\frac{2k^\nu q^\mu}{t-\mu^2}+g^{\nu\mu}\right)+efF_i\frac{q^\nu\boldsymbol{k}\gamma^\mu}{s-M_i^2}-efF_f\frac{q^\nu\gamma^\mu\boldsymbol{k}}{u-M_f^2}$$
$$+(\text{additional terms from the }\tfrac{3}{2}\text{ propagator in the }u\text{ channel which are independently gauge invariant})\,. \tag{13}$$

Here eF_π, eF_i, and eF_f are the respective charges of the pion, nucleon, and Δ. In the conventional gauge,[6] one can recognize the Feynman diagrams that lead to the various terms by their pole structure. We included explicitly the whole pion exchange in t, the whole

[10] A. C. Hearn, in *Interactive Systems for Experimental Applied Mathematics*, edited by M. Klerer and J. Reinfelds (Academic, New York, 1968), pp. 79–90.

[11] J. A. Campbell, J. Comput. Phys. 2, 412 (1968).

[12] Particle Data Group, Rev. Mod. Phys. 41, 109 (1969).

nucleon exchange in s and parts of the Δ exchange in u. In order to maintain gauge invariance, one has to add a contact (seagull) diagram that contributes the $g^{\nu\mu}$ term in (13).

The contributions of the set of electric Born terms shown explicitly in Eq. (13) to the amplitudes B_i in Eq. (9) are given by

$$\begin{aligned} B_2^{\mathrm{EB}}(s,t) &= 2ef[F_i/(s-M_i^2)+F_f/(u-M_f^2)]\,, \\ B_5^{\mathrm{EB}}(s,t) &= -efF_\pi\,, \\ B_9^{\mathrm{EB}}(s,t) &= -ef[F_i/(s-M_i^2)+F_f/(u-M_f^2)]\,. \end{aligned} \tag{14}$$

As in the case of $\gamma N \to \pi N$, the sum of the electric Born terms is composed of subsets which are independently gauge invariant. Such sets were combined in brackets in Eq. (13). By definition, the MPE amplitude consists of that part of the Born terms which is necessary and sufficient to complete the pion exchange to a gauge-invariant expression. The MPE amplitude is therefore

$$I_{\mathrm{MPE}}^{\nu\mu} = efq^\nu\left(\frac{2q^\mu F_\pi}{t-\mu^2}+\frac{2p_i^\mu F_i}{s-M_i^2}+\frac{2p_f^\mu F_f}{u-M_f^2}\right) - efF_\pi\left(\frac{2k^\nu q^\mu}{t-\mu^2}+g^{\nu\mu}\right) \tag{15}$$

and contributes the following to the invariant amplitudes of Eq. (9):

$$\begin{aligned} B_2^{\mathrm{MPE}}(s,t) &= 2ef[F_i/(s-M_i^2)+F_f/(u-M_f^2)]\,, \\ B_5^{\mathrm{MPE}}(s,t) &= -efF_\pi\,. \end{aligned} \tag{16}$$

We note that, similarly to what happened in $\gamma N \to \pi^\pm N$, considering the MPE means keeping only those terms in (9) that have an explicit $(t-\mu^2)$ pole. The MPE has a specific behavior with s and t that reduces to the one required by the low-t theorem at $t=\mu^2$. That is evident from (16) since at $t=\mu^2$, one has $u-M_f^2 = -(s-M_i^2)$ and $F_f=F_i-F_\pi$ by charge conservation.

There are several important differences between the $\gamma N \to \pi^\pm\Delta$ and the $\gamma N \to \pi^\pm N$ Born terms. First of all, we find here that the MPE does give a finite contribution in the forward direction. This turns out to be

$$(s-M^2)^2\frac{d\sigma}{dt}\bigg|_{t_{\min};\mathrm{MPE}} = \frac{(M_i+M_f)^2e^2f^2}{24\pi}\,, \tag{17}$$

which, for $\gamma p \to \pi^-\Delta^{++}$, is 530 μb GeV^2. This number agrees with the measured $d\sigma/dt$ and leads us therefore to believe that the MPE is also the dominant contribution to the invariant amplitude in this case. The experimental fit is further discussed in Sec. V. It turns out that the finite contribution in the forward direction is due to the $g^{\nu\mu}$ term in the MPE (15). This term has no counterpart in the πN photoproduction.

The second important difference is that the $\boldsymbol{k}$ terms in Eq. (13) that contribute to B_9 lead to a wrong s behavior (constant $d\sigma/dt$) and therefore have to be disregarded. Thus we see how important it is to separate the MPE from the Born terms.

The importance of a minimal gauge-invariant extension of the one-pion exchange was first pointed out by Stichel and Scholz,[13] who predicted the right shape for $\gamma p \to \pi^-\Delta^{++}$. We used a different method to introduce and justify this concept. Bender, Dosch, and Rothe[14] used also a one-pion-exchange model but chose an arbitrary way to arrive at a gauge-invariant expression. They did not work with the MPE and were led therefore to a cross section which vanished in the forward direction unless amended by a fixed pole. For

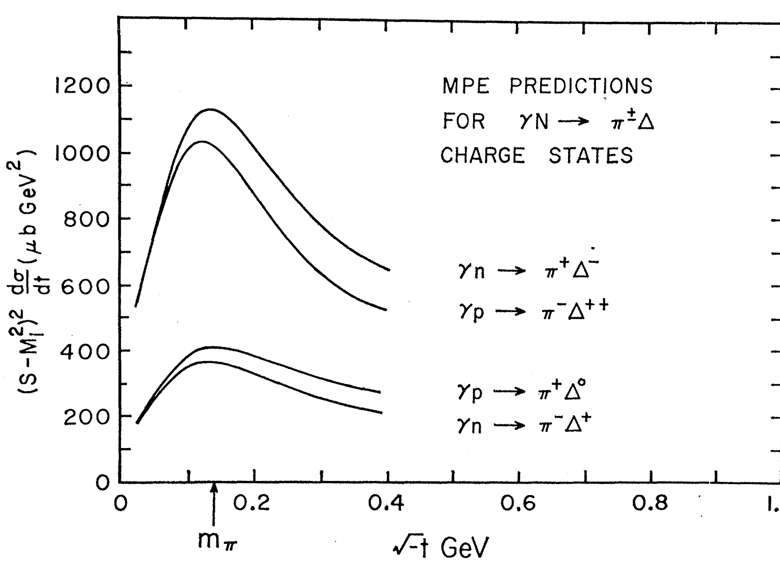


FIG. 2. MPE predictions for small momentum transfers at $E_{\gamma\,\mathrm{lab}}=16$ GeV.

[13] P. Stichel and M. Scholz, Nuovo Cimento **34**, 1381 (1965).
[14] I. Bender, H. G. Dosch, and H. J. Rothe, Nuovo Cimento **63A**, 162 (1969); E. Gotsman, Phys. Rev. **186**, 1543 (1969).

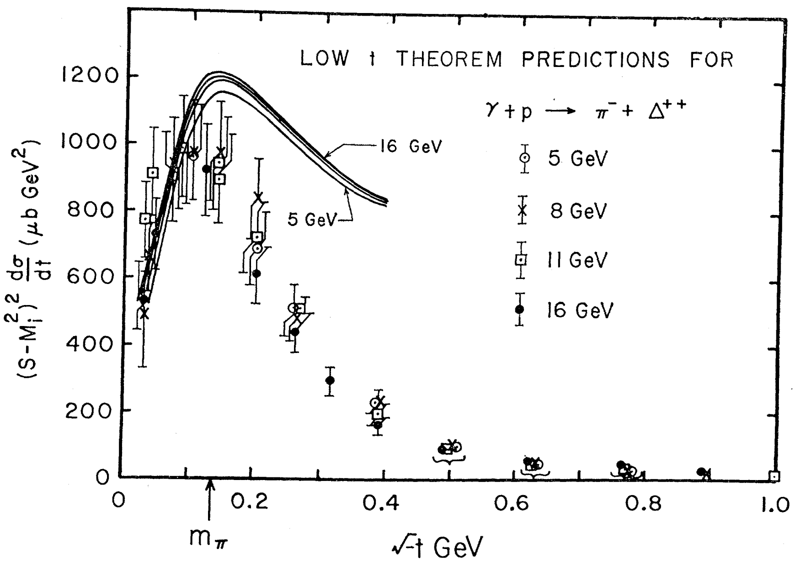


FIG. 3. Low-t-theorem predictions for small momentum transfers. Experimental points are from Ref. 15.

general Regge-pole fits of this reaction, we refer the reader to the papers quoted in Ref. 14.

V. COMPARISON OF PREDICTIONS FOR $\gamma N \rightarrow \pi^{\pm}\Delta$ WITH EXPERIMENTAL DATA

In this section we shall compare the MPE and low-t-theorem predictions with experimental results for $\gamma N \rightarrow \pi^{\pm}\Delta$ along the lines discussed in Sec. I.

We consider first only the MPE contribution to $d\sigma/dt$. There are four possible charge states of the hadrons, namely,

(a) $\gamma p \rightarrow \pi^- \Delta^{++}$,

(b) $\gamma n \rightarrow \pi^+ \Delta^-$,

(c) $\gamma p \rightarrow \pi^+ \Delta^0$,

(d) $\gamma n \rightarrow \pi^- \Delta^+$.

Their various predicted shapes due to MPE alone are plotted in Fig. 2. The similarities in magnitude between (a) and (b) and between (c) and (d) are due to isospin invariance. We note the characteristic dip-bump structure in all the curves. The decrease in magnitude in (c) and (d) is a simple isospin effect in $\pi N\Delta$ coupling which relates the coupling constants by

$$f_{p\pi^-\Delta^{++}} = f_{n\pi^+\Delta^-} = \sqrt{3} f_{p\pi^+\Delta^0} = \sqrt{3} f_{n\pi^-\Delta^+}. \tag{18}$$

In Sec. IV we found that the MPE amplitude satisfies the required low-t theorem. Nevertheless, it does have some dependence on t that is different from a simple pole with residue fixed by the low-t theorem. To compare the two cases, we have plotted the $d\sigma/dt$ curves in Fig. 3 for the strict low-t-theorem form of the amplitudes B_2 and $B_4 k\cdot P + B_5$ given at the pion pole,

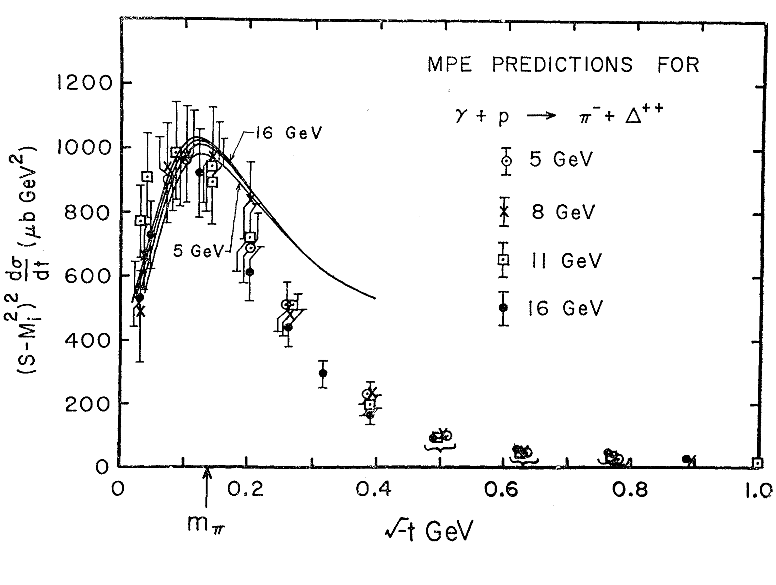


FIG. 4. MPE predictions for small momentum transfers. Experimental points are from Ref. 15.

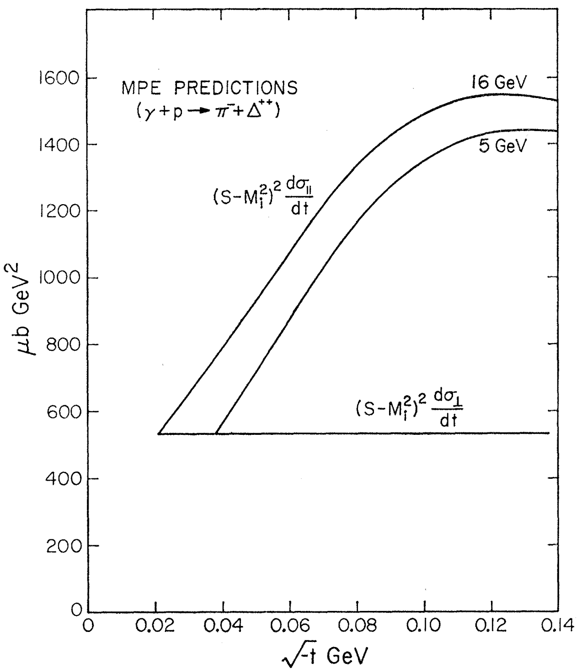


FIG. 5. MPE predictions for incident photons polarized perpendicular and parallel to the production plane.

viz.,

$$B_2^{Lt}(s,t)=2efF_\pi/(s-M_i^2)\,,$$
$$B_4^{Lt}(s,t)k\cdot P+B_5^{Lt}(s,t)=-efF_\pi\,, \tag{19}$$

for the process $\gamma p\to\pi^-\Delta^{++}$. The predictions are identical for $\gamma n\to\pi^+\Delta^-$ and multiplied by $\frac{1}{3}$ [see Eq. (18)] for $\gamma p\to\pi^+\Delta^0$ and $\gamma n\to\pi^-\Delta^+$. The general behavior is obviously the same as that predicted by MPE, but the exact details in the shape near $-t\approx\mu^2$ are different. It is hard to say exactly up to what point in t one can take the MPE approximation seriously. The difference between Figs. 2 and 3 should give us a feeling for the range of validity of our approach.

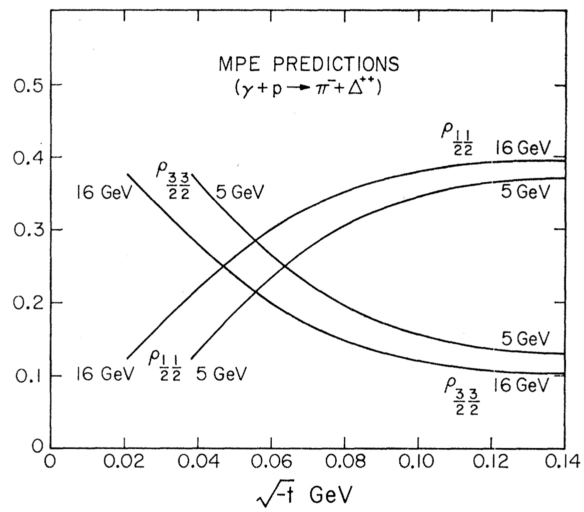
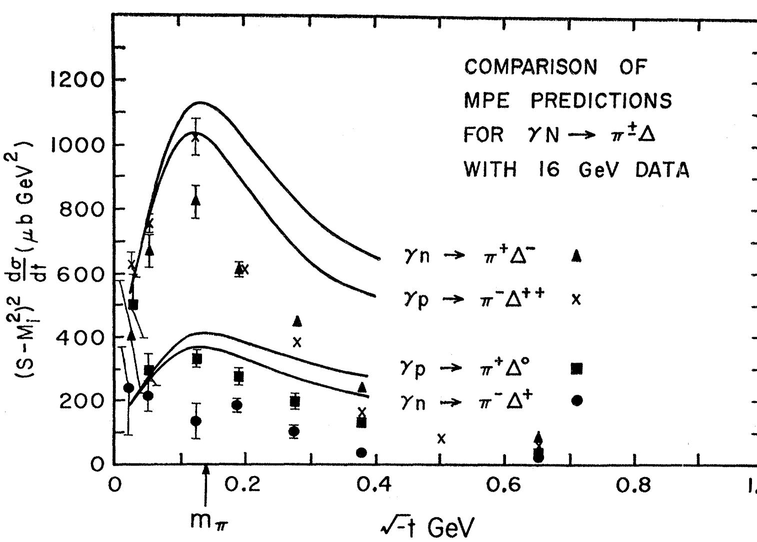


FIG. 6. MPE predictions for the spin-density matrix elements of the outgoing Δ.

Since good experimental data are available for $\gamma p\to\pi^-\Delta^{++}$,[15] we shall consider it in somewhat more detail. The MPE predicts essentially constant $s^2d\sigma/dt$ (as can be seen in Fig. 4), and that is also found to be the experimental situation. In the forward direction, the data agree quite well with the MPE calculation. Hence, one can argue that the contribution of the other amplitudes must be relatively small and that the bump structure has to be seen in $d\sigma/dt$. This is, of course, the case as already noted in Ref. 15. Comparison with

FIG. 7. Comparison of MPE predictions for small momentum transfers with 16-GeV preliminary data of Ref. 16.



[15] A. M. Boyarski, R. Diebold, S. D. Ecklund, G. E. Fischer, Y. Murata, B. Richter, and W. S. C. Williams, Phys. Rev. Letters **22**, 148 (1969).

experiment is shown in Fig. 4. We conclude that the bump in case (a) is due to the MPE and, therefore, is the manifestation of the low-t theorem.

The identification of the dominant amplitude can be checked by the behavior of polarization experiments. In Fig. 5 we show the MPE predictions for the photoproduction cross sections by photons polarized perpendicular ($d\sigma_\perp/dt$) and parallel ($d\sigma_{\parallel}/dt$) to the production plane. The two are equal in the forward direction because of the azimuthal symmetry at this point. $d\sigma_\perp/dt$ depends on the $g^{\nu\mu}$ term and remains constant while $d\sigma_{\parallel}/dt$ follows from $d\sigma/dt=\frac{1}{2}(d\sigma_\perp/dt+d\sigma_{\parallel}/dt)$. This pattern of behavior was suggested by Harari.[7] Figure 6 shows the $\rho_{1/2,1/2}$ and $\rho_{3/2,3/2}$ s-channel spin-density matrix elements of the outgoing Δ. The contact term favors $\rho_{3/2,3/2}$ and leads to a 3:1 ratio of $\rho_{3/2,3/2}:\rho_{1/2,1/2}$ in the forward direction. The other terms lead to a strong $\rho_{1/2,1/2}$ and this accounts for the rapid variation near the forward direction. Confirmation of such a behavior will demonstrate the remarkable smallness of the other amplitudes.

Recently, data have become available for the reactions (b)–(d).[16] These are compared with MPE calculations in Fig. 7. We may conclude from the results that, although the shape predictions are consistent with the present experimental information, the interpretation is by no means definite yet. In particular, the error bars are largest in the region of the greatest predictive power of the low-t theorems.

In conclusion, we can say that although we may still be far from a general understanding of the structure of photoproduction amplitudes, it proves to be very useful to follow the predictions of the low-t theorems for the low-t structure. That is particularly true in $\gamma p\rightarrow\pi^-\Delta^{++}$, where the MPE seems to be the dominant term. It still has to be understood why the other amplitudes are so small, but it is certainly comforting to know that one finds a term that has to be there.

ACKNOWLEDGMENTS

We would like to thank B. Manny for assistance in numerical computations and our colleagues in Austin, SLAC, and Caltech for many stimulating discussions.

[16] A. M. Boyarski, R. Diebold, S. D. Ecklund, G. E. Fischer, Y. Murata, B. Richter, and M. Sands, in *Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies* (Daresbury Nuclear Physics Laboratory, Daresbury, England, 1969), and private communication.

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Dips at Nonsense Wrong-Signature Points

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A model for residues of odd- and even-signature trajectories is presented which successfully accounts for the presence of dips in the differential cross sections of two-body reactions near the forward directions.

I. INTRODUCTION

THE differential cross section for two-body or quasi-two-body final-state collisions often exhibits a dip at $t\sim -0.6$ GeV^2. This dip has normally been associated with the vanishing of the residues of the ρ and ω even-signature trajectories at the nonsense wrong-signature point, corresponding to the above t value.[1] However, the seemingly erratic presence or absence of this dip in different reactions has cast doubt on this explanation. We shall offer an interpretation which does rely on the vanishing of the above residues at $t=-0.6$ GeV^2. Through the use of duality and the existence of exotic channels, we shall be able to determine for which reactions dips should or should not be observed.[2] Before stating the results of our model, we shall indicate the assumptions which are implicit in the model.

(i) Dominant features of two-body reactions are given by the Regge-pole model. For the region of momentum transfers of interest, the trajectories considered will be the Pomeranchukon P and the quartet of vector and tensor trajectories ρ, ω, P', and A_2.

(ii) Departures from the pure Regge-pole model (with the above trajectories) are significant only in the region where contributions of the dominant trajectories vanish. These deviations may be due to cuts or lower-lying trajectories. In cross sections, these deviations will manifest themselves in their interference with the dominant terms.

(iii) The dominant non-Pomeranchukon trajectories

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[1] For a recent review of Regge-pole lore see G. E. Hite, Rev. Mod. Phys. 41, 669 (1969).

[2] A previous use of duality and the existence of exotic channels to account for these dips was presented by J. Finkelstein, Phys. Rev. Letters 22, 362 (1969).