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Fixed-u Finite-Energy-Sum-Rule Calculations and the CERN πN Amplitudes*

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It is shown that Regge asymptotic πN scattering amplitudes constructed from the CERN model-independent experimental (EXP) phase shifts possess better convergence properties than the same amplitudes constructed from the CERN resonance-model dispersion-relation (TH) phases used previously in fixed-u finite-energy sum rules. The EXP phases provide a better description of the πN data since they are fitted to them directly, while the TH phases are only an attempt to approximate the EXP phases by a resonance model satisfying dispersion-relation constraints. However, substituting the EXP phases for the TH ones does not greatly alter the sum-rule results. The calculations are also extended to higher moments and suggestions are made for improving the calculation.

IN a recent paper Barger, Michael, and Phillips[1] used fixed-u finite-energy sum rules (FESR) to show the consistency of standard representations of the low-energy parts of the s- and t-channel contributions to πN scattering with Regge fits to baryon exchange at high energies. For the low-energy s-channel integral they used the CERN dispersion-relation-smoothed TH amplitudes[2] and resonance saturation for the low-energy t-channel integral. However, since the TH amplitudes were the result of an attempt to approximate the EXP amplitudes by a resonance model satisfying dispersion-relation constraints, it should not be surprising that the EXP amplitudes are a better fit to the data, as has been found to be the case.[3] We have found that, in addition, the poorer-fitting TH amplitudes seem to bear little relation to the Regge amplitudes at the finite-energy upper limit of the low-energy FESR integral. This is disturbing since the derivation of the FESR relies upon each amplitude being at approximately its Regge value beyond the finite-energy point. As can be seen in Fig. 1, however, the CERN EXP amplitudes are a considerable improvement over the TH amplitudes in this respect. Thus on two accounts the EXP amplitudes are superior to the TH amplitudes, especially for use in sum rules.

We have reevaluated the fixed-u FESR of Barger *et al.*, with the TH amplitudes replaced by the EXP ones. These sum rules are of the form

$$\frac{1}{16\pi}\int_{-x_0}^{x_0} X^n \,\mathrm{Disc} f^{\tau P}(X,u)dX = \frac{1}{16\pi}\int_{-x_0}^{x_0} X^n \,\mathrm{Disc} f^{\tau P}(X,u)_{\mathrm{Regge}}dX\,,$$

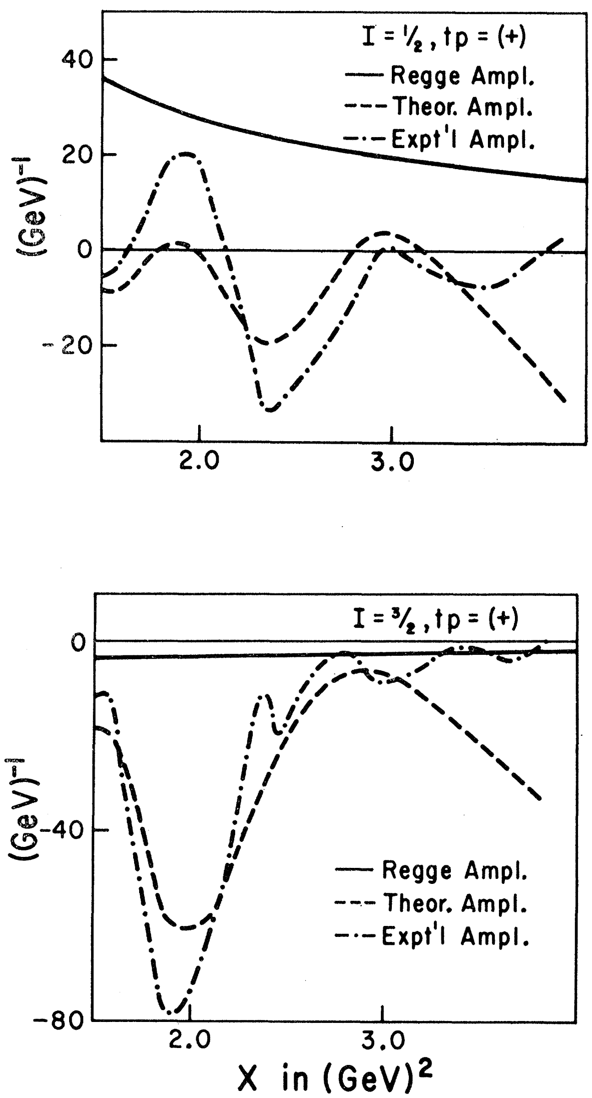

FIG. 1. Imaginary parts of the $u=0$ Regge, TH, and EXP $\tau P=(+)$ amplitudes (see text) in the neighborhood of the finite-energy point X_0. At this zero value of u, the corresponding amplitudes for $\tau P=(-)$ are simply the negative of those shown.

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[1] V. Barger, C. Michael, and R. J. N. Phillips, Phys. Rev. 185, 1852 (1969).
[2] A. Donnachie, R. G. Kirsopp, and C. Lovelace, CERN Report No. Th838 Addendum (unpublished).
[3] A. D. Brody, D. W. G. S. Leith, B. S. Levi, B. C. Shen, D. Herndon, R. Longacre, L. Price, A. H. Rosenfeld, and P. Söding, Phys. Rev. Letters 22, 1401 (1969); see also C. Lovelace and A. Donnachie, Phys. Rev. D 1, 956 (1970).

where

$$X=\tfrac{1}{2}(s-t).$$

As with Barger *et al.*, the t-channel contributions were calculated using the approximation of resonance saturation with σ, ρ, f, and g poles. While Barger *et al.* calculated only the lowest nonvanishing moment, namely, $n=1$, we have also computed some higher moments. Our results for the fixed-u FESR moments are shown in Fig. 2, along with the results using the TH amplitude. Although the TH and EXP amplitudes are substantially different (Fig. 1), they produce remarkably similar integrated curves (Fig. 2). One may say that Barger, Michael, and Phillips's conclusions are essentially unaltered, viz., that the low-energy moments are reasonably consistent with the moments obtained from a Regge description of the u-channel N_α and N_β amplitudes at high energies.

It should be noted that Barger *et al.* fixed the coupling constants for the f ang g mesons by requiring that the lowest nonzero moments of the N_γ, N_δ, Δ_α, and Δ_β amplitudes be very small. Our EXP-amplitude predictions for such moments, using the values of Barger *et al.* for the f and g coupling constants, are in agreement with this constraint. Thus our EXP curves in Fig. 2 should be regarded as predictions rather than statistical fits. Furthermore, any differences between the TH and EXP curves are due solely to differences in the TH and EXP s-channel amplitudes. We have reported only the N_α and N_β cases, since the Δ_γ and Δ_δ moments have small magnitudes and large uncertainties.

The agreement between the left- and right-hand sides of the N_α FESR is good, particularly in light of the rather crude t-channel input. Notice the rapid deterioration of the fit for higher moments. This is undoubtedly because of the rapid increase with moment number in the weight of singularities which are intercepted at high values of $|X|$, particularly the $N\bar{N}\rightarrow\pi\pi$ physical region ($t\geq 4m_N^2$), which has been neglected.

The N_β FESR is well satisfied in the restrictive sense that both the low- and high-energy terms are small compared to their values in the N_α case. In no case, however, does the low-energy physical-amplitude moment reproduce the zero at the mass squared of the nucleon, postulated to account for the absence of the parity partner of the nucleon. Replacing the 437-MeV σ meson of Barger *et al.* by the more plausible ϵ meson (765–900 MeV) can have virtually no effect on this situation. This is because both the σ and ϵ are intercepted at small values of X and hence carry very little weight in the sum rules, a fact borne out in our calculations.[4] Furthermore, the small low-energy physical-

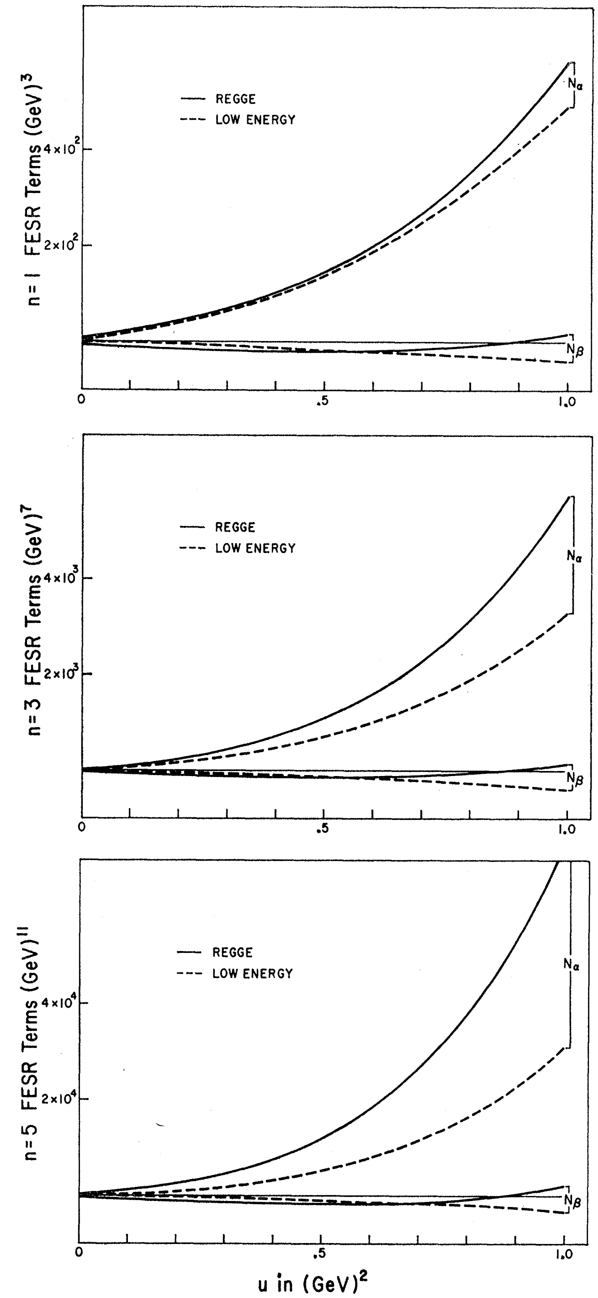

FIG. 2. Contributions to the FESR for u-channel isospin $\frac{1}{2}$, corresponding to N_α and N_β Regge exchange. Solid curves denote the moments of the Regge amplitudes; dashed curves denote the moments of the corresponding low-energy physical amplitudes. Even moments are required to vanish. If the FESR were exactly satisfied, the solid and dashed curves for individual exchanges would coincide. The TH and EXP amplitude *moments* are indistinguishable on the scale shown.

amplitude term in the N_β FESR is produced by the *addition* of small negative s- and t-channel contributions rather than through *cancellation* between large terms of opposite sign which might be sensitive to small adjustments in one of the terms. There is then no

[4] For increasing positive values of u, the dominance of the t-channel contribution to the sum rule by high-mass terms can be summarized as $\sigma\ll\rho\ll f\ll g$. This is true even for small moment number n. The relative magnitudes of these contributions could essentially be reversed, however, by applying the sum rule at large negative values of u. The weighting would then be similar to that in dispersion relations.

refinement in the representation of the low-energy processes which seems likely to reproduce this postulated feature of the N_β Regge term. In the end this situation cannot be regarded as a limitation of the sum rules, since the high-energy data were represented exclusively by Regge poles. The inclusion of Regge cuts might sufficiently modify the u dependence of the N_β Regge term so as to displace or even eliminate the zero.

Regarding the convergence properties of the s-channel amplitudes near X_0, it should be noted that while the EXP amplitudes are to be preferred over the TH amplitudes, one could hope for better convergence properties then even the EXP amplitude possesses. However, the state of the description of the t-channel processes is so elementary that probably little can be accomplished by extending X_0 beyond the limit of available phase-shift analyses until higher t-channel states are taken into account.

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Continuous-Moment Sum Rules and Pion Photoproduction Data

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Continuous-moment sum rules with different weight functions are studied for the single-pion photoproduction process. Even granting the usual ambiguities on the high-energy side, it seems that presently available very low-energy data may be in considerable error.

A CLASS of dispersion sum rules called finite-energy sum rules[1] and their powerful generalizations, continuous-moment sum rules (CMSR),[2–5] have been studied extensively in recent years. The left-hand side of such sum rules contains integrals over purely imaginary parts or combinations of real and imaginary parts of the scattering amplitudes, multiplied by suitable weight functions. The right-hand side usually consists of contributions due to one or more Regge poles, Regge cuts, etc. The procedure that is usually adopted is to predict Regge trajectory parameters by substituting the available low-energy data. It is generally believed that the right-hand side has a number of ambiguities regarding the number of Regge poles, Regge cuts, nonleading poles, etc. Also, in the case of pion photoproduction, the energy up to which low-energy data are available may be too low for the Regge behavior to dominate. Thus it would seem that one can hardly make any statement about implications for the low-energy data from the analysis of the high-energy side. In the present note we suggest that, if one studies a number of sum rules weighting different parts of the low-energy data, one can draw some inferences about the low-energy data, even granting the ambiguities on the high-energy side. As a particular case, we have studied the single-pion photoproduction sum rules.

For the sake of definiteness, we will consider the two amplitudes

$$F_1^- = A_1^- - 2mA_4^- \quad \text{and} \quad F_2^-/(\mu^2 - t) = A_1^- + tA_2^- \qquad (1)$$

in the usual notation for photoproduction processes.[6] It can be readily shown that the A_2 trajectory contributes to F_1^- and the π trajectory to F_2^-. Also the pion conspirator, if it exists, will contribute to F_1^- but the A_2 contribution will be dominant near $t=0$.

Then as shown in I and II, for both of these amplitudes one can get a CMSR of the form

$$\frac{1}{\pi}\int_{\nu_0}^{\bar{\nu}} d\nu(\nu^2-\nu_0^2)^{-\gamma}[\cos\pi\gamma \,\mathrm{Im}F(\nu,t) + \sin\pi\gamma\,\mathrm{Re}F(\nu,t)] + r(\nu_0^2-\nu_P^2)^{-\gamma} = -\frac{\alpha(t)\beta(t)\sin\pi(\frac{1}{2}\alpha-\gamma)}{2\pi\sin\frac{1}{2}\pi\alpha(\alpha-2\gamma)}\frac{(\bar{\nu})^{\alpha-2\gamma}}{(\nu_1)^{\alpha-1}}. \qquad (2)$$

Here we have

$$\nu=(s-u)/4m, \quad \nu_0=\mu+(t+\mu^2)/4m, \qquad (3)$$

and γ is a number less than $+1$. We have assumed that

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