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Electromagnetic Final-State Interactions and Tests of Time-Reversal Invariance in Nuclear Beta Decay. II*

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Expressions are given for the electromagnetic final-state interaction contribution to D, the triple-product correlation coefficient, in the nuclear β-decay transitions $\frac{1}{2}^{\pm}\rightarrow\frac{3}{2}^{\pm}$, $\frac{3}{2}^{\pm}\rightarrow\frac{5}{2}^{\pm}$, and $\frac{5}{2}^{\pm}\rightarrow\frac{7}{2}^{\pm}$ to all orders of $Z\alpha/v_e$ in the Coulomb interaction. The contribution to D from the final-state magnetic scattering is investigated to order α.

I. COULOMB INTERACTION

In a recent paper,[1] henceforth denoted as I, we gave expressions for the electromagnetic final-state interaction contribution to D, the triple-product correlation coefficient, in the nuclear β-decay ($N_I\rightarrow N_F+e^-+\bar{\nu}_e$) transitions $\frac{1}{2}\rightarrow\frac{1}{2}$, $\frac{3}{2}\rightarrow\frac{3}{2}$, $\frac{3}{2}^{\pm}\rightarrow\frac{1}{2}^{\pm}$, $\frac{5}{2}^{\pm}\rightarrow\frac{3}{2}^{\pm}$, and $1^+\rightarrow 0^+$ to all orders of $Z\alpha/v_e$ in the Coulomb interaction. We emphasized that allowed Gamow-Teller transitions ($\Delta J=1$, no parity change, and the Fermi matrix element $M_F=0$) were the best place to look for an experimental measurement of D regardless of whether or not time-reversal invariance is broken in the semileptonic weak-interaction Hamiltonian.[2] In this section we give similar results for the transitions $\frac{1}{2}^{\pm}\rightarrow\frac{3}{2}^{\pm}$, $\frac{3}{2}^{\pm}\rightarrow\frac{5}{2}^{\pm}$, and $\frac{5}{2}^{\pm}\rightarrow\frac{7}{2}^{\pm}$. For the usual introduction, notation, and methods of calculation, the reader is referred to I.

If we consider the initial nucleus polarized with $|m_I|=J_I$, then for each of these transitions we find a common form for the electromagnetic (EM) and time-reversal violating (TRV) contributions to D. With $W=E_e+E_\nu$ we have

$$\xi=[\xi_0|g|^2]+\xi_1\,\mathrm{Re}(gf_M^*)\left(\frac{2}{M_pE_e}\right)(E_eE_\nu-p_e^2)+\xi_2\frac{|f_M|^2}{2M_p^2}\left(p_e^2+E_\nu^2-\frac{p_e^2E_\nu}{E_e}\right),\tag{1}$$

$$\begin{aligned}D^{\mathrm{EM}}(\beta^{\mp})\xi=&\pm D_1\frac{\mathrm{Re}(gf_M^*)p_e}{M_pE_e}(E_e+m_e)\left[\sin(\eta_1-\eta_4)+\left(\frac{p_e}{E_e+m_e}\right)^2\sin(\eta_2-\eta_3)\right]\\&+D_2\frac{|f_M|^2p_e}{M_p^2E_e}\Big\{m_eE_\nu\sin(\eta_1-\eta_2)-\tfrac{1}{2}p_e^2[\sin(\eta_1-\eta_4)+\sin(\eta_2-\eta_3)]\\&+\tfrac{1}{4}E_\nu(E_e+m_e)\left[\sin(\eta_1-\eta_4)+\left(\frac{p_e}{E_e+m_e}\right)^2\sin(\eta_2-\eta_3)\right]\Big\},\end{aligned}\tag{2}$$

and

$$D^{\mathrm{TRV}}(\beta^{\mp})\xi=\pm D_3\frac{2p_eW\,\mathrm{Im}(gf_M^*)}{M_pE_e}.\tag{3}$$

Case $\frac{1}{2}^{\pm}\rightarrow\frac{3}{2}^{\pm}$. For this case we take our nuclear matrix element to be $J_\mu=J_\mu^\dagger(\frac{3}{2}\rightarrow\frac{1}{2};\,g^*,\,f_M^*)$. Hence, we have

$$J^\mu=\bar{u}_\alpha(p_F)\left[\gamma^\mu(1-g\gamma_5)-\frac{if_M}{2M_p}\sigma^{\mu\nu}q_\nu\right]\gamma_5\gamma^\alpha u(p_I),$$

* Research supported in part by the U. S. Atomic Energy Commission.

[1] J. C. Brodine, Phys. Rev. D 1, 100 (1970); 2, 627(E) (1970).

[2] See also C. W. Kim and H. Primakoff, Phys. Rev. 180, 1502 (1969); H. H. Chen, *ibid.* 185, 2007 (1969).

TABLE I. Values of the coefficient D_0^{EM} in the equation $D^{EM}\approx(D_0^{EM})(f_M E_e/W)$ for various cases of Gamow-Teller β-decay transitions given in this paper and in I. $|g|$ is obtained from the ft values using $-G_A/G_V=1.20$ and $ft(n\to p+e^-+\bar{\nu}_e)=1180$ sec.

Transition	(Z)	ΔJ	$\log_{10}ft$[a]	$\lvert g\rvert$	W (MeV)	$10^3\times(D_0^{EM})$
$Si^{31}\to P^{31}+e^-+\bar{\nu}_e$	(15)	$\frac{3}{2}\to\frac{1}{2}$	5.51	0.070	1.99	1.3
$Br^{77}\to Se^{77}+e^++\nu_e$	(34)	$\frac{3}{2}\to\frac{1}{2}$	5.36	0.083	0.85	1.0
$As^{77}\to Se^{77}+e^-+\bar{\nu}_e$	(34)	$\frac{3}{2}\to\frac{1}{2}$	5.76	0.0522	1.20	2.7
$Br^{83}\to Kr^{83}+e^-+\bar{\nu}_e$	(36)	$\frac{3}{2}\to\frac{1}{2}$	5.13	0.108	1.45	1.7
$P^{33}\to S^{33}+e^-+\bar{\nu}_e$	(16)	$\frac{1}{2}\to\frac{3}{2}$	5.05	0.084	0.76	0.17
$Se^{81}\to Br^{81}+e^-+\bar{\nu}_e$	(35)	$\frac{1}{2}\to\frac{3}{2}$	4.72	0.122	2.11	0.69
$Ge^{69}\to Ga^{69}+e^++\nu_e$	(31)	$\frac{5}{2}\to\frac{3}{2}$	6.40	0.025	1.72	5.7
$Ni^{65}\to Cu^{65}+e^-+\bar{\nu}_e$	(29)	$\frac{5}{2}\to\frac{3}{2}$	6.56	0.021	2.61	11.6
$Zn^{65}\to Cu^{65}+e^++\nu_e$	(29)	$\frac{5}{2}\to\frac{3}{2}$	7.34	0.0085	0.84	8.5
$Ni^{63}\to Cu^{63}+e^-+\bar{\nu}_e$	(29)	$\frac{5}{2}\to\frac{3}{2}$	6.56	0.021	0.58	2.9
$I^{121}\to Te^{121}+e^++\nu_e$	(52)	$\frac{5}{2}\to\frac{3}{2}$	5.05	0.118	1.64	2.6
$Sn^{123}\to Sb^{123}+e^-+\bar{\nu}_e$	(51)	$\frac{3}{2}\to\frac{5}{2}$	5.27	0.075	1.77	2.6
$Sn^{121}\to Sb^{121}+e^-+\bar{\nu}_e$	(51)	$\frac{3}{2}\to\frac{5}{2}$	5.00	0.102	0.89	1.0
$Te^{127}\to I^{127}+e^-+\bar{\nu}_e$	(53)	$\frac{3}{2}\to\frac{5}{2}$	5.66	0.048	1.20	3.0
$Xe^{133}\to Cs^{133}+e^-+\bar{\nu}_e$	(55)	$\frac{3}{2}\to\frac{5}{2}$	5.58	0.0525	0.86	2.0
$Cu^{67}\to Zn^{67}+e^-+\bar{\nu}_e$	(30)	$\frac{3}{2}\to\frac{5}{2}$	6.30	0.023	1.00	2.7
$Co^{61}\to Ni^{61}+e^-+\bar{\nu}_e$	(28)	$\frac{7}{2}\to\frac{5}{2}$	5.20	0.100	1.73	1.6
$Nb^{97}\to Mo^{97}+e^-+\bar{\nu}_e$	(42)	$\frac{9}{2}\to\frac{7}{2}$	5.35	0.084	1.78	3.1
$Nb^{95}\to Mo^{95}+e^-+\bar{\nu}_e$	(42)	$\frac{9}{2}\to\frac{7}{2}$	5.08	0.114	0.67	0.89

[a] Reference 3.

where $g=(-G_A/G_V)M_{GT}/\sqrt{8}$. We find $\xi_0=8$, $\xi_1=8/3$, $\xi_2=8/3$, $D_1=2/3$, $D_2=4/9$, and $D_3=\frac{2}{3}$. Using Eq. (3) from I, dropping terms in $|f_M|^2$ from $D^{EM}\xi$ and terms in f_M from ξ, we have to first order in $Z\alpha/v_e$

$$D^{EM}(\beta^{\mp})\approx\frac{\mathrm{Re}(gf_M^*)Z\alpha E_e}{24|g|^2M_p}\left[3+\left(\frac{m_e}{E_e}\right)^2\right]. \tag{4}$$

Case $\frac{3}{2}^{\pm}\to\frac{5}{2}^{\pm}$. We take our nuclear matrix element to be $J_\mu=J_\mu^\dagger(\frac{5}{2}\to\frac{3}{2};\,g^*,\,f_M^*)$. Hence, we have

$$J^\mu=\bar{u}_{\alpha\beta}(p_F)\left[\gamma^\mu(1-g\gamma_5)-\frac{if_M}{2M_p}\sigma^{\mu\nu}q_\nu\right]\gamma_5\gamma^\beta u^\alpha(p_I)\,,$$

where $g=(-G_A/G_V)M_{GT}/\sqrt{6}$. We find $\xi_0=6$, $\xi_1=19/10$, $\xi_2=19/10$, $D_1=9/10$, $D_2=\frac{3}{5}$, and $D_3=9/10$. To first order in $Z\alpha/v_e$, we find

$$D^{EM}(\beta^{\mp})\approx\frac{3\,\mathrm{Re}(gf_M^*)Z\alpha E_e}{40|g|^2M_p}\left[3+\left(\frac{m_e}{E_e}\right)^2\right]. \tag{5}$$

Case $\frac{5}{2}^{\pm}\to\frac{7}{2}^{\pm}$. We take our nuclear matrix element to be $J_\mu=J_\mu^\dagger(\frac{7}{2}\to\frac{5}{2};\,g^*,\,f_M^*)$. Hence, we have

$$J^\mu=\bar{u}_{\alpha\beta\delta}(p_F)\left[\gamma^\mu(1-g\gamma_5)-\frac{if_M}{2M_p}\sigma^{\mu\nu}q_\nu\right]\gamma_5\gamma^\delta u^{\alpha\beta}p(_I)\,,$$

where $g=(-G_A/G_V)4M_{GT}/\sqrt{3}$. We find $\xi_0=16/3$, $\xi_1=34/21$, $\xi_2=34/21$, $D_1=20/21$, $D_2=40/63$, and $D_3=20/21$. To first order in $Z\alpha/v_e$, we find

$$D^{EM}(\beta^{\mp})\approx\frac{5\,\mathrm{Re}(gf_M^*)Z\alpha E_e}{28|g|^2M_p}\left[3+\left(\frac{m_e}{E_e}\right)^2\right]. \tag{6}$$

In Table I we show values of the coefficient D_0^{EM} in the equation

$$D^{EM}\approx(D_0^{EM})(f_ME_e/W)$$

for various $\Delta J=1$ transitions which might be of some experimental utility. D_0^{EM} is the maximum value of D^{EM} given by Eqs. (1)–(3). The D's for the transitions $J_I=N+\frac{1}{2}\to J_F=N-\frac{1}{2}$ have been given in Eqs. (10)–(12) of I. The behavior of D as a function of p_e for these transitions is essentially identical to that shown in Figs. 1 and 2 of I. Table II shows the maximum value of D^{EM} for some $\Delta J=1$ boson transitions.[1]

II. MAGNETIC SCATTERING

In I and in Sec. I of this paper we calculated the Coulomb part of D^{EM} with phase shifts for the scattering of a relativistic electron with a spinless target. If the final nucleus has spin $S_F\neq0$, then it may have a magnetic moment $\boldsymbol{\mu}_F=\mu_Ng_F\mathbf{S}_F$, where μ_N is the nucleon magneton $e/2M_p$. The "magnetic moment" is usually denoted $\mu_F=g_FS_F$; hence, $\boldsymbol{\mu}_F=(\mu_N\mu_F/S_F)\mathbf{S}_F$. The scattering of the electron charge ($e=|e|$) off the final nuclear magnetic moment yields a contribution to D^{EM} called D^{MAG} since there is an interaction potential

$$V(\mathbf{r})=+e\boldsymbol{\alpha}\cdot\mathbf{A}(\mathbf{r})\,,$$

where $\mathbf{A}(\mathbf{r})=[\boldsymbol{\mu}_F\times\mathbf{r}]/r^3$. $D^{MAG}(\frac{1}{2}\to\frac{1}{2})$ has been calculated by Callan and Treiman[4] and $D^{MAG}(\frac{3}{2}\to\frac{3}{2})$ has been calculated by Chen.[5] We will obtain D^{MAG} for the Gamow-Teller transitions of Sec. I.

[3] E. J. Konopinski, *The Theory of Beta Radioactivity* (Clarendon, Oxford, England, 1966), pp. 151 and 157.
[4] C. G. Callan and S. B. Treiman, Phys. Rev. **162**, 1494 (1967).
[5] H. H. Chen, Phys. Rev. **185**, 2003 (1969).

TABLE II. Values of the coefficient D_0^{EM} in the equation $D^{\mathrm{EM}}\approx(D_0^{\mathrm{EM}})(E_e/W)$ for various cases of boson Gamow-Teller β-decay transitions. $|F_A|$ is obtained from the ft value using $-G_A/G_V=1.20$ and $ft(n\rightarrow p+e^-+\bar{\nu}_e)=1180$ sec.

Transition	(Z)	ΔJ	$\log_{10}ft$[a]	$\lvert F_A\rvert$	$\lvert F_M/F_A\rvert$[b]	W (MeV)	$10^3\times(D_0^{\mathrm{EM}})$
$P^{32}\rightarrow S^{32}+e^-+\bar{\nu}_e$	(16)	$1\rightarrow 0$	7.9	0.0063	50[c]	2.22	5.7
$P^{34}\rightarrow S^{34}+e^-+\bar{\nu}_e$	(16)	$1\rightarrow 0$	5.2	0.14	1	5.61	0.28
$B^{12}\rightarrow C^{12}+e^-+\bar{\nu}_e$	(6)	$1\rightarrow 0$	4.1	0.50	4.7	13.9	1.2
$N^{12}\rightarrow C^{12}+e^++\nu_e$	(6)	$1\rightarrow 0$	4.2	0.45	4.7	16.9	1.3
$F^{18}\rightarrow O^{18}+e^++\nu_e$	(8)	$1\rightarrow 0$	3.62	0.866	6.1	1.16	0.17
$Cu^{64}\rightarrow Zn^{64}+e^-+\bar{\nu}_e$	(30)	$1\rightarrow 0$	5.3	0.125	1	1.08	0.11
$Cu^{66}\rightarrow Zn^{66}+e^-+\bar{\nu}_e$	(30)	$1\rightarrow 0$	5.3	0.125	1	3.14	0.31
$In^{114}\rightarrow Sn^{114}+e^-+\bar{\nu}_e$	(50)	$1\rightarrow 0$	4.4	0.355	19[c]	2.51	8.6
$Mn^{56}\rightarrow Fe^{56}+e^-+\bar{\nu}_e$	(26)	$3\rightarrow 2$	7.2	0.014	1	3.36	0.28
$Na^{22}\rightarrow Ne^{22}+e^++\nu_e$	(10)	$3\rightarrow 2$	7.4	0.011	1	1.05	0.032
$Co^{60}\rightarrow Ni^{60}+e^-+\bar{\nu}_e$	(28)	$5\rightarrow 4$	7.5	0.010	1	0.85	0.084

[a] Reference 3.
[b] When the value of $|F_M|$ is unknown, we have taken $|F_M/F_A|=1.0$.
[c] Large experimental uncertainty.

Let $G_1=\mu_F/Z$, $\mathbf{j}=\mathbf{l}+\mathbf{S}_e$, and $\mathbf{J}=\mathbf{S}_F+\mathbf{j}$. In order to incorporate the magnetic scattering into our calculation, we must replace the phase shifts $\eta(j^\pi)$ of I by the total phase shifts $\epsilon(Jjl)\approx\eta(jl)+\sigma(Jjl)$, where the σ's are the magnetic-scattering phase shifts. A well-known formula from elementary quantum mechanics for the phase shifts due to a potential leads in the Born approximation to the generalized result

$$\sigma^\omega(Jjl)\approx\sin\sigma^\omega(Jjl)$$
$$\approx\frac{-p_e(E_e+m_e)}{N^2}\int\Psi_{Jjl}{}^{\dagger\omega}(\mathbf{r})V(\mathbf{r})\Psi_{Jjl}{}^{\omega}(\mathbf{r})d^3r\,,\quad(7)$$

where Ψ_{Jjl} is the total state function of the final nucleus-electron system. Following Messiah,[6] we find $(k=|\mathbf{p}_e|)$

$$|\Psi_{Jjl}{}^\omega\rangle=\sum_{m,m_F}\langle S_Fjm_Fm|JM\rangle|S_Fm_F\rangle|\psi_{kjml}{}^\omega\rangle\,,$$

with the spherical wave electron states given by

$$\psi_{kjml}{}^\omega(\mathbf{r})=\frac{1}{r}\begin{pmatrix}F_{kl}(r)\mathcal{Y}_{lj}{}^m(\hat{r})\\ iG_{kl'}(r)\mathcal{Y}_{l'j}{}^m(\hat{r})\end{pmatrix},$$

where $F_{kl}(r)=Nrj_l(kr)$,

$$G_{kl'}(r)=[N\omega k/(E_e+m_e)]rj_{l'}(kr),$$

$l=j+\frac{1}{2}\omega$, $l'=j-\frac{1}{2}\omega$, and $\omega=\pm$ when $j=l\mp\frac{1}{2}$. The normalization

$$\langle\psi_{k'jml}{}^\omega|\psi_{kjml}{}^\omega\rangle$$
$$=\delta(k-k')=\int_0^\infty[F_{kl}F_{k'l}{}^*+G_{kl'}G_{k'l'}{}^*]dr$$

yields $N=(2/\pi)^{1/2}k[(E_e+m_e)/2E_e]^{1/2}$. With the nuclear matrix elements given in I and in Sec. I above, we find the following possible states $|J_{jl}\rangle^\omega$ of the system: $|0\frac{1}{2}0\rangle^-=|1\rangle$, $|0\frac{1}{2}1\rangle^+=|3\rangle$, $|1\frac{1}{2}0\rangle^-=|2\rangle$, $|1\frac{1}{2}1\rangle^+=|4\rangle$, $|1\frac{3}{2}1\rangle^-=|5\rangle$, and $|1\frac{3}{2}2\rangle^+=|6\rangle$, where we have taken the case $S_F=\frac{1}{2}$. Since we are not interested in terms in D^{MAG} proportional to $(W/M_p)^2$, we set $f_M=0$ above and ignore the bottom components of the nuclear spinors; then, the weak interaction does not initially produce states $|5\rangle$ and $|6\rangle$. However, our electromagnetic S matrix is not diagonal in representation $|1\rangle\cdots|6\rangle$. If we diagonalize in a representation $|\mathrm{I}\rangle\cdots|\mathrm{VI}\rangle$, we find that the state mixing of $|2\rangle$ and $|6\rangle$ and $|4\rangle$ and $|5\rangle$ is so small that diagonalization is not necessary. If $S_{26}(1^+)=i\mu e^{i(\epsilon_2+\epsilon_6)}$, we find $\mu=(\sqrt{2}/6)(G_1Z\alpha)(p_e/M_p)$, and the mixing parameter between $|2\rangle$ and $|6\rangle$ is $\lambda=(\sqrt{2}/9)\times G_1[(E_e-m_e)/M_p]\lesssim3\times10^{-4}$ for all reasonable cases. It is apparent that diagonalization effects are of order $|\lambda|^2$ which can be neglected. We find $D^{\mathrm{MAG}}\propto(\sigma_1-\sigma_2)=\frac{4}{3}(G_1Z\alpha)(p_e/M_p)$ for $S_F=\frac{1}{2}$. For $S_F>\frac{1}{2}$ the σ's must be redefined. If $S_F=\frac{3}{2}$, then $\sigma_1-\sigma_2=(8/9)(G_1Z\alpha)\times(p_e/M_p)$, and if $S_F=\frac{5}{2}$, then $\sigma_1-\sigma_2=\frac{4}{5}(G_1Z\alpha)(p_e/M_p)$. The results for D^{MAG} are

$$D^{\mathrm{MAG}}(\beta^\mp;\tfrac{1}{2}\rightarrow\tfrac{1}{2})=\frac{\pm Z\alpha E_e}{4(1+3|g|^2)M_p}\times\left\{-\frac{8}{3}G_1(1\pm3g)(1\mp g)\left[1-\left(\frac{m_e}{E_e}\right)^2\right]\right\},\quad(8)$$

$$D^{\mathrm{MAG}}(\beta^\mp;\tfrac{3}{2}\rightarrow\tfrac{3}{2})=\frac{\pm Z\alpha E_e}{4[1+(5/3)|g|^2]M_p}\times\left\{-\frac{8}{9}G_1(3\pm5g)(1\mp g)\left[1-\left(\frac{m_e}{E_e}\right)^2\right]\right\},\quad(9)$$

and

$$D^{\mathrm{MAG}}(\beta^\mp;J_I=N\pm\tfrac{1}{2}\rightarrow J_F=N\mp\tfrac{1}{2})=0.\quad(10)$$

Equation (8) gives 8/9 times D^{MAG} (Callan-Treiman)[4] and Eq. (9) gives 8/9 times D^{MAG}(Chen).[5] One way of understanding Eq. (10) is to note that with our approxi-

[6] A. Messiah, *Quantum Mechanics* (Interscience, New York, 1961), Vol. II, pp. 925ff.

mation of neglecting terms in J_μ proportional to q/M_p, we get only final states with $j=j_e=j_\nu=\frac{1}{2}$. It is easy to see by conservation of angular momentum that in the transition $J_I=N\pm\frac{1}{2}\to J_F=N\mp\frac{1}{2}$ ($J_F=J_I\mp 1$), we have only one hyperfine state $J=J_F\pm\frac{1}{2}$. Since there can be no interference, Eq. (10) follows immediately. Hence, we find for any pure Gamow-Teller transition that

$$|D^{\mathrm{MAG}}|\lesssim|D^{\mathrm{COUL}}|(|g|/|f_M|)(R/Z)\,, \qquad (11)$$

where $R\sim O(W/M_p)$. D^{MAG} can be neglected compared to D^{COUL} as long as $|f_M|$ is not too small compared to $|g|$.[7] If we use experiments on $1\to 0$ transitions as a guide, we expect[8] that $|f_M|\gtrsim|g|$ and $|D^{\mathrm{MAG}}|\ll|D^{\mathrm{COUL}}|$.

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[7] There will also be a contribution $D^{\mathrm{MAG}}\sim D^{\mathrm{COUL}}(R/Z)$ from weak magnetism Gamow-Teller interference and another contribution $D^{\mathrm{MAG}}\sim D^{\mathrm{COUL}}(|g|/|f_M|)R/(AZ)$ from the bottom components of the nuclear spinors. These may also be neglected.

[8] We expect that $|f_M/g|$ for fermion transitions is of order $|F_M/F_A|$ (see Ref. 1) for boson transitions. Experimentally we have $|F_M/F_A|\approx 4.7$ for B^{12}, $N^{12}\to C^{12}+e+\nu_e$; $|F_M/F_A|\approx 40$ for $P^{32}\to S^{32}+e^-+\nu_e$; and $|F_M/F_A|\approx 6$.

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Chiral-Symmetry-Breaking Corrections to the $K\to 3\pi$ Soft-Pion Theorem

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First-order corrections, due to chiral symmetry breaking, to the $K\to 3\pi$ soft-pion theorem are calculated. The corrections turn out to be independent of any assumption of specific properties of the chrial-symmetry-breaking Hamiltonian.

The success of soft-pion theorems[1] derived from the hypothesis of partially conserved axial-vector current (PCAC) and current algebra has an elegant and simple explanation in the notion that there exists an underlying broken $SU(3)\otimes SU(3)$ symmetry for strong interactions.[2] In this approach the low-energy (or soft-pion) theorems, which are only approximate in the real world, would become exact in the symmetry limit where axial-vector currents are conserved and the pseudoscalar-meson masses vanish. Such a symmetry does not manifest itself in the multiplets of particles, as does $SU(3)$, but through the appearance of eight Goldstone bosons (massless in the symmetry limit). The language of approximate symmetry is useful in that it not only gives a precise meaning to PCAC but can provide a scheme for keeping track of corrections to PCAC approximation. The picture that emerges is that the hadronic Hamiltonian can be decomposed as follows:

$$H=H_0+\epsilon H'\,,$$

where H_0 is invariant under $SU(3)\otimes SU(3)$, and H' breaks the symmetry. ϵ is small so that symmetric-theory predictions approach the real world. In the symmetry limit when $\epsilon\to 0$, the axial-vector currents are conserved, and the symmetry is realized by the appearance of eight massless pseudoscalar mesons. Further, the basic symmetry is broken according to the pattern

$$SU(3)\otimes SU(3)\to SU(2)\otimes SU(2)\to SU(2)\,.$$

Assuming that major deviations from the predictions of symmetric theory may be computed by working to the lowest order in ϵ, there have been computed two types of corrections to the results of the symmetric theory, namely, those which are independent of any specific

[1] See, for example, S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968); R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969), where references to original literature can be found.

[2] The original suggestion that PCAC is related to broken chiral symmetry is due to Nambu and his collaborators. See Y. Nambu and D. Lurié, Phys. Rev. **125**, 1429 (1962), and earlier papers quoted therein. The first paper relating the modern work on current algebras to chiral symmetry seems to be S. Weinberg, Phys. Rev. Letters **16**, 163 (1966). See also S. Weinberg, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 253, where other references can also be found. This point of view has recently been clearly stated by R. F. Dashen, Phys. Rev. **183**, 1245 (1969); R. F. Dashen and M. Weinstein, *ibid.* **183**, 1261 (1969); **188**, 2330 (1969).