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Rescattering, Omnès Equation, and Zeros in Resonance Peaks*

L. Resnick
Physics Department, Carleton University, Ottawa, Canada
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The production of a resonance in a collision is studied from a semiphenomenological point of view as a rescattering effect. The framework is an integral equation of the Muskhelishvili-Omnès type, with the rescattering parameters (e.g., phase shift) assumed known. In particular, the possibility of structure (e.g., zeros) in the line shape is investigated. Coupled singular equations are also examined, and an explicit solution is obtained in a particular case.

I. INTRODUCTION

In this paper, we study the mass spectrum of a resonance produced in a collision, or inelastic experiment [Fig. 1(a)]. The point of view is semiphenomenological, in that a resonance R in the elastic scattering $D+E\rightarrow R\rightarrow D+E$ is assumed, with no attempt being made to calculate the dynamics of this. Rather, the phase shift is assumed known, as well as the basic nonresonant production amplitude T^0 (which might be a Born term, as a meson-exchange diagram, for example), and these are used to calculate the amplitude for production of the resonance [Fig. 1(b)]. The model takes T^0 as the inhomogeneous term in a linear integral equation of the Muskhelishvili[1]-Omnès[2] type, and thus satisfies two-body elastic unitarity in the DE subsystem. There is no attempt made to satisfy three-body unitarity by including particle C. Some account is taken of inelastic effects in the DE rescattering system insofar as coupled two-body channels are examined.

The model is a well-known method[3,4] for describing final-state interactions. In photoproduction and weak processes, the linearization of the unitarity relation results from an obvious physical and mathematical approximation—one expands in powers of a small coupling constant and keeps only lowest order in the small coupling, but all orders in the strong interactions. In the present purely hadronic process, which is a five- (or more-) point function, we similarly write $\text{Im}(T) = e^{-i\delta}\sin\delta\, T$, where δ is the two-body DE rescattering phase shift, and obtain a solution linear in T^0. The justification for ignoring iterations of T^0 in the unitarity relation is less clear now. One appealing argument is that one is dealing with high energies, so that AB interact once, producing C and the DE subsystem with large relative velocities, whereas D and E have relatively low c.m. energy and their rescattering can be important. Thus, one can view the process as a sort of impulse approximation, with C leaving the interaction region quickly, while D and E remain inside each other's force range for a relatively long time. Further discussion and justification of the use of this model in purely hadronic processes is given in Ref. 4. We shall verify below that the model works as expected where it ought to—namely, it leads to a normal resonance peak for the production amplitude T, if the phase shift δ is resonant and T^0 is relatively constant in the resonance region [the case $\delta(\infty)=0$ below]. This tends to lend *a posteriori* credence to the model for the following reason. On physical grounds, we demand $T=T^0$ if $\delta=0$. The model guarantees this in the limit $\delta\rightarrow 0$. So the model is certainly a reasonable method of adding a two-body unitarity correction term for δ very small. But now the same equation is seen to give the desired result even in the case of a resonant δ. In any case, the question of the applicability of this model to a given hadronic process is independent of the features to be discussed below. The crucial assumption is that compared to the DE rescattering, T^0 is to be considered "weak"; this may be because of a truly small coupling constant (electro-

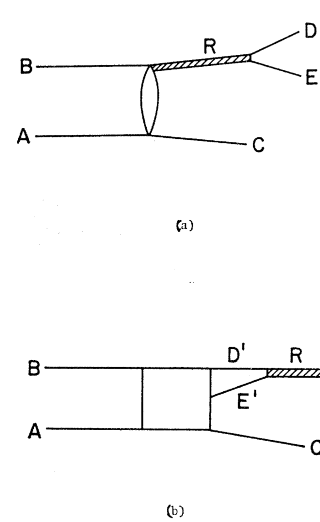

Fig. 1. (a) Inelastic collision $A+B\rightarrow C+R\rightarrow C+D+E$. (b) Model for the process in (a). Here, the square represents T^0, the nonresonant production process $A+B\rightarrow C+D'+E'$. This is followed by the rescattering $D'+E'\rightarrow D+E$, or final-state interaction, in the resonant state R. If there is absorption, D' and E' may differ from D and E.

* Supported in part by a grant from the National Research Council of Canada.

[1] N. I. Muskhelishvili, *Singular Integral Equations* (P. Noordhoff, Groningen, Holland, 1953).

[2] R. Omnès, Nuovo Cimento 8, 316 (1958).

[3] M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).

[4] J. Gillespie, *Final State Interactions* (Holden-Day, San-Francisco, 1964), and references cited herein, especially Ref. [18].

magnetic or weak interaction), a small "effective" coupling constant for some particular hadronic processes, or because of kinematic reasons (the impulse-approximation argument mentioned above).

The single-channel case is first examined, and it is found that in addition to the expected single peak, there are solutions which have a zero near the resonant energy (defined here as the point where $\delta=\frac{1}{2}\pi$). The coupled-channel problem is then formulated, and a particular case of it is solved in closed form. The possible ambiguities attendant upon these solutions are also discussed.

This work was originally undertaken with the A_2 meson in mind. This resonance appears to display a double-peaked structure, with a dip, or zero, in the middle, and to be well fitted with a dipole formula.[5–7] The basic motivation here was to see whether such a structure could also be obtained as an interference effect between the production mechanism and the rescattering amplitude.[8–10] That is, given a δ such that $\sin^2\delta/q^2$ is a typical single-peaked Breit-Wigner line, can one obtain a zero via the model outlined? The answer to this question is yes. However, the resulting line shape is not a good fit to the experimental A_2 distribution, so this model is not a possible explanation for the A_2 splitting. This is discussed further in the conclusions. Rather, we view this model as interesting in its own right, as a description of how interference with the production mechanism can drastically modify the shape of a single-pole Breit-Wigner resonance peak.

II. SINGLE CHANNEL

The physical motivation having been outlined in the Introduction, we write the dispersion and unitarity relations for the amplitude $T(s)$ as follows (s is the energy squared of the DE system in its c.m. system):

$$T_+(s)=T^0(s)+\frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds'}{s'-s-i\epsilon}\,\mathrm{Im}T_+(s') \tag{1}$$

and

$$\mathrm{Im}T_+(s)=e^{-i\delta(s)}\sin\delta(s)\,T_+(s). \tag{2}$$

So the basic equation of the model is the Omnès equation[2]

$$T_+(s)=T^0(s)+\frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds'}{s'-s-i\epsilon}\times e^{-i\delta(s')}\sin\delta(s')\,T_+(s'). \tag{3}$$

[5] B. French, *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 109.

[6] J. V. Beaupre *et al.*, Phys. Rev. Letters **21**, 1849 (1968).

[7] T. J. Gajdicar and J. W. Moffat, Phys. Rev. **181**, 1875 (1969).

[8] Interference effects have also been examined in Refs. 9 and 10.

[9] G. Goldhaber, Phys. Rev. Letters **19**, 976 (1967).

[10] G. Fogli *et al.*, Nuovo Cimento Letters **1**, 352 (1969).

The solution is[1,2]

$$T_+(s)=e^{i\delta(s)}\left[T^0(s)\cos\delta(s)+\frac{\Omega(s)}{\pi}P\int_{s_0}^{\infty}\frac{ds'}{s'-s}\,\frac{\sin\delta(s')}{\Omega(s')}T^0(s')\right], \tag{4}$$

$$\Omega(s)\equiv\exp\left(\frac{1}{\pi}P\int_{s_0}^{\infty}\frac{ds'}{s'-s}\delta(s')\right). \tag{5}$$

$P\int$ means the Cauchy principal value.

We assume $\delta(s)\to n\pi$ as $s\to\infty$ (n an integer) sufficiently rapidly, so that Eqs. (3) and (4) do not require any subtractions. By convention, $\delta(s_0)=0$. If $n\neq 0$, a single subtraction is necessary to define Ω, but this subtraction point cancels in Eq. (4), so this solution is *independent* of the subtraction point. We will call the expression (4) the "fundamental" solution. Since $e^{i\delta}\sin\delta\to 0$ for $s\to\infty$, we may demand on reasonable physical and mathematical grounds [from Eq. (3)] that any solution T_+ satisfy $T_+-T^0\to 0$ as $s\to\infty$. Since any two solutions of Eq. (3) differ by a solution of the homogeneous equation [obtained from Eq. (3) by setting $T^0=0$], the most general solution of Eq. (3) is the fundamental solution Eq. (4) plus the general solution to the homogeneous equation. With the above condition that the homogeneous solution vanish for $s\to\infty$, it is shown in Ref. 1 that Eq. (4) is the unique solution to the problem for $n\leq 0$, while for $n>0$ the general solution is the one that augments Eq. (4) by $P_{n-1}(s)e^{i\delta(s)}\Omega(s)$, with P_{n-1} a polynomial of degree $n-1$. In fact, for $n<0$, Eq. (4) violates the condition $T_+-T^0\to 0$ as $s\to\infty$, unless T^0 obeys certain necessary subsidiary conditions which are fairly restrictive. Furthermore, our interest is in the resonant case with δ starting at 0 and rising through $\frac{1}{2}\pi$. So the case $n<0$ will be ignored, and only $n\geq 0$ considered (in fact, only $n=0$ and $n=1$ will be considered explicitly). For now, only the fundamental solution Eq. (4) will be examined, the "homogeneous" term $P_{n-1}e^{i\delta}\Omega$ being dropped. The case of $P_{n-1}\neq 0$ will be considered in Sec. IV.

What does Eq. (4) look like? It is instructive first to recover the usual single peak. This occurs when $n=0$ [$\delta(\infty)=0$]. For a sharp resonance, $\delta(s)$ rises rapidly through $\frac{1}{2}\pi$ at M^2, say, and we suppose it reaches a maximum value $\lesssim\pi$, after which it begins to fall. It cannot go to 0 too fast, by causality,[11] so $\delta(s)$ decreases through $\frac{1}{2}\pi$ at a value of s, say, s_h, much higher than M^2. We note that $\sin\delta(s_h)=1$. The value of $\Omega(s)$ is consequently very large near M^2 [of order $(s_h-s_0)/(M^2-s_0)$] where[12] it has a sharp peak; and it is $\lesssim 1$ everywhere

[11] E. P. Wigner, Phys. Rev. **98**, 145 (1955).

[12] By taking s_h sufficiently large for a given T^0, we seemingly can make $|T(M^2)|$ arbitrarily large, violating unitarity in a given partial wave. All this means is that the approximation neglecting iterations of T^0 is no longer valid. The point is, $|T(M^2)|$ can be as large as the unitarity limit for $|T^0(M^2)|$ much smaller than this.

else. In fact, $\Omega \ll 1$ near s_h. Then $(\sin\delta)/\Omega$ has a large broad maximum near s_h, and is small elsewhere. For a mildly varying T^0, the integral in (4) is thus positive for $s \simeq M^2$, and very slowly varying with s in this vicinity. The zero occurs for $s \simeq s_h \gg M^2$. Since $\cos\delta(M^2)=0$, $T(s) \simeq \text{const} \times e^{i\delta(s)}\Omega(s)$ near M^2, and $T(s)$ displays the single peak expected.

In fact, we can prove the following rigorously. Take T^0 as exactly constant. Then T^0 factors out of (4). But we can now solve the mathematical problem of Eq. (3) in an alternative way to obtain

$$T_+(s) = T^0 e^{i\delta(s)}\Omega(s). \tag{6}$$

[This is obtained by the same methods as before,[1,2] noticing that $T(s)$ has now only the right-hand cut, since $T^0=\text{const.}$] By the uniqueness of the solution,[13] we therefore have the mathematical identity

$$\cos\delta(s)+\Omega(s)\frac{1}{\pi}P\int_{s_0}^{\infty}\frac{ds'}{s'-s}\,\frac{\sin\delta(s')}{\Omega(s')}=\Omega(s). \tag{7}$$

So we have the following desirable features in the solution (4). If $\delta \equiv 0$ (no rescattering), then $T(s)=T^0(s)$, as demanded by the physical model. While for $e^{i\delta}\sin\delta$ resonating at M^2, $\delta \to 0$ at ∞, and for $T^0(s)$ smooth and approximately constant in the relevant region, $|T(s)|^2$ also displays a single-peaked sharp resonance, as expected.

We now consider $n \neq 0$, and will in fact consider only $n=1$ [$\delta(\infty)=\pi$]. (We want a positive, rapidly rising δ near M^2, so this is the simplest case, and is adequate for our purposes.) Then δ rises rapidly to π and stays there. Equation (5) is now defined in a once-subtracted form with an arbitrary subtraction point; or equivalently, in (4), $\Omega(s')$ in the denominator of the integrand is subtracted at the point s, and the $\Omega(s)$ in front of the integral is replaced by unity. The convergence of the integrals in (3) and (4) is unaffected, since $\sin\delta \to 0$ in the present case as well. The subtracted $\Omega(s')$ again peaks sharply at M^2, as does $\sin\delta$. However, now $\sin\delta(s')/\Omega(s')$ is peaked near M^2. [Ω tries to follow[14] $\sin\delta$, but if we take $\delta=\pi$ strictly for $s \geq s_1$, then we see that $\sin\delta=0$ for $s=s_0$ and $s=s_1$, while $\Omega(s_0)\neq 0$ and $\Omega(s_1)\neq 0$, with $\Omega \sim 1/|s|$ as $|s| \to \infty$.] Then for $T^0(s') \simeq \text{const}$ over the region in which δ changes rapidly from 0 to π, the integral vanishes near M^2 (since of course the Cauchy principal-value integral vanishes at the midpoint of a symmetric distribution). Since $\cos\delta$ also vanishes at M^2, the result is that $T(s)$ has a zero near M^2. So we conclude that though $|e^{i\delta}\sin\delta/q|^2$ displays a single peak at M^2, $|T(s)|^2$ in fact has a zero near there. We interpret this as interference between the nonresonant production amplitude T^0 and the rescattering amplitude.

In the present case, there is a relation analogous to (7). The alternative solution with the correct boundary

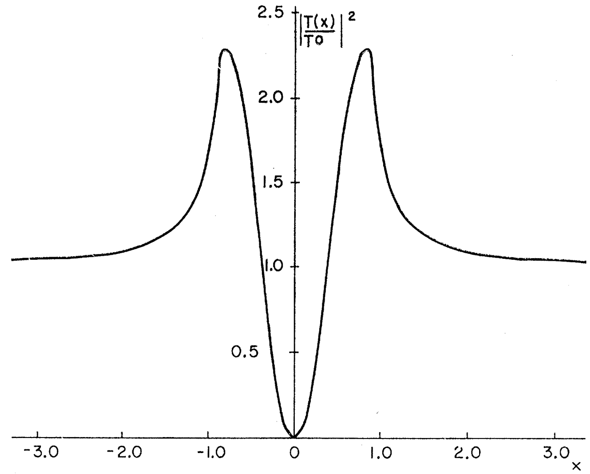

FIG. 2. $|T(x)|^2$ for the symmetric linear phase shift, and constant T^0.

conditions when T^0 is strictly constant is $T^0 \times (C'-s)e^{i\delta(s)}\Omega(s)$, where C' is an arbitrary constant, and the subtraction in Ω is such that $s\Omega(s) \to 1$ at ∞. Adding the nontrivial solution to the homogeneous equation to arrive at the general solution to the inhomogeneous equation, we get

$$\cos\delta(s)+\Omega(s)\frac{1}{\pi}P\int_{s_0}^{\infty}\frac{ds'}{s'-s}\,\frac{\sin\delta(s')}{\Omega(s')}=(C-s)\Omega(s), \tag{8}$$

where C is a constant. We now see that C must be chosen as the zero of the left-hand side. For $T^0(s)$ not exactly constant, the left-hand side of (8) must be used, with $T^0(s')$ inside the integral.

We have evaluated directly the left-hand side of Eq. (8) for several simple cases. In each case, $\delta(s)=\pi$ for $s \geq s_1$, and it is easy to prove that one gets a linear scaling law. That is, the magnitude is independent of the absolute values of s_0 and s_1, so if one makes a linear transformation $s=s_0+\frac{1}{2}(s_1-s_0)(x+1)$, with $-1 \leq x \leq 1$ as the relevant domain of the new variable, the resulting curves are universal for a given functional form of $\delta(s) \to \delta(x)$. Further, if $\sin\delta$ is even in x, i.e., $\delta(x)+\delta(-x)=\pi$, then so is $\Omega(x)$, and hence the integral vanishes at $x=0$; while $\delta(0)=\frac{1}{2}\pi$, so $\cos\delta(0)=0$ and thus $T(x=0)=0$. The first example is the linear one with

$$\begin{aligned}\delta(x)&=\tfrac{1}{2}\pi(x+1), && -1\leq x\leq 1\\ &=\pi, && x\geq 1.\end{aligned} \tag{9}$$

This is shown in Fig. 2. A second example is the quadratic phase shift, defined by

$$\begin{aligned}\delta(x)&=\tfrac{1}{2}\pi(x+1)^2, && -1\leq x\leq 0\\ &=\pi-\tfrac{1}{2}\pi(x-1)^2, && 0\leq x\leq 1\\ &=\pi, && x\geq 1.\end{aligned} \tag{10}$$

[13] See Ref. 4, pp. 37 and 59.

[14] J. D. Jackson, Nuovo Cimento 34, 1644 (1964), Appendix B.

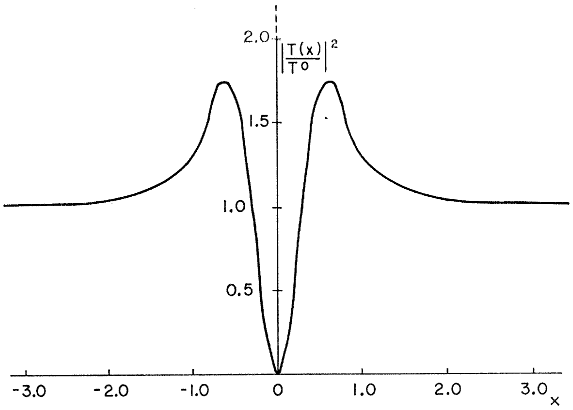

FIG. 3. $|T(x)|^2$ for the symmetric quadratic phase shift and constant T^0.

This is shown in Fig. 3. Also an "asymmetric Breit-Wigner phase shift" was tried. This is defined as follows: For $1.300-0.045\leq s\leq 1.300+0.045$, $\delta(s)=\frac{1}{2}\pi-\tan^{-1}\times[(1.300-s)/0.045]$. This gives a Breit-Wigner phase shift between the $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ points, and the numbers chosen are not unconnected with the A_2. (This makes s the energy instead of the energy squared, but this is not an important point here.) s_0 was chosen at 0.900 (the $\rho\pi$ threshold) and s_1 at 2.000 arbitrarily. The curve for δ was smoothly interpolated by eye to s_0 and s_1. The result of the calculation, transformed to the interval $-1\leq x\leq 1$, is shown in Fig. 4. A symmetric Breit-Wigner phase shift, also examined, gives a result similar to Fig. 2.

We see that in all cases, a zero is obtained. In the symmetric cases, two symmetric peaks are obtained, while in the asymmetric case there is a considerable distortion. (The zero is relatively stable. It is shifted, in terms of s, upwards from $M=1.300$ by $\lesssim 0.010$.) However, in all cases, $|T/T^0|^2\to 1$, not 0, on the extremities. And the peak height-to-background ratio is quite small (≈ 2). This is an intrinsic, dynamical background.

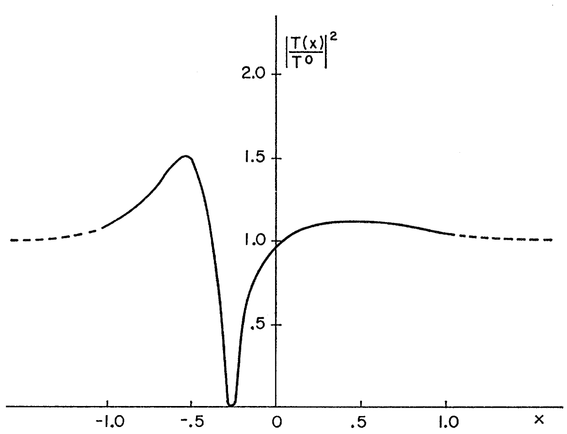

FIG. 4. $|T(x)|^2$ for the asymmetric Breit-Wigner phase shift and constant T^0.

III. TWO COUPLED CHANNELS

We define the general unitary, time-reversal invariant 2×2 rescattering amplitude

$$\begin{bmatrix} f_1 & f_{12} \\ f_{12} & f_2 \end{bmatrix} \equiv \begin{bmatrix} \frac{1}{2}i(1-\eta e^{2i\delta_1}) & \frac{1}{2}(1-\eta^2)^{1/2}e^{i(\delta_1+\delta_2)} \\ \frac{1}{2}(1-\eta^2)^{1/2}e^{i(\delta_1+\delta_2)} & \frac{1}{2}i(1-\eta e^{2i\delta_2}) \end{bmatrix} \tag{11}$$

and generalize the fundamental equation (3) to

$$\begin{aligned} T_+(s)&=T^0(s)+\frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds'}{s'-s-i\epsilon}\times[f_1^*(s')T_+(s')+f_{12}^*(s')U_+(s')],\\ U_+(s)&=U^0(s)+\frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds'}{s'-s-i\epsilon}\times[f_{12}^*(s')T_+(s')+f_2^*(s')U_+(s')]. \end{aligned} \tag{12}$$

For simplicity, we have here neglected differences in threshold and phase space for the two channels. It is clear how one would consider three or more channels.

We define

$$\begin{aligned} t(z)&=\frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds'}{s'-z}[f_1^*(s')T_+(s')+f_{12}^*(s')U_+(s')],\\ u(z)&=\frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds'}{s'-z}[f_{12}^*(s')T_+(s')+f_2^*(s')U_+(s')]; \end{aligned} \tag{13}$$

then

$$T_+(s)=T^0(s)+t_+(s)\,,\quad U_+(s)=U^0(s)+u_+(s)\,.$$

t and u are given by the coupled Hilbert problem

$$\begin{gathered} at_+-t_-=c+2if_{12}^*u_+\,,\\ bu_+-u_-=d+2if_{12}^*t_+\,,\\ a\equiv\eta e^{-2i\delta_1},\quad c\equiv 2i(f_1^*T^0+f_{12}^*U^0)\,,\\ b\equiv\eta e^{-2i\delta_2},\quad d\equiv 2i(f_{12}^*T^0+f_2^*U^0)\,. \end{gathered} \tag{14}$$

The second equation in (14) can be formally solved for u as an integral transform of the right-hand side, so $u_+=I_b(d)+I_b(2if_{12}^*t_+)$. I_b represents a linear homogoneous integral operator, essentially as defined in Eq. (4). This can be substituted in the first equation, whose formal solution is

$$t_+=I_a(c+2if_{12}^*I_b(d))+I_a(2if_{12}^*I_b(2if_{12}^*t_+))\,. \tag{15}$$

Equation (15) is an integral equation for t_+ alone, and can be reduced to a Fredholm equation. Equations (14) may be solved by such techniques, but this leads to a numerical solution of Fredholm equations, and is less transparent than is desirable here. By considering the special case $a=b$ (or $\delta_1=\delta_2=\delta$ if $\eta\neq 0$), the equations can be explicitly solved in closed form. This simplification is made hereafter. With the definitions

$$\begin{aligned} V&=T+U\,, & V^0&=T^0+U^0\,, & f&=f_1+f_{12}\,,\\ W&=T-U\,, & W^0&=T^0-U^0\,, & g&=f_1-f_{12}\,, \end{aligned} \tag{16}$$

Eqs. (12) become uncoupled on adding and subtracting:

$$V_+=V^0+\frac{1}{\pi}\int_{s0}^{\infty}\frac{ds'}{s'-s-i\epsilon}f^*V_+\,,$$

$$W_+=W^0+\frac{1}{\pi}\int_{s0}^{\infty}\frac{ds'}{s'-s-i\epsilon}g^*W_+\,.$$

In (12), the spectral functions $\mathrm{Im}T_+=f_1{}^*T_++f_{12}{}^*U_+$ and $\mathrm{Im}U_+=f_{12}{}^*T_++f_2{}^*U_+$ are each real. Hence the sum $\mathrm{Im}V_+=f^*V_+$ is also real. Let $\theta\equiv\arg(f)$ and for simplicity consider only $0\leq\theta\leq\pi$. Then $f=|f|e^{i\theta}$ and $V_+=|V_+|e^{i\arg(V_+)}$. The reality of f^*V_+ then requires that $\arg(V_+)=\theta+m\pi$ ($m=$integer) or $\mathrm{Im}V_+=(-1)^m|f||V_+|$. But $\mathrm{Im}V_+=\mathrm{Im}(|V_+|e^{i\arg V_+})=\mathrm{Im}(|V_+|e^{i\theta+im\pi})=(-1)^m|V_+|\sin\theta$. Therefore, $|f|=\sin\theta$. Similarly, if $\arg(g)=\phi$, $0\leqslant\phi\leqslant\pi$, then $|g|=\sin\phi$. The equations are therefore

$$\begin{aligned} V_+&=V^0+\frac{1}{\pi}\int_{s0}^{\infty}\frac{ds'}{s'-s-i\epsilon}e^{-i\theta}\sin\theta\, V_+\,,\\ W_+&=W^0+\frac{1}{\pi}\int_{s0}^{\infty}\frac{ds'}{s'-s-i\epsilon}e^{-i\phi}\sin\phi\, W_+\,, \end{aligned} \tag{17}$$

and are immediately soluble. It is convenient to define

$$\gamma\equiv\cos^{-1}\eta\,,\quad 0\leq\gamma\leq\tfrac{1}{2}\pi\quad\text{for } 1\geq\eta\geq 0\,. \tag{18}$$

Then simple algebra gives

$$\begin{aligned} \theta&=\arg(f_1+f_{12})=\delta+\tfrac{1}{2}\gamma\,,\\ \phi&=\arg(f_1-f_{12})=\delta-\tfrac{1}{2}\gamma\,. \end{aligned} \tag{19}$$

We further define

$$\begin{aligned} v&\equiv\cos\theta V^0+\frac{\Omega_\theta}{\pi}P\int_{s0}^{\infty}\frac{ds'}{s'-s}\frac{\sin\theta(s')}{\Omega_\theta(s')}V^0(s')\,,\\ w&\equiv\cos\phi W^0+\frac{\Omega_\phi}{\pi}P\int_{s0}^{\infty}\frac{ds'}{s'-s}\frac{\sin\phi(s')}{\Omega_\phi(s')}W^0(s')\,. \end{aligned} \tag{20}$$

Ω_θ and Ω_ϕ (and below Ω_δ and Ω_γ) are as defined in Eq. (5), with the appropriate angle. The solutions to the equations are (with real T^0 and U^0)

$$\begin{aligned} &V=e^{i\theta}v\quad\text{and}\quad W=e^{i\phi}w\,,\\ &|T|^2=\tfrac{1}{4}(v^2+w^2+2vw\eta)\,,\\ &|U|^2=\tfrac{1}{4}(v^2+w^2-2vw\eta)\,. \end{aligned} \tag{21}$$

Let us examine (21) for the following particular circumstances: $U^0=0$, $T^0\simeq$const in the domain of interest (when $\sin\theta$, $\sin\phi\neq 0$). Also $\eta\simeq 1$, so $1-\eta\simeq\frac{1}{2}\gamma^2\ll 1$, and we may expand in powers of γ. Since $V^0=W^0=T^0$, $v\simeq w$ to lowest order in γ, as follows from Eqs. (19) and (20). Defining $\Delta\equiv w-v$, we expect $\Delta\equiv O(\gamma)$ from Eq. (20). Then we obtain from (21)

$$\begin{aligned} 4|T|^2&=4v^2+4v\Delta+[\Delta^2-2v^2(1-\eta)]-2v\Delta(1-\eta)\,,\\ 4|U|^2&=\qquad\qquad\;\,[\Delta^2+2v^2(1-\eta)]+2v\Delta(1-\eta)\,, \end{aligned} \tag{22}$$

in ascending powers of γ through γ^3. Keeping the leading contributions for each gives

$$\begin{aligned} &|T|^2\simeq v^2\,,\\ &|U|^2\simeq\tfrac{1}{2}(1-\eta)v^2+\tfrac{1}{4}\Delta^2\,. \end{aligned} \tag{23}$$

Since $|U/T|^2=O(\gamma^2)$, a small branching ratio to another channel is easily managed (e.g., $T=T_{A_2\to\rho\pi}$, $U=U_{A_2\to K\bar{K}}$). The physical picture is that only the direct production of state 1 is allowed, state 2 being strongly suppressed in the nonresonant production mechanism. State 2 can be created indirectly through unitarity by the rescattering, but $\sigma_2/\sigma_1\ll 1$ if $\eta\simeq 1$. If now $\delta(\infty)=\pi$, the zero in v follows by the same arguments as in the single-channel case, and from Eq. (23), $|T|^2$ also displays the double-humped structure. However, $\frac{1}{4}\Delta^2$ is comparable to $\frac{1}{2}(1-\eta)v^2$ in magnitude, but in general has a completely different shape, so that $|U|^2$ has a mass spectrum that is quite different from that of $|T|^2$.

We have also examined Eq. (21) for the case $\theta(\infty)=\phi(\infty)=0$, and constant T^0 and U^0. Equations (7), (19), and (20) now permit us to write $v=V^0\Omega_\theta=V^0\Omega_\delta\Omega_\gamma{}^{1/2}$ and $w=W^0\Omega_\phi=W^0\Omega_\delta\Omega_\gamma{}^{-1/2}$, so that (21) becomes

$$\begin{aligned} |T|^2&=\tfrac{1}{4}\Omega_\delta{}^2[(V^0)^2\Omega_\gamma+(W^0)^2\Omega_\gamma{}^{-1}+2V^0W^0\eta]\,,\\ |U|^2&=\tfrac{1}{4}\Omega_\delta{}^2[(V^0)^2\Omega_\gamma+(W^0)^2\Omega_\gamma{}^{-1}-2V^0W^0\eta]\,. \end{aligned} \tag{24}$$

If W^0 vanishes, $|T|=|U|$, and the shape is determined by $\Omega_\delta{}^2\Omega_\gamma$, which is a single sharp peak for resonant δ and reasonable γ. In the other extreme, $V^0=W^0$ (so $U^0=0$),

$$\begin{aligned} |T|^2&=\tfrac{1}{4}(T^0)^2\Omega_\delta{}^2(\Omega_\gamma+\Omega_\gamma{}^{-1}+2\eta)\,,\\ |U|^2&=\tfrac{1}{4}(T^0)^2\Omega_\delta{}^2(\Omega_\gamma+\Omega_\gamma{}^{-1}-2\eta)\,. \end{aligned} \tag{25}$$

For δ rising to a maximum of around π, $\Omega_\delta{}^2$ always strongly dominates Ω_γ (since $\gamma\leq\frac{1}{2}\pi$), and the shape is again essentially determined by $\Omega_\delta{}^2$. For the case of pure absorption,[15] $\delta\equiv 0$, so $\Omega_\delta\equiv 1$, and the shape is

[15] This violates our earlier assumption that $0<\phi<\pi$, as is seen from Eq. (19). However, the generalization to $\phi<0$ is readily made with the result that Eq. (25) remains valid.

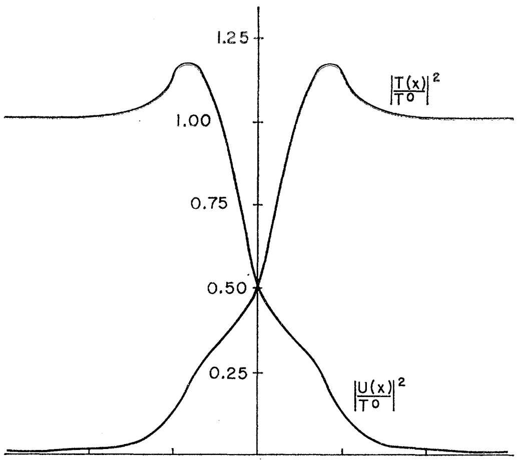

FIG. 5. $|T(x)|^2$ and $|U(x)|^2$ for the two-channel case with pure absorption ($\delta\equiv 0$, $\gamma=\cos^{-1}\eta$ symmetric, linear, and zero at ∞) and constant T^0.

determined by the functions in the brackets of Eq. (25). The example considered was

$$\begin{aligned}\gamma(x)&=\tfrac{1}{2}\pi(1+x)\,, && -1\leq x\leq 0\\ &=\tfrac{1}{2}\pi(1-x)\,, && 0\leq x\leq 1\\ &=0\,, && x\geq 1\\ \delta(x)&\equiv 0\,, && \text{all } x.\end{aligned}\tag{26}$$

The results are shown in Fig. 5. It is seen that the two spectra are quite dissimilar. This is because the $\pm 2\eta$ term in Eq. (25) is important, because Ω_γ is not large compared to 1. This can be remedied by letting γ go to 0 more slowly for large x, but then $|T(x)|^2$ would have a single large peak, and $|U(x)|^2\simeq|T(x)|^2$ near $x=0$.

IV. UNIQUENESS OF SOLUTION; CASTILLEJO-DALITZ-DYSON POLES

We return here to the ambiguity in the solution of Eq. (3) when $n>0$. We limit the explicit discussion here to $n=1$. When $n=1$, P_0 is constant and the ambiguity rests in the term $\beta e^{i\delta}\Omega$ which may be added to the fundamental solution (4), with β an arbitrary constant. Now we demand of a reasonable physical solution that necessarily $T_+\rightarrow 0$ as $T^0\rightarrow 0$. (One can think of T^0 being governed by an effective coupling constant g and take the limit $g\rightarrow 0$.) The term $\beta e^{i\delta}\Omega$ violates this requirement, so *only* the fundamental solution *necessarily* vanishes as $g\rightarrow 0$. There is, however, the possibility of taking $\beta=\alpha T^0(M^2)$, with α arbitrary. The homogeneous term is thus

$$T_{\text{homo}}(s)=\alpha T^0(M^2)e^{i\delta(s)}\Omega(s)\,.\tag{27}$$

Now $T_{\text{homo}}\rightarrow 0$ as $g\rightarrow 0$, but this is a somewhat *ad hoc* procedure, because T_{homo} is the solution of an equation in which $T^0(s)$ does not appear, so such a form for the solution is somewhat forced. Furthermore, it represents an additive term to $T_+(s)$ which in its s dependence is completely independent of the variation of $T^0(s)$ with s. So while the presence of T_{homo} is not categorically ruled out, and it is certainly allowed mathematically, physically it would seem to correspond to a mechanism of making the *DE* rescattering system that bypasses the specific production mechanism T^0. If a specific experiment involves a definite initial state, with a unique T^0 leading to the final state, T_{homo} should be absent on physical grounds.

Mathematically, the fundamental solution may be uniquely prescribed by boundary conditions at $s\rightarrow\infty$. We assume always that $T^0(s)$ is bounded by a constant at ∞ (it may decrease). For $\delta(s)=\pi+O(s^{-p})$, $p>0$, $\Omega(s)\rightarrow s^{-1}$, so $(\sin\delta)/\Omega\rightarrow s^{1-p}$ and from the requirement that Eq. (4) be well defined, in its unsubtracted form, we demand $p>1$. [If $0<p\leq 1$, a single subtraction is necessary. The change in Eq. (4) is that the integral is written in subtracted form, and a term similar to T_{homo} is added. The constant, however, is essentially T_+-T^0 evaluated at the subtraction point; it necessarily vanishes as $g\rightarrow 0$.] Then it is easily verified that for Eq. (4), $|T_+-T^0|=O(s^{-p'})$, where $p'=\min(p,2)>1$. That is, for the fundamental solution, $|T_{\text{fund}}-T^0|=o(s^{-1})$ while $|T_{\text{homo}}|=O(s^{-1})$. So the fundamental solution is uniquely characterized by this statement: It converges to T^0 fastest as $s\rightarrow\infty$. (This statement is true for $n>1$ also.) That is, Eq. (3) with the boundary condition "$T_+\rightarrow T^0$" does not specify a unique solution to the problem if $n>0$. However, Eq. (3) with the boundary condition "$T_+\rightarrow T^0$ as fast as possible" does specify a unique solution. We may, of course, adopt this latter asymptotic condition in order to have a unique solution. However, this mathematical condition would not seem to be accompanied by a compelling physical reason. Since $\sin\delta\sim s^{-p}$ with $p>1$, and $T_+\rightarrow T^0$, Eqs. (1) and (2) show that

$$\pi s(T^0(s)-T_+(s))\rightarrow\int_{s0}^{\infty}ds\,\mathrm{Im}T_+(s)<\infty\,.$$

So the fundamental solution is also specified by the equivalent condition

$$\int_{s0}^{\infty}ds\,\mathrm{Im}T_+(s)=0\,.$$

In the language of N/D, $e^{i\delta}\Omega$ is just $\mathfrak{D}^{-1}$, where

$$\mathfrak{D}(s)\equiv\exp\left(-\pi^{-1}\int_{s0}^{\infty}\frac{ds'\,\delta(s')}{s'-s}\right)$$

is that D which has the correct unitary cut and has no zeros or poles in the finite plane. Thus, Eq. (7) (for the case $n=0$) is the familiar

$$\mathrm{Re}\mathfrak{D}(s)=1+\frac{1}{\pi}P\int_{s0}^{\infty}ds'\,\frac{\mathrm{Im}\mathfrak{D}(s')}{s'-s}\,.\tag{28}$$

Equation (8) [for $\delta(\infty)=\pi$] is the equivalent to Eq. (28) in which $\mathfrak{D}(s)\to D(s)=(s_c-s)^{-1}\mathfrak{D}(s)$. That is, D has the one pole [Castillejo-Dalitz-Dyson (CDD) pole] in the finite plane at the point $s=s_c$. Since in Eq. (8) C is the zero of the left-hand side of the equation, we may ask if C is simply the position of this CDD pole. The answer is no, since by substituting $D(s)=(s_c-s)^{-1}\mathfrak{D}(s)=(s_c-s)^{-1}e^{-i\delta}\Omega^{-1}(s)$ for $\mathfrak{D}(s)$ in Eq. (28), and using Eq. (8) we obtain

$$C=s_c+\frac{1}{\pi}P\int_{s_0}^{\infty}\frac{ds'}{s'-s_c}\frac{\sin\delta(s')}{\Omega(s')}, \tag{29}$$

so, in general, $C\neq s_c$. It might seem that $s_c=C=M^2$ is, in fact, consistent (since the integral then vanishes), but implicit in the derivation from the dispersion relation was the assumption that s_c was not on the physical cut. We can also see this another way. For $T^0=\text{const}$ and δ "symmetric," we saw that $C=M^2$, where $\delta(M^2)=\frac{1}{2}\pi$. For T^0 slowly varying in the resonance region and δ "reasonable," the zero of T_{fund}, Eq. (4), will be close to M^2. Let it be the point $s=\hat{s}$; $T_+(\hat{s})=0$. The question is whether $\hat{s}$ can be a pole of the function $D_{\text{el}}(s)$, where

$$\frac{f(s)}{\rho(s)}=\frac{e^{i\delta(s)}\sin\delta(s)}{\rho(s)}=\frac{N(s)}{D_{\text{el}}(s)}; \tag{30}$$

ρ is phase space. Suppose $D_{\text{el}}(s)$ has a pole at $\hat{s}$, and has the unitary cut

$$D_{\text{el}}(s)=1+\frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds'}{s'-s}\sigma(s')+\frac{d}{s-\hat{s}}\equiv\bar{D}(s)+\frac{d}{s-\hat{s}}. \tag{31}$$

σ is a weight function. $\bar{D}$ has the cut, but no pole. If $N(s)$ is regular at $\hat{s}$, then $f(\hat{s})=0$, contradicting the given phase shift since

$$\frac{f(\hat{s})}{\rho(\hat{s})}\simeq\frac{f(M^2)}{\rho(M^2)}=\frac{i}{\rho(M^2)}. \tag{32}$$

Suppose that N has a pole at $\hat{s}$. Then $N(s)=\bar{N}(s)+n/(s-\hat{s})$. We note that $\bar{N}(s)$ is regular at $\hat{s}$. Now $\lim_{s\to\hat{s}}N(s)/D_{\text{el}}(s)=n/d\neq0$. However, f/ρ is complex at $\hat{s}$, so n/d is complex. So at least one of n and d is not real. It follows that at least one of N or D, and hence f/ρ, is not a real analytic function, since near the pole the discontinuity does not equal the imaginary part. We conclude that from the Schwarz reflection principle, if $\sin\delta\neq0$ (i.e., $\delta\neq$ multiple of π) at a point on the physical cut, there is no CDD pole in the elastic amplitude at that point. A CDD pole can be "at ∞," at an unphysical point, or only at those physical points where, in fact, the elastic scattering amplitude (in the one-channel problem) actually vanishes.

Since $n=[\delta(\infty)-\delta(0)]/\pi=1$, Levinson's theorem tells us there is one "net" CDD pole, i.e., the number of poles of D_{el} minus the number of zeros is one. So there is at least one CDD pole. If we choose $D_{\text{el}}(s)=\mathfrak{D}(s)$, there are no zeros or poles in the finite plane. However, $\mathfrak{D}(s)\sim s$ at ∞, and the integral of the logarithmic derivative contributes exactly $2\pi i$ from the circle at ∞ on closing the Cauchy contour around the cut. So Levinson's theorem is satisfied by this pole at ∞. $\mathfrak{D}$ may be multiplied by any rational function, which can introduce equal numbers of poles and zeros and interchange the singularities in $\ln D$ between ∞ and finite points. For example, if $D_{\text{el}}(s)=s^{-1}\mathfrak{D}(s)$, s^{-1} has a pole at the origin and a zero at ∞, so it has *no net* poles and does not change the content of the Levinson theorem. However, now $D_{\text{el}}\sim1$ at ∞, and the CDD pole has been shifted from ∞ to the origin.

The point is that if δ is such that $\sin\delta\neq0$ at any finite point on the physical cut, and $\delta(\infty)=\pi$, the CDD pole in D_{el} for the elastic rescattering amplitude N/D_{el} *must be either* at ∞ *or at an unphysical point.* The choice $D_{\text{el}}=\mathfrak{D}$ would correspond to the former. Whereas for the same δ, and constant T^0, the 0 in T_+ *must occur at a finite point* and can readily be on the physical cut. This follows from the boundary condition $T\to T^0=\text{const}$. So if $T(s)=T^0D_{\text{prod}}(s)^{-1}$, then necessarily this imposes $D_{\text{prod}}(s)\to1$ at ∞, so $D_{\text{prod}}=(C-s)^{-1}\mathfrak{D}$ because $\mathfrak{D}\sim s$ at ∞.

Another way of seeing the difference is to note that whatever the position of the CDD pole in D_{el}, its position and the assumed left-hand singularities of N determine the phase shift δ. But this is assumed given *a priori* in this problem (either as a result of such a dynamical calculation, or from experiment). *δ is fixed and is independent of T^0 by the assumption at the outset of the weakness of the coupling* However, the zero in T_+ [Eq. (4)] certainly depends on T^0; the position can be varied by varying the shape of $T^0(s)$.

We may also note that by changing very slightly the assumed $\delta(s)$ and $T^0(s)$, one can arrange to keep $\hat{s}\simeq M^2$, and $\delta(M^2)=\frac{1}{2}\pi$, but have δ cross zero just above threshold (it starts negative) or else have it overshoot above resonance and go asymptotically to π from above. Now the CDD pole in D_{el} occurs precisely at the point where $\sin\delta=0$ and is in the physical region, but is still distinct from the zero in T_+.

Finally, one may consider one other example which is admittedly contrived and for that reason has not been emphasized, but which is quite illuminating. Let $\delta(\infty)=0$, so that by Levinson's theorem, there are *no* net CDD poles in the elastic rescattering amplitude, and also the inelastic problem has the *unique* solution Eq. (4). There are *no ambiguities*. Let δ rise to π, remain near π until a large value of s, and then slowly decrease through $\frac{1}{2}\pi$ to 0. But now, instead of T^0 remaining constant, suppose it is nearly constant in the resonance region, but decreases quickly to 0 in the region where $\delta\simeq\pi$. Then in Eq. (4), the large-s region, where $\sin\delta/\Omega$ had previously contributed the major part to the integral, is eliminated by the factor T^0 in the integrand.

The resonant region behaves very much like the case $\delta(\infty)=\pi$. [The factors $\Omega(s)$ and $\Omega(s')$ have different asymptotic properties in the two cases, but differ in the region s, s' small by factors which are only slowly varying.] So, effectively, the s dependence is very nearly the same as in the case $\delta(\infty)=\pi$, and T_+ has a zero near M^2. But this is now a unique result; there is no CDD ambiguity in the elastic problem. It is therefore quite clear that the zero in T_+ is not simply the putting in by hand of a CDD pole at a particular point; it is a nontrivial dynamical effect depending on the given δ and T^0.

The source of this difference between the elastic and production amplitudes is the basic assumption, Eq. (2). This represents inelastic unitarity; it states $\mathrm{Im}T_+=0$ if $\delta=0$, but allows $\mathrm{Re}T_+\neq 0$. It is also linear, and hence sets no scale, that is, we may transform $T_+\to\lambda T_+$, and Eq. (2) remains valid. The elastic unitarity condition for $D+E\to D+E$ is $\mathrm{Im}f=|f|^2$ with $f=e^{i\delta}\sin\delta$, so that $\delta=0\to|f|=0$. It is nonlinear, and sets a scale.

V. CONCLUSIONS

We have studied some simple examples of the production of a resonance in a linear-integral-equation formulation, both for a single channel and for two coupled channels. We assumed the experimental peaking to come about because of rescattering, or final-state interactions, with the basic production amplitude T^0 varying relatively slowly in the resonance region. The rescattering amplitude was assumed to be known and to exhibit a single unsplit resonance. The behavior of the total production amplitude was seen to depend on the high-energy behavior of the phase shift and of T^0. A zero in the cross section near the resonance, with peaks on either side, can readily be understood. It is possible to have either symmetric or skewed distributions, depending on the detailed behavior of δ and T^0.

A more realistic numerical calculation than was carried out here (for the A_2 for instance) would (i) start with a reasonable physical model for T^0 and U^0, (ii) include phase space, (iii) relax the $\delta_1=\delta_2$ condition, and (iv) if required, include three or more channels (e.g., $\rho\pi$, $\eta\pi$, and $K\bar{K}$ for the A_2). The coupled equations would then be solved numerically. This would be worth doing if first there were qualitative agreement with experiment. This is not so in the case of the A_2, for the following reasons:

(1) The A_2 seems to be well fitted by the dipole formula $(E-M)^2/[(E-M)^2+(\frac{1}{2}\Gamma)^2]^2$. This expression approaches 0 for $|E|\to\infty$. The present model led to Eq. (8), $(M-E)\Omega(E)$ (setting $s\to E$). This approaches 1 for $|E|\to\infty$, as is also apparent in Figs. 2–4. There is an intrinsic, non-negligible dynamical background in the present model such that (i) peaking on either side of the zero may be present but the peak to background ratio is strictly limited (and is $\simeq 2$) and (ii) the dip in the middle is definitely lower than the asymptotic, background cross section. We consider this the strongest argument against this model as an explanation of the split A_2. One might attempt to force T_+ to go to zero for E large by considering a T^0 which is peaked near the resonance and has a width of the order of the over-all resonance width (the two peaks and the dip). However, this goes against the spirit of this model, and would begin to resemble the model of two interfering resonances which are degenerate in mass, but different in widths. This latter model also gives an adequate fit to the A_2, but the present model was an attempt to get a fit with a single monopole resonance.

(2) The symmetry of the double-humped structure in the model can be achieved but not in a natural, automatic fashion. It is sensitive to δ and T^0.

(3) Finally, while the possibility of identical symmetric spectra in two channels was not ruled out (with a dominant branching ratio to a particular channel), the several cases considered in Sec. III do not make this very likely, whereas the $K\bar{K}$ mode of the A_2 also appears to be symmetrically split.[16]

We believe, however, that similar zeros or dips may occur in other reactions through this mechanism, and that this should be kept in mind. In particular, it is important that the background to any such dips, or split peaks, be carefully examined. We argue that the asymptotic value of $\delta(s)$ need not be zero *a priori*. If it is nonzero, it only implies the existence of a CDD pole in the elastic rescattering amplitude. This would violate no important physical principle. For example, in the case of the A_2, this pole might be merely the result of neglecting the $\eta\pi$ channel. The result is only a possible ambiguity in the dispersive formulation of the production problem. For example, if $\delta(\infty)=\pi$ and one does not want to buy the more restrictive asymptotic boundary condition that would make the problem unique, the most general result is a T_+ with one arbitrary real parameter. For $T^0=\mathrm{const}$, one would have

$$\begin{aligned} T_+ &= T_{\mathrm{fund}}+T_{\mathrm{homo}} \\ &= T^0(C-s)e^{i\delta}\Omega+\alpha T^0 e^{i\delta}\Omega \\ &= T^0(C+\alpha-s)e^{i\delta}\Omega. \end{aligned} \tag{33}$$

If a fit can be found with $C'=C+\alpha$ a free parameter, one would have a one-parameter fit to the experiment, which is not an empty result. (In the present case, if α is chosen $\neq 0$, the symmetric distributions in Figs. 2 and 3 become skewed, and the fit to the A_2 even worse.) Also, as pointed out at the end of Sec. IV, it is possible to get a dip even with $\delta(\infty)=0$ for some δ and T^0, the result being unique.

The interference of a resonance with a coherent background is also considered in Refs. 9 and 10. These treatments are more general in that arbitrary amplitude and phase of the background are allowed. But they are

[16] M. Aguilar-Benitez *et al.*, Phys. Letters **29B**, 62 (1969); R. Baud *et al.*, *ibid.* **31B**, 397 (1970).

also more phenomenological than the present work, which, though it treats the resonance as phenomenologically given, is nevertheless a dynamical model of how it is produced in a particular experiment. For example, in the present particular model the dependence of the phase shift on the higher energies, beyond the resonance region, can markedly affect the resonance region. After this work was completed, several other calculations[17,18] came to our attention which also distinguish the production of a resonance from its formation in elastic scattering. The work of Aitchison and Graves-Morris in particular is parallel to the present study, and these authors reach similar conclusions.

[17] I. J. R. Aitchison and P. R. Graves-Morris, in Proceedings of the Birmingham Three-Body Conference, Birmingham, England, 1969 (unpublished); and Nucl. Phys. **B14**, 683 (1969).
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Phenomenological Model for a Possible Failure of the Pomeranchuk Theorem*

HIDEZUMI TERAZAWA
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 20 May 1970)

The author proposes a phenomenological model for a possible failure of the Pomeranchuk theorem. This predicts the following relations among asymptotic total cross sections: $\sigma_T(\pi^-p)-\sigma_T(\pi^+p)=\sigma_T(K^-n)-\sigma_T(K^+n)$, $\sigma_T(K^+p)=\sigma_T(K^+n)$, $\frac{2}{3}[\sigma_T(K^-n)-\sigma_T(K^+n)]\leq\sigma_T(K^-p)-\sigma_T(K^+p)\leq 2[\sigma_T(K^-n)-\sigma_T(K^+n)]$, $\sigma_T(pp)=\sigma_T(pn)$, and $\frac{2}{3}[\sigma_T(\bar{p}n)-\sigma_T(pn)]\leq\sigma_T(\bar{p}p)-\sigma_T(pp)\leq 2[\sigma_T(\bar{p}n)-\sigma_T(pn)]$. All of these are consistent with the recent Serpukhov data.

THE data of the recent Serpukhov experiment performed by the IHEP-CERN collaboration[1] tempt us to consider a possible failure of the Pomeranchuk theorem.[2] This theorem asserts the asymptotic equality of particle and antiparticle total cross sections. Eden[3] has pointed out that a number of conventional assumptions about strong interactions at high energy are unproven from basic principles and therefore should be subjected to further experimental measurements. These include the following assumptions: (1) The ratio of the real part of a forward scattering amplitude to the imaginary part multiplied by $\ln s$ tends to zero; (2) elastic scattering dominates exchange scattering near the forward direction; (3) the breaking of $SU(3)$ symmetry becomes unimportant at high energies. The assumptions (1) and (2) are crucially needed to prove the Pomeranchuk theorem and the Okun-Pomeranchuk rules,[4] respectively. Horn[5] has offered a possible interpretation of the Serpukhov data, associating the sharp change in the trend of the total cross sections plotted against $s^{-1/2}$ with an "ionization point" for baryon resonances, and predicts that both conventional Regge expansions and the Pomeranchuk theorem would fail from there on. Which of the assumptions can we incriminate for the violations of the Pomeranchuk theorem? In this note we propose a phenomenological model for the violation of the theorem and obtain its direct consequences.

Consider asymptotic total cross sections $\sigma_T(AB)$ for scattering reactions of the type $A+B\rightarrow$ any final states. We may separate $\sigma_T(AB)$ into two parts:

$$\sigma_T(AB)=\sigma_T^{(P)}(AB)+\sigma_T^{(1)}(AB). \quad (1)$$

The first part $\sigma_T^{(P)}(AB)$ obeys not only the Pomeranchuk theorem but also the Okun-Pomeranchuk rules. This means that it satisfies the following symmetry properties:

$$\sigma_T^{(P)}(AB)=\sigma_T^{(P)}(\bar{A}B)=\sigma_T^{(P)}(A\bar{B})=\sigma_T^{(P)}(\bar{A}\bar{B}) \quad (2)$$

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[1] J. V. Allaby *et al.*, IHEP-CERN collaboration, Phys. Letters **30B**, 500 (1969).
[2] I. Ia. Pomeranchuk, Zh. Eksperim. i Teor. Fiz. **34**, 725 (1958) [Soviet Phys. JETP **7**, 499 (1958)]. As for the progress made afterward, see T. Kinoshita, in *Perspectives in Modern Physics*, edited by R. E. Marshak (Wiley, New York, 1966), p. 211. Recently Kinoshita has proved that either or both of the relations $\sigma_T(AB)=\sigma_T(\bar{A}B)$ and $d\sigma(AB)/dt=d\sigma(\bar{A}B)/dt$ for $t=-c(\ln s)^{-2}$ should hold asymptotically [T. Kinoshita, Phys. Rev. D (to be published)].
[3] R. J. Eden, Phys. Rev. D **2**, 529 (1970). See also O. V. Dumbrais and N. M. Queen, JINR (Dubna) Report No. E2-4965, 1970 (unpublished).
[4] L. B. Okun and I. Ia. Pomeranchuk, Zh. Eksperim. i Teor. Fiz. **30**, 307 (1956) [Soviet Phys. JETP **3**, 307 (1956)].
[5] D. Horn, Phys. Letters **31B**, 30 (1970). Lam and Truong claimed that the origin of the violation can be the electromagnetic interaction [W. S. Lam and T. N. Truong, Phys. Letters **31B**, 307 (1970)].