Asymmetry and Background in $\eta \to 3\pi$

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We have considered the problem of the asymmetry observed in the decay $\eta \to \pi^+\pi^-\pi^0$. Bernstein, Feinberg, and Lee suggested that an asymmetry in the average energies of the charged pions in this decay would be proof of C violation in the electromagnetic interaction among hadrons. Recall, however, that the η is identified as a peak in the invariant-mass distribution of three pions from a reaction like $\pi^- p \to \pi^+\pi^-\pi^0 n$. Yuta and Okubo proposed that the observed effect is due to interference between the η channel for this reaction and direct 3π production background processes. We have calculated the background size and its possible contributions to the asymmetry using a simple model which takes into account only tree diagrams (no closed loops) with constant vertices and ignores the effects of spin. We can account for 40% of the background observed in the experiment of Gormley *et al.* but find that the largest asymmetry which can be produced through interference is 10^{-5}, as compared to the experimentally observed 1.5×10^{-2}. We conclude that if the observed asymmetry persists, then charge-conjugation invariance is violated by the decay. These results agree with the analysis of Stein of the experimental background. He concluded that the background could account for an asymmetry of at most 2.7×10^{-3}.

I. INTRODUCTION

In 1965 Bernstein, Feinberg, and Lee[1] pointed out that no experimental evidence existed which confirmed the conservation of the charge-conjugation quantum number C in the electromagnetic interaction among hadrons. These authors suggested that the decays of the η meson provided an excellent place to search for a C-violating interaction and particularly noted that the confirmation of an asymmetry between the average energies of the charged pions in the decay $\eta \to \pi^+\pi^-\pi^0$ would be proof of C violation. Several experiments[2] have sought since that time to determine the asymmetry

$$A \equiv \frac{N(\omega_+ > \omega_-) - N(\omega_- > \omega_+)}{N(\omega_+ > \omega_-) + N(\omega_- > \omega_+)}. \tag{1}$$

Here $N(\omega_+ > \omega_-)$ ($N(\omega_- > \omega_+)$) is the number of decays in which the energy of the positive (negative) pion is greater than the energy of the negative (positive) pion. The most precise determination of A (the experiment having the best statistics) has been done by Gormley *et al.*[3] in 1968. They found

$$A = 0.015 \pm 0.005. \tag{2}$$

We are particularly interested in this experiment in which the reaction $\pi^- p \to \pi^+\pi^-\pi^0 n$ at an incident pion momentum of 720 MeV/c was studied.

The observed asymmetry results from interference in the η-mass region between that part of the amplitude for $\pi^- p \to \pi^+\pi^-\pi^0 n$ which is even under exchange of the final π^+ and π^- and that part which is odd. If the odd part is also due to the η resonance, then charge-conjugation invariance is violated. However, Yuta and Okubo[4] have pointed out that the asymmetry might be due to interference between some direct production mode or background amplitude and the amplitude for the η resonance, which in this picture is entirely even under interchange of π^+ and π^-. These authors found that if the background amplitude (1) accounts for all of the experimentally observed background, (2) is entirely odd under interchange of the charged pions, and (3) is purely imaginary, then an asymmetry of about 0.015 could be accounted for without requiring C violation. It becomes very important then to determine what processes might contribute to the background and if they could account for the observed asymmetry.

II. PROCEDURE

We have calculated the background size and its possible contributions to the asymmetry using an extremely naive model which takes into account only tree diagrams (no closed loops) with constant vertices and ignores the effects of spin. We show that the graphs of Fig. 1 account for about 40% of the observed background. These graphs, however, are completely symmetric with respect to interchange of π^+ and π^- and so contribute nothing to the asymmetry. We believe that

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no other diagrams are as large as these.[5] We also show that diagrams like those shown in Figs. 2 and 3 can contribute no more than $\sim 10^{-5}$ to the asymmetry as compared to the experimentally observed 0.015 ± 0.005. Other diagrams which can contribute to the asymmetry are believed to contribute no more than these diagrams.[5]

III. EXPERIMENTAL QUANTITIES AND DEFINITION OF VARIABLES

The reaction we are considering is $\pi^- p \to \pi^+\pi^-\pi^0 n$ at an incident pion momentum $|\mathbf{k}| = 450$ MeV/c $= 3.2\, m_\pi$ in the c.m. system. The total c.m. energy is $W = \sqrt{s} = 1500$ MeV $= 10.7\, m_\pi$, and the final momentum of the nucleon is $|\mathbf{k}'| = 126$ MeV/c $= 0.9\, m_\pi$ in the c.m. system when $\sqrt{s_{3\pi}} = m_\eta$. The cross section for the reaction $\pi^- p \to \eta n \to \pi^+\pi^-\pi^0$ is about 0.5 mb at this energy and the background observed in the experiment requires a direct-production cross section of about 0.05 mb. The subenergies $s_{\alpha\beta\cdots}$ are defined as

$$s_{\alpha\beta\cdots} = (p_\alpha + p_\beta + \cdots)^2, \tag{3}$$

and the ranges on the appropriate variables for this reaction are

$$4m_\pi^2 \leqslant s_{\pi\pi} \leqslant 9m_\pi^2, \tag{4a}$$

$$60m_\pi^2 \leqslant s_{\pi N} \leqslant 76m_\pi^2, \tag{4b}$$

and

$$75m_\pi^2 \leqslant s_{\pi\pi N} \leqslant 95m_\pi^2. \tag{4c}$$

IV. CALCULATIONS

A. General

In order to estimate the size of the various background contributions in the η-mass region, we assume a constant matrix element and modify the four-body phase-space integral, by use of δ functions, to include an integration over s_η from $(m_\eta - \frac{1}{2}\Delta)^2$ to $(m_\eta + \frac{1}{2}\Delta)^2$. The result is

$$\sigma_B \cong \frac{1}{8^2 3\sqrt{3}} \frac{1}{(2\pi)^4} \frac{(m_\eta - 3m_\pi)^2}{s} \frac{|\mathbf{k}'|}{|\mathbf{k}|} m_\eta \Delta \langle |T_B|^2 \rangle_{\mathrm{av}}. \tag{5}$$

Here $|\mathbf{k}|$ and $|\mathbf{k}'|$ are the initial and final nucleon momenta in the c.m. system, $s_\eta = m_\eta^2$, $\langle |T_B|^2 \rangle_{\mathrm{av}}$ is the average of $|T_B|^2$, the invariant matrix element for the

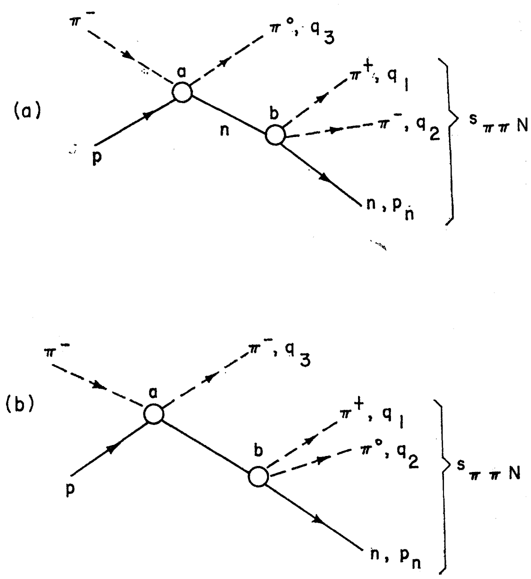

FIG. 2. Principal nonsymmetric background.

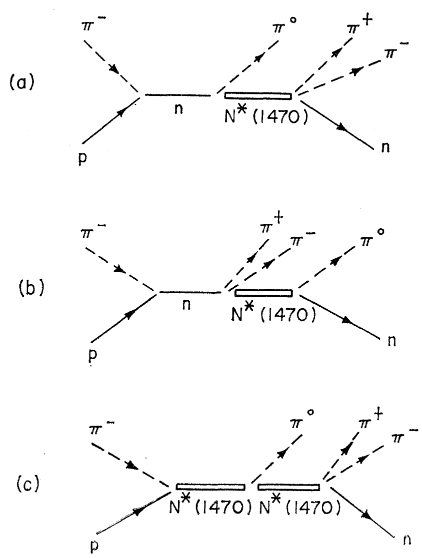

FIG. 1. Principal background.

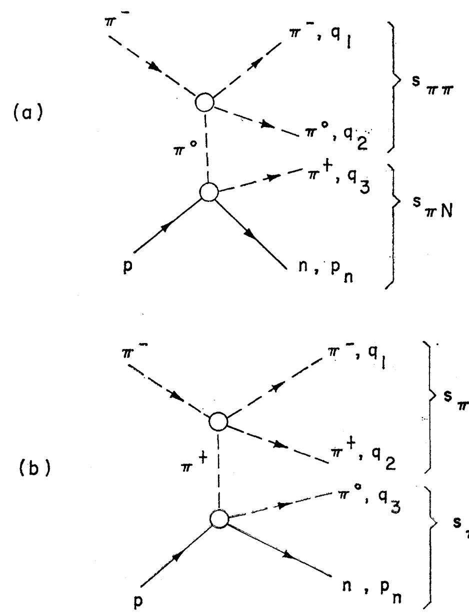

FIG. 3. One-pion exchange.

[5] K. Taggart, Ph.D. thesis, Case Western Reserve University, 1970 (unpublished).

background, over the ranges of the subenergies, and Δ is the experimental resolution.

B. Principal Background

The matrix element for those diagrams shown in Fig. 1 is written as

$$T_B=-m_N^2(\tfrac{1}{3}\sqrt{2})g_{\pi NN}g_{N^*N\pi}g_{N^*N\pi\pi}[(s-m_N^2)^{-1}\times(s_{\pi\pi N}-m_{N^*}^2+i\Gamma_{N^*}m_{N^*})^{-1}+(s-m_N^2)^{-1}\times(s_{\pi N}-m_{N^*}^2+i\Gamma_{N^*}m_{N^*})^{-1}+(s-m_{N^*}^2+i\Gamma_{N^*}m_{N^*})^{-1}\times(s_{\pi\pi N}-m_{N^*}^2+i\Gamma_{N^*}m_{N^*})^{-1}]. \quad (6)$$

When we square this (ignoring the second term since the resonance is very far from its mass) and average over all the subenergies as independent variables, we find

$$\langle|T_B|^2\rangle_{\rm av}\cong(\tfrac{1}{3}m_N^2\sqrt{2})^2g_{\pi NN}^2g_{N^*N\pi}^2g_{N^*\pi\pi}^2\times\left\{\left(\frac{1}{(s-m_N^2)^2}+\frac{1}{\Gamma_{N^*}^2m_{N^*}^2}\right)\left(\frac{1}{\Gamma_{N^*}m_{N^*}}\times\frac{\frac{1}{4}\pi}{s_{\pi\pi N}(\max)-s_{\pi\pi N}(\min)}\right)+\frac{2}{(s-m_N^2)\Gamma_{N^*}m_{N^*}}\times\frac{(\frac{1}{4}\pi)^2+0.81}{[s_{\pi\pi N}(\max)-s_{\pi\pi N}(\min)]^2}\right\}. \quad (7)$$

Equations (5) and (7) along with the values of $g_{\pi NN}$, $g_{N^*N\pi}$, and $g_{N^*N\pi\pi}$ given in Table I give the result

$$\sigma_B\sim 2\times10^{-2}\ \text{mb}. \quad (8)$$

We have assumed that the coupling for $N^*N^*\pi$ is the same as for $NN\pi$. The small value for $g_{\pi NN}^2$ is a result of approximating threshold pion-nucleon scattering by the s-channel nucleon-pole diagram. The usual value of $g_{\pi NN}^2$ assumes pseudoscalar coupling, and since we have ignored spin in our treatment, its use here would be incorrect.

C. Principal Nonsymmetric Background

We estimate the size of the largest nonsymmetrical part of the background from the diagrams shown in Fig. 2. The matrix element for either diagram is

$$T_B=\frac{g_a}{s_{\pi_1\pi_2N}-m_N^2}\left\{\frac{g_b(\pi_1,N)m_N^2}{s_{\pi_1N}-m_\Delta^2+i\Gamma_\Delta m_\Delta}+\frac{g_b(\pi_2,N)m_N^2}{s_{\pi_2N}-m_\Delta^2+i\Gamma_\Delta m_\Delta}\right\}, \quad (9)$$

with

$$g_a^2=16\pi s\sigma_{\pi N} \quad (10a)$$

TABLE I. List of coupling constants.

Coupling	Value	Source
$g_{\pi NN}^2$	6.3	Very-low-energy πN scattering
$g_{N^*N\pi}^2$	19.8	$\Gamma(N^*\to N\pi)$
$g_{N^*N\pi\pi}^2$	5.0×10^4	$\Gamma(N^*\to N\pi\pi)$
$g_{\Delta N\pi}^2$	22.6	$\Gamma(\Delta\to N\pi)$

and

$$g_b^2=g_{\Delta N\pi}^2\times(\text{appropriate Clebsch-Gordan coefficients}). \quad (10b)$$

We have included the fact that π-N scattering is dominated by the $\Delta(1237)$ resonance. We again square and average over all of the subenergies as independent variables to get (the c's are Clebsch-Gordan coefficients)

$$\langle|T_B|^2\rangle_{\rm av}=\frac{16\pi s\sigma_{\pi N}(s)g_{\Delta N\pi}^2}{[s_{\pi\pi N}(\max)-m_N^2][s_{\pi\pi N}(\min)-m_N^2]}\times\left\{\frac{\frac{1}{4}\pi}{\Gamma_\Delta m_\Delta[s_{\pi N}(\max)-s_{\pi N}(\min)]}[(c_1^2c_2^2)_{\pi_1N}+(c_1^2c_2^2)_{\pi_2N}]+\frac{(c_1c_2)_{\pi_1N}(c_1c_2)_{\pi_2N}}{[s_{\pi N}(\max)-s_{\pi N}(\min)]^2}\times[2(\tfrac{1}{4}\pi)^2+2(0.91)^2]\right\}. \quad (11)$$

This gives $\sigma_B\sim1\times10^{-3}$ mb for *each* diagram when we use the value of $g_{\Delta N\pi}$ given in Table I.

D. Other Nonsymmetrical Background

Figure 3 shows the type of process Yuta and Okubo suggested might be the major contribution to the background. The matrix element for both these diagrams is

$$T_B=\left(\frac{g_{\pi^-\pi^0}+g_{\pi^-\pi^+}}{[q-(q_1+q_2)]^2-m_\pi^2}\right)\frac{g_{\pi N(\text{CEX})}m_N^2}{s_{\pi N}-m_\Delta^2+i\Gamma_\Delta m_\Delta}, \quad (12)$$

where CEX means charge exchange. Once again we average over $s_{\pi N}$ as an independent variable and find for both that

$$\langle|T_B|^2\rangle_{\rm av}\cong(1+\tfrac{3}{2})^2\times\left[\frac{\frac{1}{4}\pi(16\pi s_{\pi\pi}/m_\pi^2)[16\pi m_\Delta^4\Gamma_\Delta^2(2/m_\pi^2)]}{120m_\pi^4\Gamma_\Delta m_\Delta[s_{\pi N}(\max)-s_{\pi N}(\min)]}\right], \quad (13)$$

where we have used $(1/120m_\pi^4)$ as a reasonable value for $1/\{[q-(q_1+q_2)]^2-m_\pi^2\}$. The background cross section calculated from Eqs. (5) and (13) is then

$$\sigma_B\sim6\times10^{-4}\ \text{mb}. \quad (14)$$

E. Asymmetry

The asymmetry expected from interference between the background arising from the two diagrams of Fig. 2 may be estimated from the formula

$$A=\int_{\omega_+>\omega_-} 2\,\mathrm{Re}[T_\eta \times (\text{odd part of } T_B{}^* \text{ under } \omega_+ \leftrightarrow \omega_-)] \times \int_{\omega_+>\omega_-} |M_\eta|^2 \quad (15)$$

which becomes, after much work,[5]

$$A=\frac{1}{\frac{1}{2}\sigma_\eta\Gamma(\eta\to\pi^+\pi^-\pi^0)/\Gamma_\eta}\,\pi\,\frac{1}{4\sqrt{s}|\mathbf{k}|(2\pi)^8}\,\frac{\pi}{\sqrt{s}}|\mathbf{k}| \times\frac{(2)^2Q^2}{24\sqrt{3}}\,\frac{1}{2}\left(\sigma_\eta 16\pi\frac{|\mathbf{k}|}{|\mathbf{k}'|}\right)^{1/2} \times\left(\frac{\Gamma(\eta\to\pi^+\pi^-\pi^0)m_\eta 384\sqrt{3}\pi^2}{Q^2}\,\frac{16}{17}\right)^{1/2} G. \quad (16)$$

Here $Q=m_\eta-3m_\pi$, and we have used the form $\mathrm{Im}\bar{T}_B(\text{odd})=GX$ with $X=\sqrt{3}(T_+-T_-)/Q$. For the graph of Fig. 2(a), we find

$$\mathrm{Im}\bar{T}_B(\text{odd})=\frac{\sqrt{2}}{3\sqrt{3}}\,\frac{(16\pi s\sigma_{\mathrm{CEX}})^{1/2}}{s_{\pi\pi N}(\max)-s_{\pi\pi N}(\min)}(\ln\tfrac{5}{3}) \times\left(\frac{8\pi\Gamma_\Delta m_\Delta{}^2}{1.65m_\pi}\right)E_N Q\left(\frac{(\frac{1}{4}\pi)^2+(0.91)^2}{[s_{\pi N}(\max)-s_{\pi N}(\min)]^2}\right). \quad (17)$$

This gives an asymmetry of $A\sim 10^{-5}$, indeed very small compared to the observed value of 1.5×10^{-2}. We do not believe that any other diagrams will give a larger asymmetry since other nonsymmetric diagrams contribute even less to the background than these do.[5]

V. RESULTS

We find, then, that in our naive model the diagrams of Fig. 1 account for about 40% of the observed background in the experiment of Gormley *et al.* The largest diagrams which are nonsymmetric with respect to interchange of the charged pion energies are shown in Fig. 2. These diagrams give a cross section an order of magnitude smaller than do those shown in Fig. 1. They are really not very asymmetric either, as they can account for an asymmetry of only 10^{-5}. The diagrams shown in Fig. 3 are of the type which Yuta and Okubo proposed to be responsible for the background. We find that they are even smaller than the diagrams in Fig. 2, and hence their contribution to the asymmetry is certainly no greater than that of the diagrams of Fig. 2.

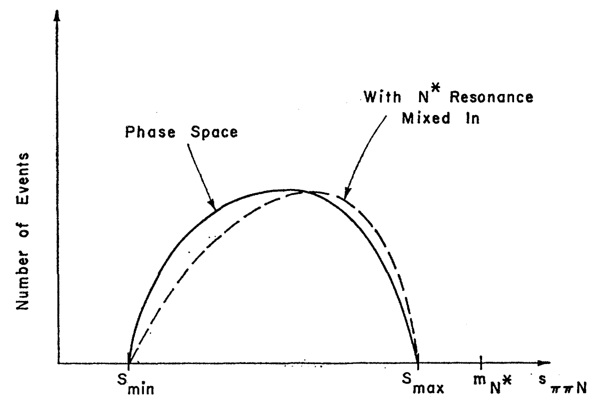

FIG. 4. $\pi^+\pi^-n$ effective-mass distribution.

In order to check the dominance of the virtual $N^*(1470)$ in the background, one should compare the experimental effective mass plot for $\pi^+\pi^-n$ with the distribution expected from phase space. The presence of the resonance should shift the maximum of the distribution toward $s_{\pi\pi N}(\max)$, as schematically shown in Fig. 4. However, since only 10% of the experimental events are background the deviation from the uncorrelated distribution would be small. In any event we feel that this question should be looked into.

VI. CONCLUSIONS

We conclude, in agreement with the experimental analysis of Stein,[6] that the background in the experiment of Gormley *et al.* is *not* asymmetric enough to cause even a small fraction of the observed asymmetry.

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[6] S. Stein, Ph.D. thesis, Columbia University, 1969 (unpublished).