New Regge Models of Asymptotic Behavior*

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The properties and predictions of several new Regge models, suggested by the Serpukhov total-cross-section data, are analyzed and compared. Forward elastic πN, KN, $\bar{K}N$, NN, and $\bar{N}N$ scattering are considered, for Regge models incorporating strong multi-Pomeranchukon cuts, or Regge dipoles, or complex secondary trajectories. Total cross sections are fitted. Forward real parts, charge-exchange scattering, and regeneration are predicted, within subtraction constants. High-energy extrapolations are shown.

I. INTRODUCTION

The πN, KN, and $\bar{K}N$ total-cross-section data from Serpukhov,[1] in the 25–70-GeV/c range, diverge remarkably from the extrapolations of conventional Regge-pole models.[2] It has long been accepted that these models need corrections, but the size of the corrections—giving an abrupt change from steadily falling σ_T to constant σ_T—is a surprise. It seems there may be something new to be learned here.

These results have therefore stimulated some interesting theoretical conjectures about the dynamical features that may govern asymptotic scattering, such as (a) strong multi-P cuts,[3–5] (b) Regge dipoles to violate the Pomeranchuk theorem,[6–8] (c) Regge dipoles to give asymptotically rising σ_T,[6] (d) Regge poles with complex α,[9] and (e) an ionization point, where resonances suddenly cease.[10]

All of these except (e) are Regge models, being expressed in terms of J-plane singularities.

In the present paper we evaluate models (a)–(d) quantitatively, for forward πN, KN, $\bar{K}N$, NN, and $\bar{N}N$ scattering amplitudes, establishing their predictions for real parts and forward charge-exchange and C— exchange scattering, and for total cross sections not yet measured. We discuss which experiments, at present or foreseeable-future accelerators, can distinguish between the models.

It is convenient to use the variable $\nu=(s-u)/4m_N$; at $t=0$, $\nu=\omega$, the total meson lab energy. The surprising characteristics of the new data are most dramatically shown by plotting σ_T against $\nu^{-1/2}$, following Feynman[11] (see Fig. 1). Conventional Regge-pole models, with $\alpha_P(0)=1$ and $\alpha_i(0)\approx 0.5$ $(i\neq P)$, give a behavior $\sigma_T\approx a+b\nu^{-1/2}$, i.e., straight lines on this plot, with particle and antiparticle cross sections converging to common limits as $\nu\rightarrow\infty$ in accordance with the Pomeranchuk theorem.[12,13] Figure 1 shows that the meson-nucleon data[14,15] up to about 25 GeV/c do behave like this. But the Serpukhov data[1] spoil this tidy picture; they remain essentially constant through the range 25–65 GeV/c, and raise two questions:

(i) Is the Pomeranchuk theorem violated?
(ii) What mechanism gives the break in σ_T?

In contrast, the Serpukhov $\bar{p}p$ data show no break up to 50 GeV/c, within the errors, raising a third question:

(iii) Are any new mechanisms also compatible with pp and $\bar{p}p$?

To discuss these questions, we separate the forward nonflip amplitude $A'(t=0)$ into even- and odd-signature parts,

$$A'^{\pm}(\pi p)=\tfrac{1}{2}A'(\pi^-p)\pm\tfrac{1}{2}A'(\pi^+p)\,, \qquad (1)$$

and similarly for $A'^{\pm}(Kp)$ and $A'^{\pm}(pp)$. The optical theorem gives $\sigma_T=\mathrm{Im}A'(t=0)/p_L$, so we can also separate σ_T into $\sigma_T^{\pm}$. The first question, about the Pomeranchuk theorem, thus refers to A'^-. The second question relates essentially to A'^+ (excluding exceptional behavior of A'^-).

Our approach is first to fit $\mathrm{Im}A'^{\pm}$ to total-cross-section data, with the various models. We then predict

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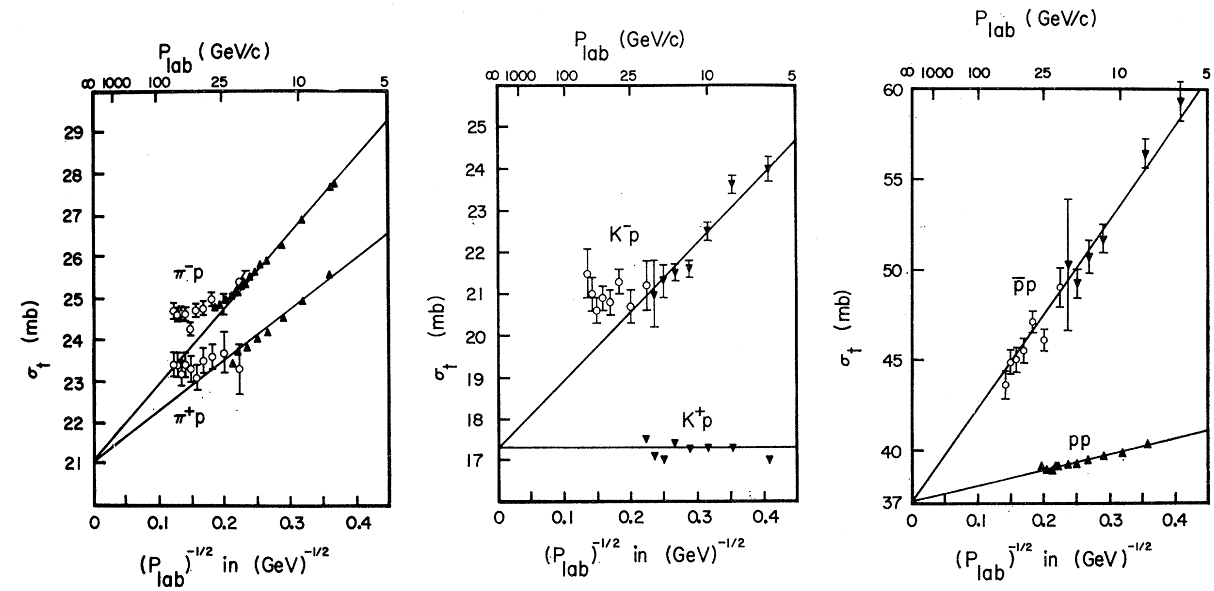

FIG. 1. Plots of total-cross-section data versus $(P_{\rm lab})^{-1/2}$. Conventional Regge-pole models give straight lines on this plot as indicated.

forward real parts—within subtraction parameters—together with forward charge-exchange and regeneration cross sections, and σ_T at higher energies. Within the framework of models (a)–(d) there is considerable freedom in the parametrization, and corresponding freedom in the extrapolations to higher energies. The fits to data that we present are therefore illustrative rather than definitive.

In the models we present, A'^{+} and A'^{-} are not treated separately in the data fitting. This is because the data are incomplete, and do not allow a complete separation in the KN and NN cases. However, the theory separates cleanly, so that in principle we can combine A'^{+} from one model with A'^{-} from another—though best-fit parameters might change a little.

II. ODD-SIGNATURE AMPLITUDE A'^{-}

The question here is, does $\sigma_T^{-}\rightarrow 0$ as $\nu\rightarrow\infty$? A single Regge pole gives a contribution of the form

$$A'^{-}(\text{pole})=i\gamma(-i\nu)^{\alpha}. \tag{2}$$

Such a term cannot violate the Pomeranchuk theorem by giving $\sigma_T^{-}\sim$constant. To get the right energy dependence requires $\alpha=1$, but then A'^{-} is real and $\sigma_T^{-}\rightarrow 0$. Regge cuts, given by continua of poles, also cannot give $\sigma_T^{-}\sim$constant. However, a Regge dipole, essentially given by the derivative $d/d\alpha$ of a pole term, has the required property when $\alpha=1$,

$$A'^{-}(\text{dipole})=i\gamma(-i\nu)^{\alpha}(\ln\nu-b-\tfrac{1}{2}i\pi)\,, \tag{3}$$

where the term b comes from the freedom to choose the scale of ν. By introducing such a term at $t=0$, we can violate the Pomeranchuk theorem.

With such an amplitude, however, the real part dominates asymptotically over the imaginary part (a general property, not limited to dipoles, as noted by Martin[13,16] and Eden[7]), so that for both elastic and exchange scattering we have $d\sigma/dt\sim(\ln\nu)^2$ at $t=0$. If σ_T tends to a finite limit as $\nu\rightarrow\infty$, this requires $d\sigma/dt$ to shrink as $(\ln\nu)^2$—faster than the $\ln\nu$ shrinking given by an ordinary Regge pole or dipole—with attendant problems as discussed by Finkelstein.[8] However, it is also conceivable that $\sigma_T^{+}\sim\ln\nu$ rather than tending to a constant as $\nu\rightarrow\infty$; this is not exactly in the spirit of the original Pomeranchuk theorem, but it is one possible explanation of the break in σ_T^{+}; in this case, plain $\ln\nu$ shrinking of $d\sigma/dt$ is enough. In any case, our present discussion is confined to $t=0$, and here it remains valid to use a dipolelike formula.

For πN scattering, there is just one odd-signature amplitude $A'^{-}(\pi p)$, defined in Eq. (1). In the KN case, however, there are two independent amplitudes that we may take to be $A'^{-}(Kp)$ and $A'^{-}(Kn)$, defined analogously. Since there are not enough data to discriminate between Pomeranchuk violation in Kp and Kn scattering, we arbitrarily consider two cases; one in which the odd-signature dipole has pure t-channel isospin $I_t=0$, and one in which both $I_t=0$ and $I_t=1$ components are present with comparable strength. In the NN case, we work with pp and $\bar{p}p$ data alone and therefore cannot separate $I_t=0$, 1 components; $I_t=0$ is arbitrarily assigned.

We get the most direct measure of odd-signature amplitudes in processes that depend on nothing else. The relevant examples are $\pi^{-}p\rightarrow\pi^{0}n$ charge exchange, depending on $A'^{-}(\pi p)$ alone, and $K_2p\rightarrow K_1p$ regenera-

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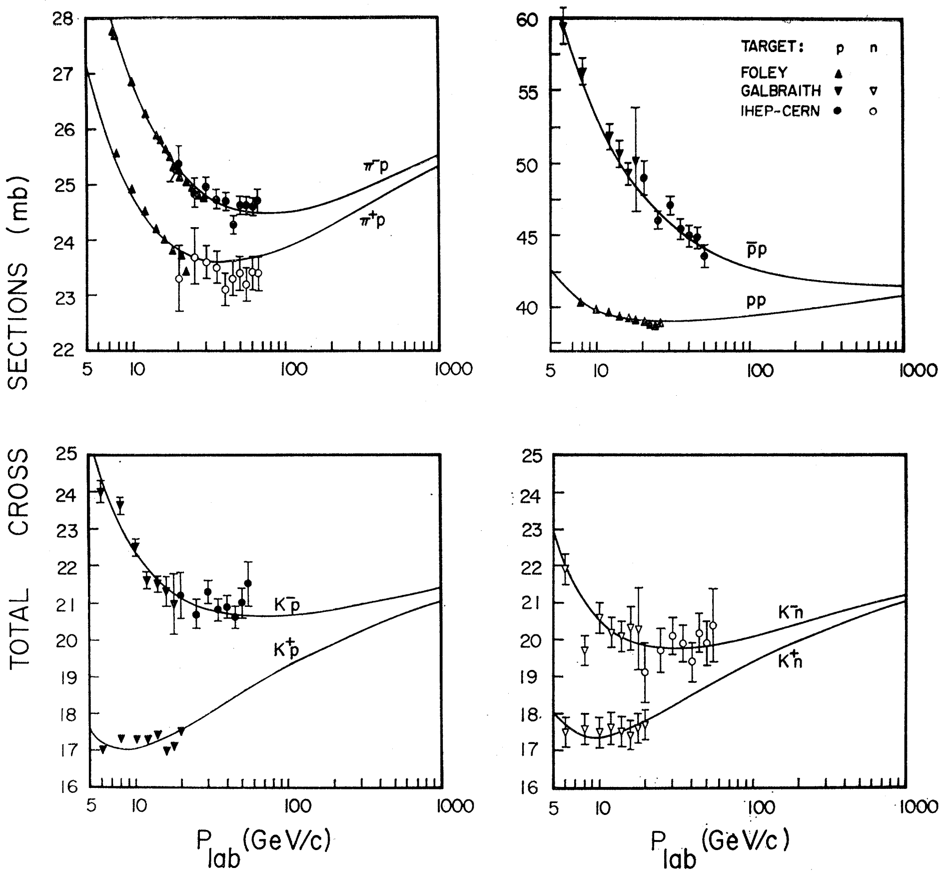

FIG. 2. Fit to σ_T data with Pomeranchuk Regge-cut model.

tion, depending on $A'(K^0p)-A'(\bar{K}^0p)$ alone, at $t=0$. $K^-p\rightarrow\bar{K}^0n$ and $K^+n\rightarrow K^0p$ charge exchanges are not pure examples, since they contain even-signature contributions too, but they are still more sensitive than elastic scattering to small odd-signature effects. When the odd-signature dipole has pure $I_t=0$, the KN charge exchanges show no anomalies of course. In the present analysis, pn and $\bar{p}p$ charge exchanges cannot be predicted.

TABLE I. Regge-cut model.

	P	P'	ω	ρ	A_2	Subtraction constant
$\alpha(0)$	1.0	0.4	0.4	0.5	0.5	
$\gamma(\pi N)$	84	177		12		
$\gamma_c(\pi N)$	-200					
$b_c(\pi N)$	-0.86					
$\gamma_s(\pi N)$						4
$\gamma(KN)$	67	78	27	5.8	3.9	
$\gamma_c(KN)$	-112					
$\gamma(NN)$	117	218	84			
$\gamma_c(NN)$	-119					

III. EVEN-SIGNATURE AMPLITUDE A'^+

The question here is: What causes the break in σ_T^+? The first possible answer is multi-Pomeranchukon cuts, added to conventional Regge poles. With presently popular prescriptions, the net effect of these cuts is a

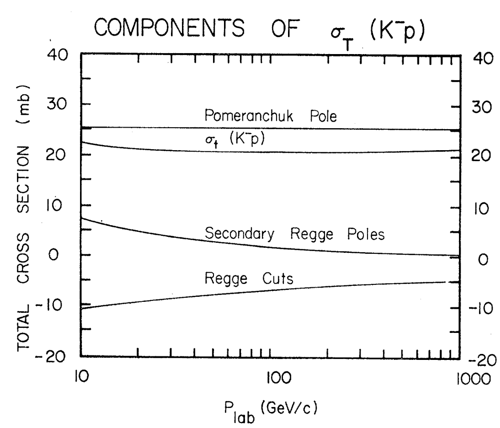

FIG. 3. Contributions of Pomeranchuk pole, secondary poles, and Regge cuts in fit to $\sigma_t(K^-p)$ data; cf. Sec. IV A.

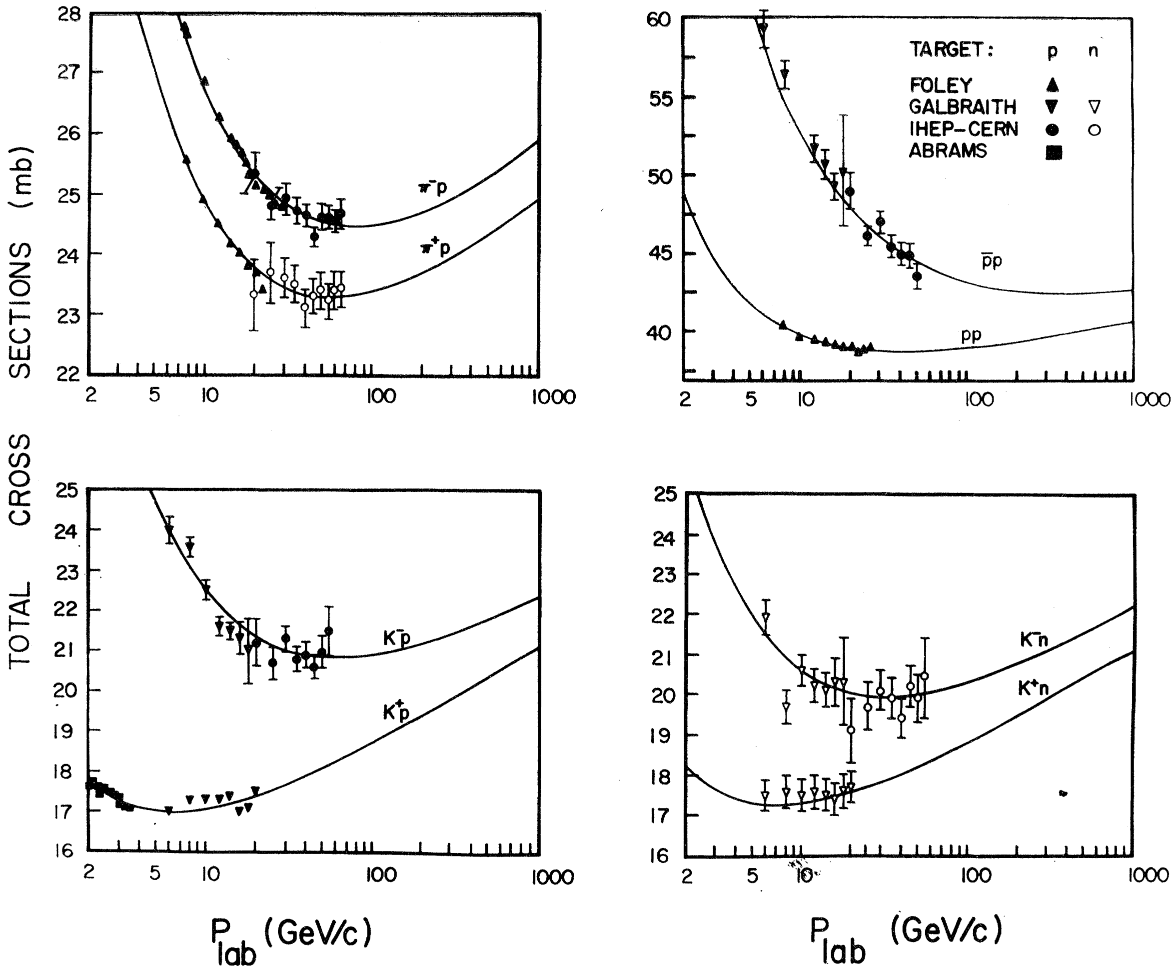

FIG. 4. Fits to σ_T data with Regge-dipole model (P and ω dipoles); cf. Sec. IV B.

negative contribution to $\sigma_T{}^+$, decreasing only logarithmically and thus more slowly than secondary Regge-pole contributions. The former therefore dominates over the latter above some energy, giving a flattening out of $\sigma_T{}^+$, followed by a very slow rise to the asymptotic limit. This behavior was predicted by Frautschi and Margolis[3] for the pp system, and later by Dean[4] for the πN system. The present authors[5] have made explicit

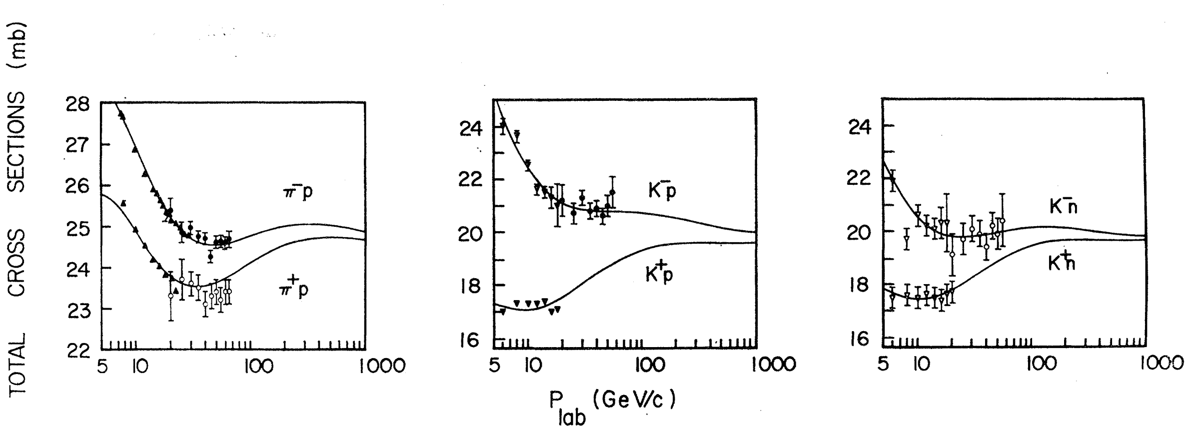

FIG. 5. Fits to σ_T data with complex-α Regge model; cf. Sec. IV C.

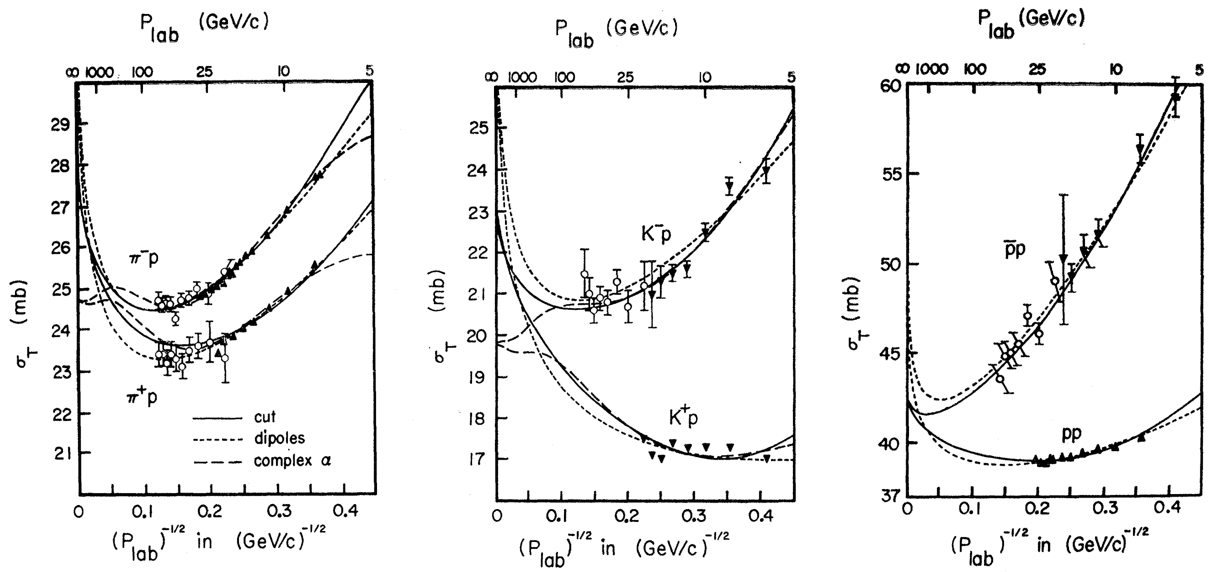

FIG. 6. Extrapolations of the cut, dipole, and complex-α models of Figs. 2, 4, and 5 to infinite momenta on a $(P_{\rm lab})^{-1/2}$ plot.

fits to the Serpukhov data, using the following parametric form for the multi-P cut contribution:

$$A'^{+}(\text{cut})=\gamma i\nu(\ln\nu-b-\tfrac{1}{2}i\pi)^{-\lambda-1}, \qquad (4)$$

where γ, b, and λ are free parameters, and $\alpha_{\rm cut}=1$ is implicit. The parameter λ is fixed at the value $\lambda=0.1$.

Another explanation is a Regge dipole. The possibility of dipolelike terms in the odd-signature amplitude is already accepted, albeit reluctantly. This motivates us to consider the effect of corresponding terms in the even-signature amplitude. Taking into account the change in signature, the analogs of Eqs. (2) and (3) are

$$A'^{+}(\text{pole})=-\gamma(-i\nu)^{\alpha}, \qquad (5)$$

$$A'^{+}(\text{dipole})=-\gamma(-i\nu)^{\alpha}(\ln\nu-b-\tfrac{1}{2}i\pi). \qquad (6)$$

TABLE II. Double-dipoles model.

	P	P'	ω	ρ	A_2	Subtraction constant
α	1.0	0.4	0.4	0.5	0.5	
$\gamma(\pi N)$		103		6		
$\gamma_D(\pi N)$	2.9			−0.6		
$b_D(\pi N)$	−15			4.5		
$\gamma_s(\pi N)$						−47
$\gamma(KN)$		53	22	5.5	4.3	
$\gamma_D(KN)$	2.9		−0.8			
$b_D(KN)$	−12					
$\gamma(NN)$		178	73			
$\gamma_D(NN)$	2.9		−1.0			
$b_D(NN)$	−29		2.8			
$\gamma_s(NN)$						−101
		Split ω-ρ dipoles				
$\gamma(KN)$		54	24	3.2	4.1	
$\gamma_D(KN)$	2.9		−0.5	−0.3		
$b_D(KN)$	−12			4.5		

With $\alpha=1$, such a dipole term gives a logarithmically increasing contribution $\sigma_T^{+}\sim\ln\nu$, but still within the Froissart bound $\sigma_T<c(\ln\nu)^2$. Such a contribution will ultimately dominate over all other Regge poles and cuts, giving a break in the behavior of σ_T^{+}.

In the KN system we can define two independent even-signature amplitudes, with $I_t=0$ and 1. For the present investigation we arbitrarily assume that the even-signature dipole has $I_t=0$ (vacuum quantum numbers). But it remains an amusing possibility that an $I_t=1$ term (with A_2 quantum numbers) may also be present; such a term would show up in KN and $\bar{K}N$ charge exchange, and could also appear in $\pi^-p\to\eta^0 n$.

In the pp, $\bar{p}p$ system we cannot distinguish between $I_t=0$, 1 contributions, and again assume pure $I_t=0$.

Yet another explanation of the break in σ_T^{+} is a complex trajectory. Chew and Snider[9] suggested, on the basis of some calculations with the multiperipheral model, that secondary boson Regge poles may occur in pairs, with conjugate complex trajectories and residues:

$$A'^{+}=-\tfrac{1}{2}[\gamma(-i\nu/\nu_0)^{\alpha}+\gamma^{*}(-i\nu/\nu_0)^{\alpha^{*}}]\nu_0^{\alpha_R}. \qquad (7)$$

The phase of γ can be made real by appropriately

TABLE III. Complex-α model.

	P	P'	ω	ρ	A_2	Subtraction constant
α_R	1.0	0.4	0.4	0.5	0.5	
α_I		1.4				
$\gamma(\pi N)$	64	3.9		11.6		
$b(\pi N)$		2.4				
$\gamma_s(\pi N)$						84
$\gamma(KN)$	51	2.6	27	5.8	3.9	
$b(KN)$		1.6				

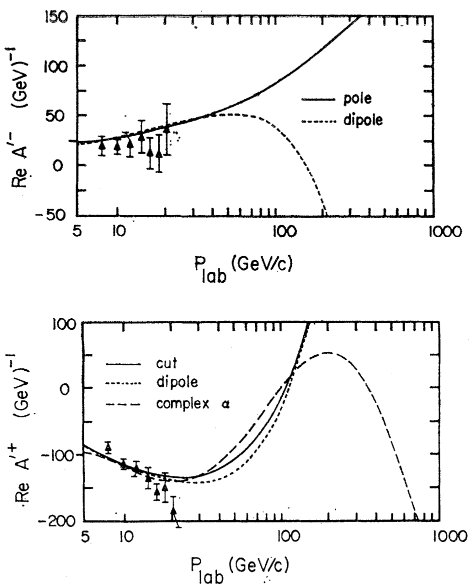

FIG. 7. Calculations of Re$A'^{\pm}$ for πN scattering with the cut, dipole, and complex-α models.

choosing the scale ν_0. The total-cross-section contribution from the complex poles is

$$\sigma_T^+(\text{pair of poles}) = \tfrac{1}{2}\gamma\nu^{\alpha_R-1} \times\{e^{\frac{1}{2}\pi\alpha_I}\sin[\tfrac{1}{2}\pi\alpha_R-\alpha_I(\ln\nu-b)] + e^{-\frac{1}{2}\pi\alpha_I}\sin[\tfrac{1}{2}\pi\alpha_R+\alpha_I(\ln\nu-b)]\}, \quad (8)$$

where $\alpha=\alpha_R+i\alpha_I$, $b=\ln\nu_0$. Clearly α_P cannot be complex, for this would make σ_T^+ negative in some regions. In a phenomenological fit to data, the complex-α pair can replace the usual P'-exchange contribution.

One might think about exchange degeneracy with an odd-signature complex pair of poles, in the KN and NN cases. This would have the merit of making $\sigma_T(K^+p)$ and $\sigma_T(pp)$ flat, but would also make σ_T^- oscillate (changing sign) with the same wavelength as the oscillations in σ_T^+, in conflict with the data. If complex poles are to play a strong role in σ_T^+, they cannot be exchange degenerate; for duality considerations they may then perhaps be grouped with the Pomeranchuk term.

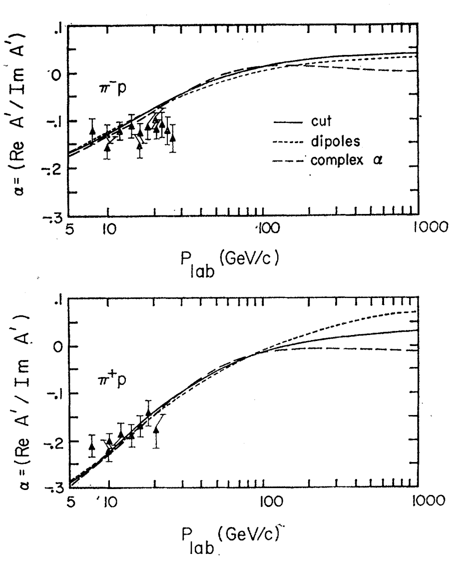

FIG. 8. Predictions for the ratio of real to imaginary parts at $t=0$ for πN scattering.

IV. MODELS AND RESULTS

A. Pomeranchuk-Cut Model

Here we assume that the break in σ_T^+ comes from multi-Pomeranchukon cuts added to conventional Regge poles. For σ_T^- we assume that the Pomeranchuk theorem holds, $\sigma_T^- \to 0$.

The Regge poles are P, P', ρ for the πN case; P, P', ω, ρ, A_2 for the KN case; P, P', ω for the pp, $\bar{p}p$ case (absorbing A_2 and ρ contributions into effective P' and ω terms). The parametrization follows Eqs. (2), (4), and (5).

In addition, for this and the other models, we introduce an additive real constant in A'^+ that fulfills the role of a subtraction constant in dispersion relations for ReA'^+; in Regge-pole language, this is equivalent to a P'' pole with intercept $\alpha_P''(0)=0$. It plays no role in σ_T.

A typical good fit to data is obtained with the parameters in Table I, and illustrated, together with predictions and extrapolations, in Figs. 2 and 6–8 (below). The separate contributions of Pomeranchuk pole, secondary Regge poles, and Pomeranchuk cut are illustrated for the fit to $\sigma_T(K^-p)$ in Fig. 3.

In any model with strong multi-P cuts, asymptotic cross sections are not approached until $\ln s \gg 1$, i.e., not until impossibly high energies for accelerators, as already emphasized in Ref. 5. In the fit shown, $\sigma_T^\infty(\pi N) = 33$ mb, $\sigma_T^\infty(KN)=26$ mb, $\sigma_T^\infty(NN)=46$ mb.

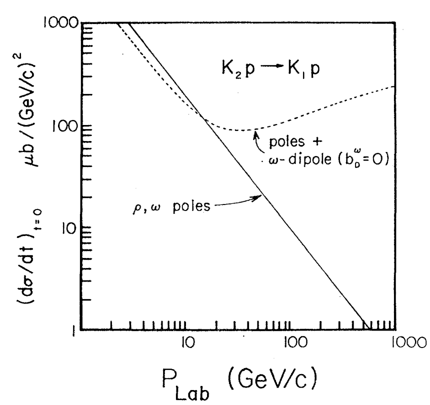

FIG. 9. Predictions for $d\sigma/dt(K_2p\to K_1p)$ at $t=0$ based on conventional poles (solid line) and Pomeranchuk-theorem violation (dashed curve).

B. Dipole Models

Here we assume that the Pomeranchuk theorem is violated by a dipole or dipoles in A'^{-}, and that the break in σ_T^+ is caused by an even-signature dipole term, parametrized according to Eqs. (3) and (6).

The even-signature dipole is taken to have $I_t=0$ in all cases. The odd-signature dipole necessarily has $I_t=1$ in the πN case. In the KN case, we consider two possibilities: (i) that it has pure $I_t=0$, and (ii) that an $I_t=1$ component is present also, related in strength to the πN case by SU_3 while the $I_t=0$ component is searched for a best fit.

In the odd-signature dipoles, the parameter b does not affect σ_T, but rather plays the role of a subtraction constant in ReA'^{-} (appearing because we now permit Re$A'^{-}\sim\nu$ lnν asymptotically). Calculations of forward real parts are discussed in Sec. V.

We also introduce the same conventional Regge poles and A'^{+} subtraction constant as in model A.

Typical good fits to data are found with the parameters in Table II. The fits are illustrated in Figs. 4 and 6–13.

C. Complex P' Model

Here we assume that the break in σ_T^+ comes from oscillations, generated by a complex conjugate pair of

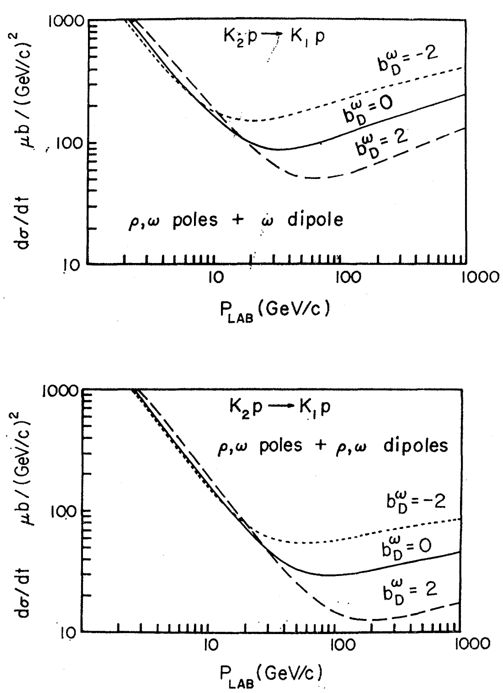

FIG. 10. Typical predictions for $d\sigma/dt(K_2p\to K_1p)$ in models with Pomeranchuk-theorem violation. The upper part illustrates a model with a pure $I_t=0$ dipole; the lower part illustrates a model with both $I_t=0$ and $I_t=1$ dipoles. The predictions are made for various choices of the subtraction constant b_D^ω. cf. Eq. (3).

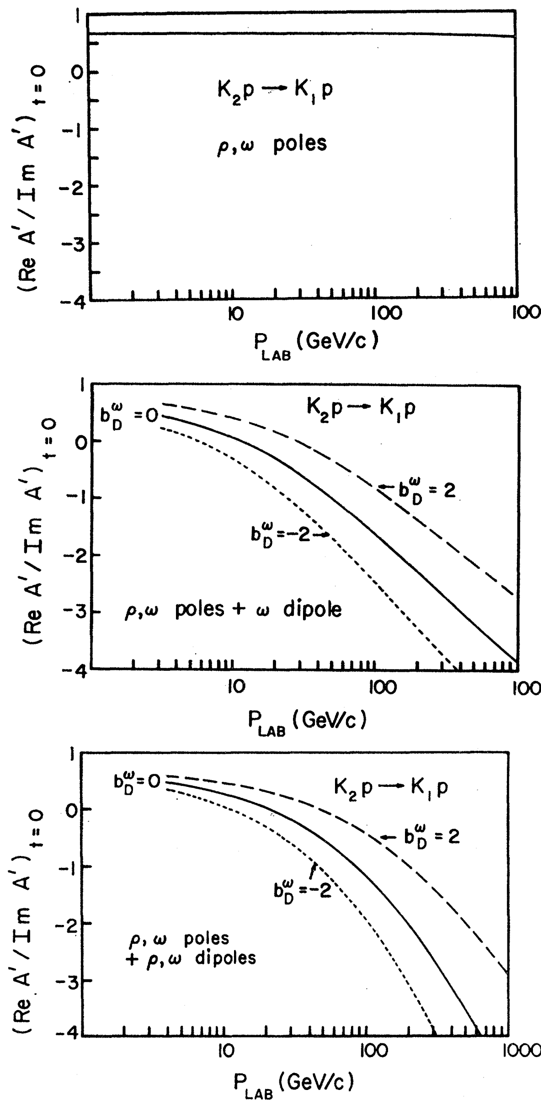

FIG. 11. Predictions for (ReA'/ImA') at $t=0$ for the $K_2p\to K_1p$ reaction, from models A and B. The three parts represent the following assumptions about the Pomeranchuk theorem: (a) no violation, (b) violation by an $I_t=0$ dipole, and (c) violation by both $I_t=0$ and $I_t=1$ dipoles.

P' Regge poles. Other Regge poles P, ω, A_2, ρ are treated conventionally, as in model A, and the Pomeranchuk theorem is not violated.

In order to produce a spectacular result, we forced the imaginary part α_I to be rather large, giving a short wavelength for the oscillations in σ_T^+ versus lnν. We could not find a good fit to the NN case, however, with this value of α_I. On the other hand, it is clear that by reducing α_I or by adding further nonoscillatory terms, a fit could also be achieved here.

A typical fit to data is given by the parameters in Table III. The results are shown in Figs. 5–8.

V. DISCUSSION

The three models described above are far from exhaustive. We can permute A'^{+} and A'^{-} independently, and also combine the physical features (e.g., cuts and dipole and complex α in A'^{+}). However, by concentrating in each case on a particular dynamical mechanism, these models serve to highlight the physical consequences of each.

The extrapolations of the three models to infinite momenta are compared in the $(P_{\rm lab})^{-1/2}$ plot of Fig. 6.

The high-energy forward real parts are essentially determined by the high-energy parametrizations for the σ_t, up to the subtraction constants. In the πN case the subtraction constants $(\gamma_s, b_D{}^{\rho})$ were adjusted to fit present data on $\mathrm{Re}A'^{\pm}$ by evaluations of dispersion relations.[6] The $\mathrm{Re}A'^{\pm}$ results for the three models are illustrated in Fig. 7. Predictions for the ratio of real to imaginary parts $\alpha(\pi^{\pm}p)$ are quite similar below 100 GeV, as shown in Fig. 8. In the range where the predictions for $\alpha(\pi^{\pm}p)$ diverge, the various model fits to σ_T also begin to diverge from one another. Thus it is not clear that $(\mathrm{Re}A'/\mathrm{Im}A')$ is a better discriminator of asymptotic behavior than σ_T. Similar comments also apply to $\mathrm{Re}A'/\mathrm{Im}A'$ in KN and NN cases, where fewer data exist below 25 GeV.

Models A and C assume that anomalous behavior is confined to A'^{+} and that the Pomeranchuk theorem is satisfied. Immediate tests of the validity of the Pomeranchuk theorem include:

i. Measurements of $\sigma_T(K^+p)$ *or* $\sigma_T(K^+n)$ *above 30 GeV/c*, to detect the predicted rise with increasing momentum. It is possible that the upward curvature in $\sigma_T(K^+N)$ could also be seen in very accurate remeasurements in the 5–25-GeV/c range.

ii. Measurements of the forward differential cross section and phase for the reaction $K_2p \to K_1p$ *above 30* GeV/c. Pomeranchuk-theorem violation causes dramatic deviations from the $(P_{\rm lab})^{-1}$ behavior of pole-model extrapolations for $d\sigma/dt$, as shown in Fig. 9. A range of predictions for $d\sigma/dt(K_2p\to K_1p)$ with Pomeranchuk-theorem violation are given in Fig. 10, for various choices of the ω-dipole subtraction constant $b_D{}^{\omega}$. Predictions for $(\mathrm{Re}A'/\mathrm{Im}A')$ at $t=0$ for the $K_2p\to K_1p$ reaction are shown in Fig. 11, without and with Pomeranchuk-theorem violations.

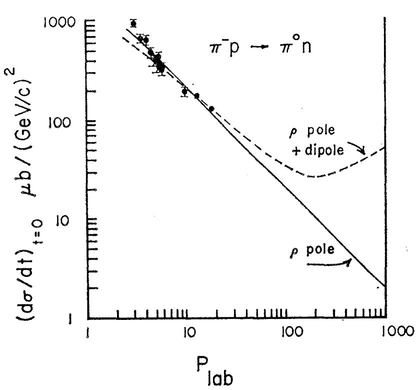

FIG. 12. Predictions for $d\sigma/dt(\pi^-p\to\pi^0n)$ at $t=0$ based on conventional ρ-Regge pole (solid line) and Pomeranchuk-theorem violation (dashed curve), from models A and B.

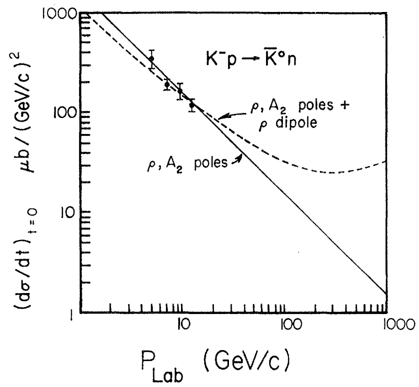

FIG. 13. Predictions for $d\sigma/dt(K^-p\to\bar{K}^0n)$ at $t=0$ based on conventional ρ and A_2 Regge poles (solid line) and Pomeranchuk-theorem violation (dashed curve), from models A and B.

iii. Measurements of $d\sigma/dt$ near $t=0$ for the charge-exchange reactions $\pi^-p\to\pi^0n$, $K^-p\to\bar{K}^0n$, *and* $K^+n\to K^0p$. Pomeranchuk-violating terms (odd-signature dipoles) with $I_t=1$ components cause the forward cross sections for these reactions to rise like $(\ln E)^2$ beyond a minimum value, which is reached at 200–300 GeV in our examples, as shown in Figs. 12 and 13. A comparison of high-energy data on $K^-p\to\bar{K}^0n$ and $K^+n\to K^0p$ would also be interesting. These amplitudes differ by the sign of A_2 exchange. The odd-signature ρ dipole interfers with A_2 constructively in one case, destructively in the other. Approximate equality of the two cross sections would be restored if there were an even-signature dipole, with A_2-like quantum numbers, degenerate with the odd-signature one. There is no evidence at all for such an object, but if it existed it should also show up in $d\sigma/dt$ near $t=0$ for $\pi^-p\to\eta n$.

All three models give fairly similar fits to $\sigma_T{}^+$ up to 70 GeV, as illustrated in Fig. 6. To discriminate between the cut, dipole, and complex-α mechanisms in $\sigma_T{}^+$, measurements at still higher momenta are needed. Studies of the elastic process $K_2p\to K_2p$ above 70 GeV can also be used to differentiate between these models for the even-signature exchanges.

Equally, it would be interesting to look for possible anomalous behavior at high energies in inelastic reactions of all kinds, such as $d\sigma/dt$ near $t=0$ for diffractive processes (P-type dipole), $\gamma p\to\pi^0p$ (ω-type dipole), etc. Such results from experiments at the new accelerators will assist in the unraveling of the asymptotic boundary conditions realized in nature.