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"Shadow" Correction in Deuterium

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The cross-section defect for high-energy hadron-deuteron scattering is calculated by two methods. The widely employed "shadow" correction is shown to give a larger value than the "eikonal" calculation. A new black-sphere derivation is presented which supports the "eikonal" result. The calculation chosen may significantly affect the analysis of experimental data.

SINCE pion-neutron scattering is inaccessible to direct measurement and, until recently, direct neutron-proton scattering experiments have been difficult to perform, the Glauber "shadow" correction[1] has been frequently used to extract hadron-neutron total cross sections from hadron-deuteron and hadron-proton data.[2,3] Unfortunately, the geometrical correction commonly applied is based upon severe restrictions which are frequently ignored. The purpose of this paper is to examine that method and to suggest a means of improving experimental analyses.

If we consider the deuteron as two totally absorbing spheres separated by a distance r considerably greater than their interaction radius R, a geometrical calculation of the total cross section for an isotropic distribution of the two spheres yields[1]

$$\sigma_d=\sigma_n+\sigma_p-\delta\sigma\,, \tag{1}$$

where

$$\delta\sigma=(\sigma_n\sigma_p/4\pi)\langle 1/r^2\rangle \tag{2}$$

and σ_n (σ_p) is the neutron (proton) total cross section.

A somewhat less restrictive expression may be obtained from a diffractionlike (eikonal) theory,[1,4] which gives an elastic scattering amplitude for particle-deuteron scattering:

$$F_{ii}=f_n(\mathbf{q})S(\tfrac{1}{2}\mathbf{q})+f_p(\mathbf{q})S(\tfrac{1}{2}\mathbf{q}) +(i/2\pi k)\int S(\mathbf{q}')f_n(\tfrac{1}{2}\mathbf{q}+\mathbf{q}')f_p(\tfrac{1}{2}\mathbf{q}-\mathbf{q}')d^2q'\,, \tag{3}$$

where

$$S(\mathbf{q})=\int|\psi(\mathbf{r})|^2e^{i\mathbf{q}\cdot\mathbf{r}}d\mathbf{r}$$

and $\mathbf{r}$ is the neutron-proton separation as above.

An expression for the total section is readily generated by application of the optical theorem, and Eq. (1) is obtained with

$$\delta\sigma=-(2/k^2)\int S(\mathbf{q})\,\mathrm{Re}[f_n(\mathbf{q})f_p(-\mathbf{q})]d^2q\,. \tag{4}$$

The restrictions on this derivation are that the nucleon velocities are small compared to that of the incident projectile, and that the energy transfers involved are small, both of which are well satisfied for high-energy elastic scattering. The derivation also assumes, however, that scattering is restricted to near-forward angles, hence only single and double scattering terms are present. This indicates that the approximation is valid for any $r>R$, but may be in error by a few percent for $r<R$, as discussed by Pumplin.[5] We conclude therefore that Eq. (4) is more valid than Eq. (2) for nucleon separations commonly considered in the deuteron.

We now make a direct comparison of the two models. If we assume a purely imaginary Gaussian amplitude for elastic hadron-nucleon scattering and a Gaussian deuteron wave function

$$f_N=(ik\sigma_N/4\pi)e^{-\beta q^2}\,,$$
$$\psi=(8\pi^{3/2}\gamma^{3/2})^{-1/2}e^{-r^2/8\gamma}\,,$$

Eq. (4) yields

$$\delta\sigma=\sigma_n\sigma_p/8\pi(2\beta+\gamma)\,. \tag{5}$$

For this deuteron wave function, it is easily seen that

$$\langle 1/r^2\rangle=3/\langle r^2\rangle=1/2\gamma\,, \tag{6}$$

whereupon Eq. (5) becomes

$$\delta\sigma=\frac{\sigma_n\sigma_p}{16\pi\beta+4\pi/\langle 1/r^2\rangle}\,. \tag{7}$$

We see that Eq. (7) reduces to Eq. (2) in the limit $r\to\infty$, as would be expected from the restrictions discussed above.

Letting $\sigma_n=\sigma_p$ for convenience and $\beta=5$ (GeV/c)$^{-2}$, a value typical of hadron-hadron scattering, we plot $4\pi\delta\sigma/\sigma^2$ against the mean-square deuteron radius for Eqs. (2) and (7) in Fig. 1. As expected, we see that the two curves converge for large $\langle r^2\rangle$. The purely geometrical calculation, however, diverges for small $\langle r^2\rangle$, revealing its lack of validity in this region.

The most frequently quoted value of $\langle 1/r^2\rangle$ for the deuteron appears to be[2,5] $\langle 1/r^2\rangle=0.03$ mb^{-1} or, in a Gaussian model, $\langle r^2\rangle=100$ mb. Comparing Eqs. (2)

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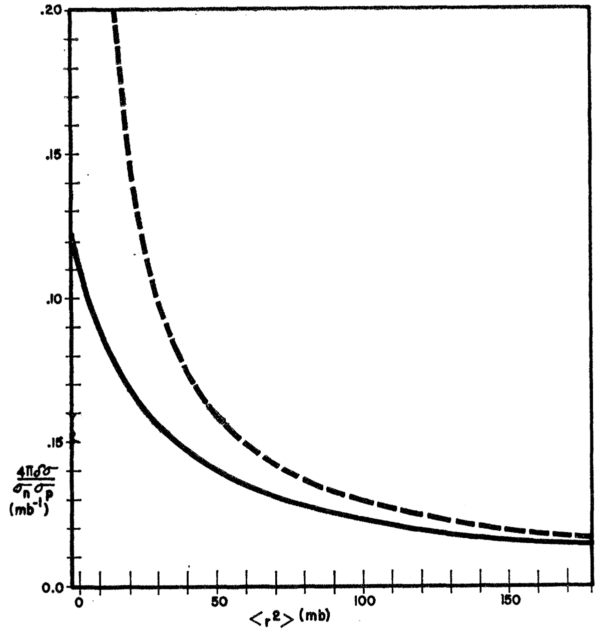

FIG. 1. Two calculations of cross-section defect, as discussed in text. Dashed curve is from Eq. (2); Solid curve is from Eq. (7).

and (7) at this value, we see that the values of $\delta\sigma$ differ by 19%.

The experimental technique usually employed to find σ_n is to measure σ_d and σ_p, calculate $\delta\sigma$ with Eq. (2), and subtract. If, however, Eq. (7) provides a more reliable calculation of $\delta\sigma$ than Eq. (2), the reported value of σ_n will be too large. An example is the recent Serpukhov pion data.[2] If we take $\sigma_p = 24.6$ mb, as obtained in the "asymptotic" region, Eq. (2) yields $\delta\sigma = 1.45$ mb, while Eq. (7) gives $\delta\sigma = 1.17$ mb, reducing the reported value of $\sigma_p - \sigma_n$ by 0.28 mb.

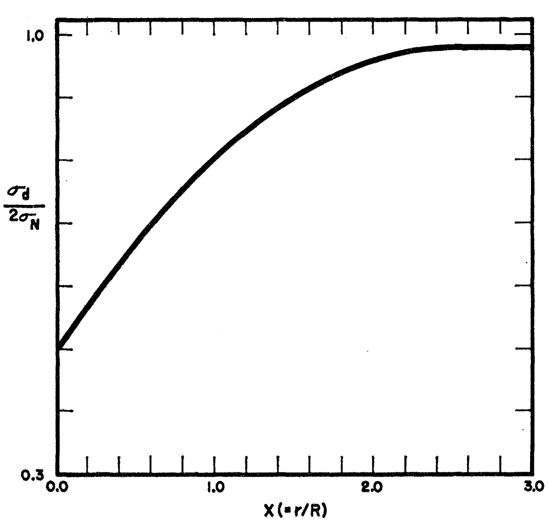

FIG. 2. Total cross section for hadron-deuteron scattering in a black-sphere model.

There is another geometrical calculation which may be performed to test the validity of the two methods above. We assume that the deuteron consists of two black spheres, each of radius r, a distance R apart, revolving about their center of mass under their mutual attraction, and permit interpenetration to any degree, thus including all orders of small-angle multiple scattering. In this case the absorption cross section is simply the area "seen" by the incident particle, and the total cross section is twice this geometrical cross section. The calculation of average area is straightforward. We first observe the spheres in the plane in which they appear to perform simple harmonic motion, average over one period, then rotate the plane of observation, averaging over this 360° rotation. The resultant double integral is rather cumbersome, but is readily evaluated numerically. The results for the total cross section are displayed in Fig. 2. The assumption $\sigma_n = \sigma_p$ is implicit in this calculation, the result being purely a measure of the geometrical shadowing.

We note that for $r \to 0$ (complete overlap), $\sigma_d \to \sigma_N$ as anticipated for black spheres, and $r \to \infty$ leads to $\sigma_d \to 2\sigma_N$, or $\delta\sigma \to 0$, in agreement with both calculations above.

Using $\sigma_p = 24.6$ mb and $\langle 1/r^2 \rangle = 0.03$ mb^{-1} as above, and assuming the strong interaction radius R to be 1.1 fm for diffraction scattering,[5] we obtain $\delta\sigma = 1.13$ mb, in excellent agreement with the result of Eq. (7) above. This result is fairly insensitive to small changes in R, as we are in the flattened portion of Fig. 2 ($X = 2.8$).

Equation (7) may be further improved by inclusion of the real part of the hadron-nucleon amplitude in Eq. (4). While these values are small, their magnitudes appear to decrease with projectile energy,[6] thus producing an energy-dependent increase in the screening correction. The experimental uncertainties in these quantities are large, but it is hoped that future determinations will increase their usefulness in this calculation.

We conclude that our new black-sphere calculation supports the contention that the more general eikonal formulation [Eq. (7)] provides a more reliable determination of the cross-section defect for hadron-deuteron scattering than the frequently used geometrical calculation [Eq. (2)], and we urge its adoption for data analysis in the future.

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