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Distribution over the Number of Prongs in Inelastic Hadronic Collisions*

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The distribution over the number of prongs in inelastic hadronic collisions can be calculated by assuming (i) that the multiplicities of produced pions form a Poisson distribution, and (ii) that all the isospin final states are equally probable. The set of distributions for $\pi^- p$ inelastic collisions have been calculated, and agree well with experiments for all energies.

To predict a Poisson distribution in the number of produced pions, Chew and Pignotti[1] used a multi-Regge-trajectory model. However, experimentally, one usually observes the distribution over the number of prongs (charged particles) in the final states. They assumed, for simplicity, that the effective meson Regge pole has the following properties. (i) It carries either isospin 0 or 1; (ii) it occurs with equal probabilities at the ends of the multi-Regge chain with isospin 0 or 1; (iii) it occurs with alternating values of the isospin along the multi-Regge line. When the Poisson distribution is augmented with a specification of the charges of the final particles, they were then able to calculate the prong distributions for pp inelastic collisions, which agree satisfactorily with experiments.

The authors believe that a Poisson distribution is the consequence of the incoherence of the produced pions. It is very general and not necessarily only valid in the multi-Regge-trajectory model. In this paper, the authors attempt to eliminate the features which are dependent on the specific Regge model, particularly the *ad hoc* isospin-independent postulates (i)–(iii). Instead, we choose the well-known *statistical hypothesis*,[2] which assumes that all the possible inelastic final states satisfying the conservation laws of total isotopic spin, charge, and baryon number are equally probable. If prong distributions resulting from a combination of the Poisson distribution and the statistical hypothesis agree with experiments, one can view the assumed properties (i)–(iii) of the effective meson Regge pole to be a good set of approximations of the statistical hypothesis.

On the other hand, Wang[3] has recently suggested that the charged pions in the inelastic final states of hadronic collisions are produced in pairs in small cells inside the nucleon and that the multiplicities of these pairs follow a Poisson distribution. It was also suggested that neutral pions are produced singly in the same cellular manner but *statistically independent* of the charged production and that the neutrals also follow a Poisson distribution. The assumption that the neutrals and prongs are produced in a statistically independent manner cannot be exactly valid because of the isospin-conservation constraint which relates the branching ratios of reactions involving charged and neutral pions. Wang's model can only provide a fair general agreement for the distribution of prongs for all energies. The model suggested by the authors satisfies isospin conservation and can be used to express all inelastic partial cross sections at various multiplicities in terms of a single one. This is not possible in Wang's model. Take $\pi^- p$ scattering as an example. Wang can only predict the distributions of the number of prongs but cannot relate the partial cross sections to each other. That is, for example, he can express the cross section of $\pi^- p$ going to four prongs plus an arbitrary number of neutral pions in terms of the cross section of $\pi^- p$ going to two prongs plus an arbitrary number of neutral pions, but cannot express the cross section of $\pi^- p \to \pi^- p\pi^+\pi^-\pi^0$ in terms of $\pi^- p \to \pi^- p\pi^+\pi^-$. Concerning neutral-pion production, he can express $\pi^- p \to \pi^0 n\pi^0\pi^0\pi^0$ in terms of $\pi^- p \to \pi^0 n\pi^0\pi^0$, but cannot express $\pi^- p \to \pi^0 n\pi^0\pi^0$ in terms of $\pi^- p \to \pi^- p\pi^+\pi^-$ because the neutral pions are assumed to be produced independently of the charged pions. In our model, all the aforementioned cross sections are related by isospin conservation and can be expressed in terms of a single one. Thus, our model has a higher predictive power. In this paper, we will only calculate the distributions over the number of prongs since one does not generally observe the neutral pions in an experiment.

Let us start with a Poisson distribution for the produced pions,

$$W_0(\bar{m},m) = (\bar{m})^m e^{-\bar{m}}/m!\,, \tag{1}$$

where m is the number of charged and neutral pions produced in the hadronic collisions and $\bar{m}$ is the mean multiplicity of produced charged and neutral pions which depends on the energy of the incident particle.

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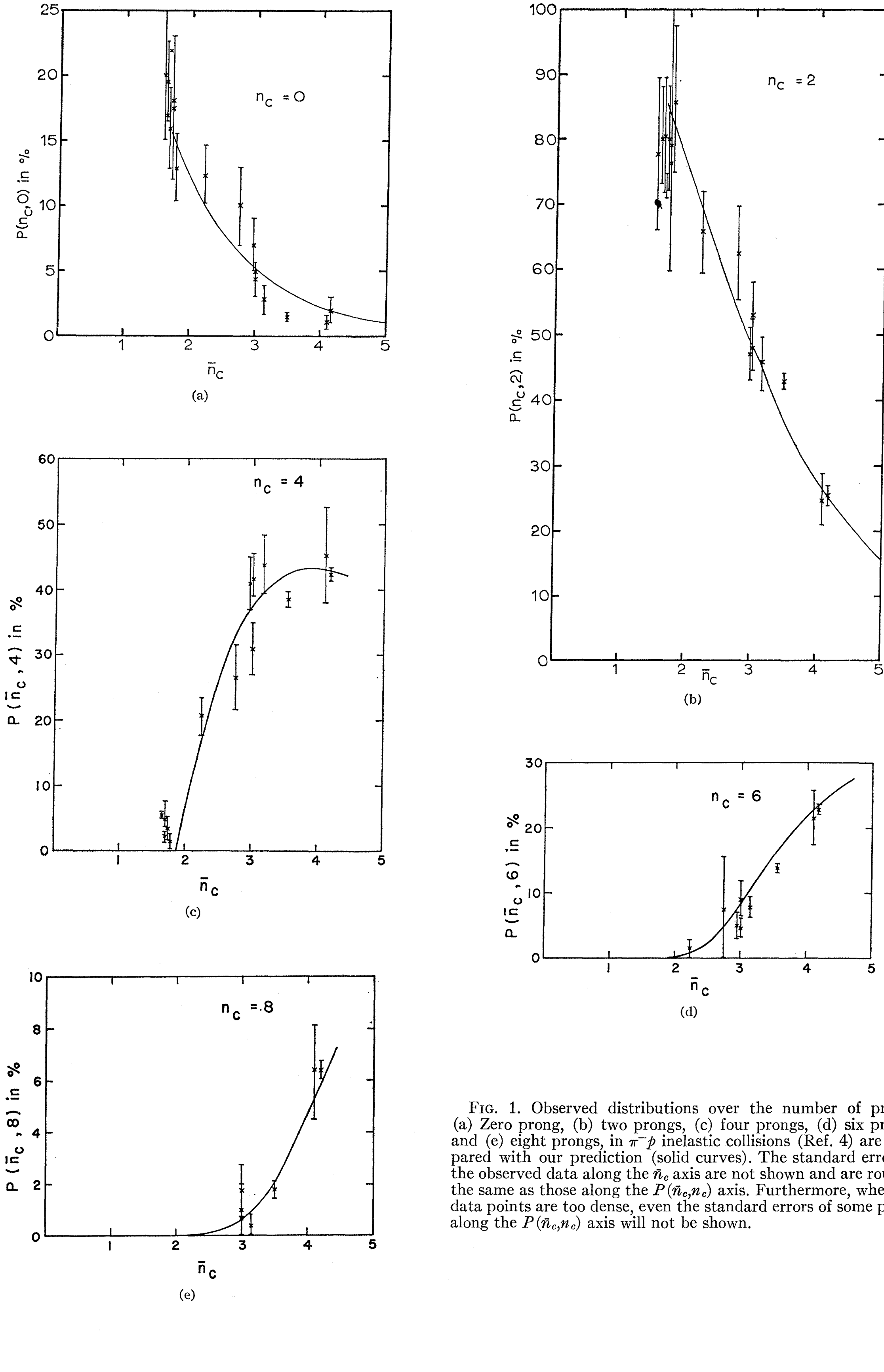

FIG. 1. Observed distributions over the number of prongs. (a) Zero prong, (b) two prongs, (c) four prongs, (d) six prongs, and (e) eight prongs, in $\pi^- p$ inelastic collisions (Ref. 4) are compared with our prediction (solid curves). The standard errors of the observed data along the $\bar{n}_c$ axis are not shown and are roughly the same as those along the $P(\bar{n}_c,n_c)$ axis. Furthermore, when the data points are too dense, even the standard errors of some points along the $P(\bar{n}_c,n_c)$ axis will not be shown.

TABLE I. Comparison of the observed distribution over the number of prongs in $\pi^- p$ inelastic collisions at 16 GeV/c with various predictions. Same notation as in Fig. 2.

No. of prongs	Distribution over the number of prongs in % Expt.	Ours	W_p	W_s
0	2.1±0.9	1.9	···	···
2	25.4±1.4	24.8	33.5	22.3
4	42.3±1.0	42.9	36.6	53.1
6	22.8±0.6	23.3	20.0	21.1
8	6.4±0.3	5.7	7.3	3.4

It is well known that models of a statistical nature cannot account for elastic diffraction scattering. Thus, our model is incapable of predicting the percentage of incident particles going through the elastic and charge-exchange channel (i.e., $m=0$). Therefore, we need to subtract out the elastic and charge-exchange portions from the Poisson distribution and renormalize Eq. (1),

$$W(\bar{m},m)=(1-e^{-\bar{m}})^{-1}(\bar{m})^m e^{-\bar{m}}/m! \quad \text{for } m\geqslant 1. \quad (2)$$

Experimentally, one measures the distribution of the number of *all* the prongs in the final states instead of the distribution for produced charged pions only. In order to compare our predicted distribution with experiment, one should relate $\bar{m}$ with the mean number of prongs (original prongs plus produced prongs) in the final states, $\bar{n}_c$. The relation is

$$\bar{n}_c=\sum_{m=1}^{\infty} a(m)W(\bar{m},m), \quad (3)$$

where $a(m)$ is the mean number of prongs in the fina states when m pions are produced. One can calculate $a(m)$ by using the statistical hypothesis and tables of numerical values of statistical weights.[2] The distribution over the total number of prongs in the final states is given by

$$P(\bar{n}_c,n_c)=\sum_{m=1}^{\infty} b(m,n_c)W(\bar{m},m), \quad (4)$$

where n_c is the total number of prongs (original prongs plus produced prongs) in the final states, and $b(m,n_c)$ is the probability of finding n_c prongs when m pions are produced. The value of $\bar{n}_c$ is fixed by $\bar{m}$ from Eq. (3). The values for $b(m,n_c)$ can be calculated by using the statistical hypothesis. In this paper, we only present numerical calculations for $\pi^- p$ inelastic collisions. The numerical results for pp, $\bar{p}p$, π^+p, and $K^\pm p$ will be published later. The predicted distribution $P(\bar{n}_c,n_c)$ for $\pi^- p$ and the corresponding experimental data[4] are shown in Fig 1. The predicted distributions agree well with experiments. Since Wang's model cannot predict the distribution for $n_c=0$ [Fig. 1(a)], he therefore subtracted out the $n_c=0$ portion and renormalized the distributions for $n_c\geqslant 2$ to 100%. Thus, a point to point comparison of our predictions with those of Wang's is not meaningful. However, one can compare our results with Wang's at the energy range where zero-prong events are unimportant. A comparison is made in Fig. 2 and Table I at $p_{\text{lab}}=16$ GeV/c because this is the energy at which the most accurate measurement[4] of $\pi^- p$ prong distribution was made. One can easily observe that our prediction has achieved a much better fit than Wang's. When compared with experiment, our prediction of $\pi^- p$ at $p_{\text{lab}}=16$ GeV/c is also substantially better than the prediction of pp by Chew and Pignotti.[1] This means that, even though their assumption about isospin independence is reasonable, the statistical hypothesis which we used is more reliable and, in fact, more fundamental.

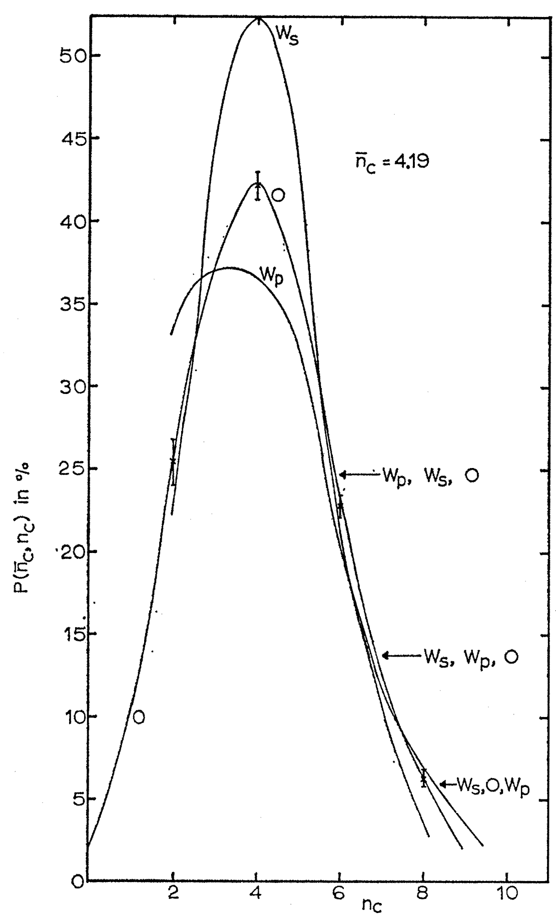

FIG. 2. Observed distribution over the number of prongs in $\pi^- p$ inelastic collisions at 16 GeV/c is compared with our prediction (denoted by O), Wang's prediction based on pair-production (denoted by W_p) and Wang's prediction based on singly produced pions (denoted by W_s).

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Conservation of energy is well satisfied in our model by having the mean multiplicity of pions as the exponent of the Poisson distribution. Compared with the Fermi model,[5] the calculation presented here is much simpler because no tedious phase-space integration is involved. Our model is expected to work well at cosmic-ray energies too. Besides, one can use the model to express all inelastic partial cross sections at various multiplicities in terms of a single one. As the new high-energy accelerators begin to accumulate data, our proposal would provide a convenient way to systematize high-energy data.

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Current Commutators, Dispersion Relations, and K_{l3} Form Factors

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We present a treatment of K_{l3} decays based on current commutators and using a covariant sum-rule formulation. Working in a general kinematical configuration and dispersing on a line (in the plane of the squared pion mass and of the momentum transfer) which belongs to a family of curves all parallel to the "steepest" (kaon rest frame) parabola, we write down definite expressions for the form factors f_+ and f_-. The relation of our procedure to other recently reported approaches is discussed.

I. INTRODUCTION

Recently, Ademollo, Denardo, and Furlan,[1] using the techniques of Ref. 2, gave a generalization of the Callan-Treiman soft-pion relation[3] between the K_{l3} form factors and those of the axial-vector decays of the K and π mesons, in which the pion mass was extrapolated to the physical value by performing a saturation of certain commutators in the kaon rest frame. In order to obtain expressions for the K_{l3} form factors as functions of the momentum transfer, it is of course necessary to work in a more general kinematical configuration. One possibility, actually used by Denardo and Komen,[4] is to saturate the relevant commutators by sandwiching them between the vacuum and a K-meson state of arbitrary energy E. The intergration path is then a curve in the plane of the squared pion four-momentum (q^2) and of the momentum transfer (k^2), which belongs to a family of parabolas (labeled by E) always passing through the Callan-Treiman point [$q^2=0$, $k^2=(\text{kaon mass})^2$]. In this paper, applying the generalized sum-rule formalism given in Ref. 2, we shall consider another possible way of getting the momentum-transfer dependence of the K_{l3} form factors. In order to work covariantly, we shall disperse on a line belonging to a set of parabolas all parallel to the steepest one (that corresponding to the kaon rest frame).

In Sec. II, we start from current- and field-algebra commutators and we derive a set of sum rules, the evaluation of which is treated in Sec. III, where we find, in the one-particle saturation scheme, definite expressions for the K_{l3} form factors $f_\pm$. An alternative procedure, based on the use of dispersion relations for $f_\pm$ in k^2 at $q^2=(\text{pion mass})^2$, together with a current-algebra sum rule and a field-algebra (superconvergent-type) relation, is developed in Sec. IV. The results of this work, as well as its relation to Refs. 1 and 4, are discussed in Sec. V.

II. GENERALIZED SUM RULES

In this section we use the procedure proposed by Fubini and Furlan[2] to derive sum rules for the invariant weak amplitudes related to the K_{l3} decays in the same way as it was done for axial-vector–nucleon-scattering amplitudes in Ref. 5. We start by defining the quantities

$$T_{\mu\nu}=\int d^4z\, e^{iqz}\theta(z_0)\times\langle 0|[A_\mu{}^{(3)}(z),V_\nu{}^{(K^+)}(0)]|K^-(p)\rangle, \quad (2.1)$$

$$U_\mu=i\int d^4z\, e^{iqz}\theta(z_0)\times\langle 0|[D_A{}^{(3)}(z),V_\mu{}^{(K^+)}(0)]|K^-(p)\rangle, \quad (2.2)$$

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