PHYSICAL REVIEW D VOLUME 2, NUMBER 7 1 OCTOBER 1970

Lack of a Burnett-Kroll Theorem for Soft-Pion Emission*

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(Received 19 May 1970)

It is shown that, in the context of the reaction $NN \to NN\pi$, when more than one nucleon pole graph contributes, the soft-pion analog of the Burnett-Kroll extension of the Low soft-photon theorem cannot be established.

SOME time ago, Low showed that the first two terms in the expansion of the photon emission amplitude of $O(k^{-1})$ and $O(k^0)$ are completely determined by the on-shell amplitude for the nonradiative process.[1,2] This theorem was recently extended by Burnett and Kroll, who found that to $O(k^0)$ the squared, spin-averaged radiative amplitude can be expressed in terms of the squared, spin-averaged nonradiative amplitude and derivatives thereof.[3,4] This result enables calculations of radiative cross sections and rates to be carried out more compactly.[5] On the other hand, it also means that one cannot learn more from a study of a radiative, unpolarized cross section to $O(k^0)$ than is already present in the elastic unpolarized cross section.

In view of the close analogy between soft photons and soft pions,[2] it is natural to ask if a similar extension can be found for the soft-pion emission theorem.[6] We find that, in general, this cannot be done.

The soft-pion theorem, as applied, e.g., to the process $NN \to NN\pi$, states that the leading term of $O(q^0)$ in an expansion of the production amplitude in powers of pion momentum is given by the nucleon pole contributions with axial-vector $NN\pi$ coupling.[6] In the photon case, $NN \to NN\gamma$, these nucleon pole terms are the source of the infrared divergence, i.e., the leading term of $O(k^{-1})$ in the Low expansion. It is instructive to recall why, in this order, the radiative amplitude is proportional to the elastic amplitude. For the case of $np \to np\gamma$, where the γ is emitted (in this order) only from the proton, one has

$$T_\gamma{}^{\mathrm{IR}} = \bar{u}_2 \mathfrak{M} \frac{\gamma\cdot(p_1-k)+m}{(p-k)^2-m^2}\gamma\cdot\epsilon u_1 + \bar{u}_2\gamma\cdot\epsilon\frac{\gamma\cdot(p_2+k)+m}{(p_2+k)^2-m^2}\mathfrak{M}u_1$$
$$=\left(-\frac{p_1\cdot\epsilon}{p_1\cdot k}+\frac{p_2\cdot\epsilon}{p_2\cdot k}\right)u_2\mathfrak{M}u_1+O(k^0)\,, \tag{1}$$

where $T_{\mathrm{el}}=\bar{u}_2\mathfrak{M}u_1$ represents the elastic np amplitude. Since $T_\gamma{}^{\mathrm{IR}}$ is proportional to T_{el}, the Burnett-Kroll theorem follows trivially at this level.

Crucial in the above result was the use of

$$(\gamma\cdot p+m)\gamma\cdot\epsilon u(p,s)=2p\cdot\epsilon u(p,s)\,. \tag{2}$$

In the case of soft-pion emission, however, the vertex $\gamma\cdot\epsilon$ is replaced by $\gamma_5\gamma\cdot q$ and things are no longer so simple; the γ matrices do not fall away. Nevertheless, one can show that *if the pion can only be emitted from one external leg*, a result analogous to the Burnett-Kroll theorem does hold. (An example of such a reaction is $np \to pp\pi^-$ at threshold; the $\gamma_5\gamma\cdot q$ vertex and the restriction to threshold means only preemission graphs contribute and the π^- can thus only be emitted from the incident neutron.[7]) For, to $O(q^0)$, the amplitude involves a factor

$$[(\gamma\cdot p+m)/2p\cdot q]\gamma_5\gamma\cdot qu(p,s)\,, \tag{3}$$

and the squared amplitude, summed over the spin s, has a factor[8]

$$(\gamma\cdot p+m)\gamma_5\gamma\cdot q(\gamma\cdot p+m)\gamma\cdot q\gamma_5(\gamma\cdot p+m) =4[\mu^2m^2-(p\cdot q)^2](\gamma\cdot p+m)\,. \tag{4}$$

The disappearance of all γ matrices but $(\gamma\cdot p+m)$ is just what is needed so that the squared, spin-averaged pion-radiative matrix element be proportional to the squared, spin-averaged matrix element for the elastic scattering. This proportionality can, in fact, be seen explicitly in the expression given in Ref. 7 for the threshold production cross section for $np\to pp\pi^-$, where the combination of pp invariant functions[9] that appears is just the same as for unpolarized pp elastic scattering, viz.,

$$|F_S+F_V|^2+3|F_T+F_A|^2\,. \tag{5}$$

This point was not commented on in Ref. 7.

On the other hand, when the pion can be emitted from more than one leg (i.e., more than one nucleon pole graph contributes), then the trick used in Eq. (4)

* Work performed under the auspices of the U. S. Atomic Energy Commission.
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does not work simply because of the interference terms.[10] As an example, in $pp \to pp\pi^0$ at threshold, where the π^0 can be preemitted from either incoming proton, the combination of pp invariant functions which appears is[7]

$$3|F_S+F_V|^2-2\,\mathrm{Re}(F_S+F_V)(F_T^*+F_A^*)+11|F_T+F_A|^2, \quad (6)$$

quite different from the elastic combination given in Eq. (5). A similar statement holds for a process such as $p\alpha \to p\alpha\pi^0$ at higher energies, where the π^0 can be emitted by the proton either before or after the $p\alpha$ scattering vertex. Again, the interference term between the pre- and post-emission graphs gives a different combination of the invariant functions than is measured in an unpolarized $p\alpha$ elastic-scattering experiment.

To summarize, we have seen that in contrast to the case of soft-photon emission the measurement of an unpolarized pion-radiative cross section can, as seen in the comparison of Eq. (6) with Eq. (5), give information on the elastic-scattering amplitudes that is unavailable from unpolarized elastic-scattering experiments.

The author acknowledges helpful conversations with H. W. Fearing, P. A. M. Gram, M. E. Schillaci, M. D. Shuster, and D. A. Zollman.

[10] That the interference terms do not cause any trouble in the photon case, even in $O(k^0)$, can be seen quite explicitly in Eq. (14) of Ref. 4.

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Regge Poles and the Pauli Principle*

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(Received 11 April 1969)

In a Regge-pole model of large-angle pp scattering, Huang and Pinsky take the Pauli principle into account by adding t- and u-channel Regge poles. We point out that this procedure is consistent with the finite-energy sum rules. In a recent note, Pinsky points out some regularities in the experimental data and contends that they favor a different model in dealing with the Pauli principle, namely, some kind of "duality" between the t and u channels. We show that the regularities he noticed are also reproduced by the original "additive" model.

IN their model of large-angle pp scattering, Huang and Pinsky[1] take the Pauli principle into account by adding the contributions of a t- and u-channel Regge pole, in analogy with Feynman graphs. It is known, however, that a simple addition of Regge-pole contributions from two different channels may be counting the same contribution twice, as is the case in the so-called interference model for πp scattering, in which the scattering amplitude is taken to be the sum of t- and s-channel Regge-pole contributions. That double counting is committed and revealed by examining the finite-energy sum rules (FESR).[2] In that case, there is a duality between t- and s-channel Regge poles, in the sense that a t-channel Regge pole already includes some contributions of the s-channel Regge poles and vice versa. It was argued in Ref. 1 that the criticism of the interference model may not apply to the t-u additivity, because the third channel (s channel) has no resonances. We wish to supplement that argument here and to comment on a paper by Pinsky,[3] which contains a criticism of the model of Ref. 1.

The criticism of the interference model based on FESR is not relevant to the model of Ref. 1, because the latter is in fact consistent with FESR. To demonstrate this in a simplified form we write a Veneziano representation[4] for a scalar amplitude $A(t,u)$, which is antisymmetric in t and u, and which has no pole in the variable $s=4m^2-t-u$:

$$A(t,u)=\beta(\alpha_t-\alpha_u)\Gamma(1-\alpha_t)\Gamma(1-\alpha_u)/\Gamma(2-\alpha_t-\alpha_u)$$
$$=\beta\left[\frac{\Gamma(2-\alpha_t)\Gamma(1-\alpha_u)}{\Gamma(2-\alpha_t-\alpha_u)}-\frac{\Gamma(2-\alpha_u)\Gamma(1-\alpha_t)}{\Gamma(2-\alpha_t-\alpha_u)}\right], \quad (1)$$

where β is a constant. The fact that this is consistent with FESR can be shown in a manner similar to that employed in Ref. 4. Note that the first term in (1) alone contains the leading t-channel Regge pole α_t, and the second term alone contains the leading u-channel Regge pole α_u, but the daughter Regge poles α_t-k α_u-k, $(k=1, 2, 3, \ldots)$ enter into both terms. In

* Work supported in part through funds provided by the Atomic Energy Commission under Contract No. At(30-1)2098.

† National Science Foundation Predoctoral Fellow.

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