the simple-$SU(3)$ relations between the vertices (in a pure pole model) to predict the $\rho_{ij}$ and $d\sigma/dt$ for the reaction $K^+p \to K^0\Delta^{++}$ from those of reactions (i) and (ii).

We believe that the experimental evidence favors the "Argonne"[8] recipe for generating cuts, in which the absorptive correction only causes moderate changes from the pure pole term and where the dips are a result of the ghost-eliminating factors present in the pole amplitude.

PHYSICAL REVIEW D VOLUME 2, NUMBER 7 1 OCTOBER 1970

# Dynamical Enhancement of Symmetry Breaking in $K_{l3}$ Decay*

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(Received 18 May 1970)

The possible dynamical enhancement of the second-order symmetry-breaking corrections to a recently derived expression for the $K_{l3}$ form factors is investigated.

RECENTLY, Dashen and Weinstein[1] derived a theorem for the form-factor ratio $\xi(0)$ in $K_{l3}$ decay. It states that

$$\lambda_+\frac{m_K^2-m_\pi^2}{m_\pi^2}+\xi(0)=\frac{1}{2}\left(\frac{f_K}{f_\pi}-\frac{f_\pi}{f_K}\right)+O(\epsilon^2), \quad (1)$$

where $\xi(0)$ and $\lambda_+$ are defined in the usual manner, viz.,

$$\xi(0)=f_-(0)/f_+(0),$$

$\lambda_+$ is defined by

$$f_+(t)=f_+(0)(1+\lambda_+t/m_\pi^2),$$

and the $f_i$ are defined by

$$\langle 0|A_\mu{}^{K^-}|K^+\rangle=f_Kp_\mu{}^K,$$
$$\langle 0|A_\mu{}^{\pi^-}|\pi^+\rangle=f_\pi p_\mu{}^\pi.$$

The $O(\epsilon^2)$ terms are those of second order in the $SU_3\times SU_3$ symmetry-breaking parameter $\epsilon$.

If one uses the values $\lambda_+=0.06$ and $\xi(0)\simeq-0.9\pm0.4$, given by the latest $X_2$ collaboration experiment,[2] and the value $f_K/f_\pi=1.23\pm0.05+O(\epsilon^2)$, then the left-hand side of (1) is $\sim-0.16\pm0.4$ and the right-hand side $\sim0.23\pm0.05$, so that the theorem is indeed satisfied, but just within the quoted errors. It may thus be important to obtain an estimate of the neglected $O(\epsilon^2)$ terms, for if they are for some reason dynamically enhanced and of such a sign as to spoil the agreement of (1) with experiment, then the procedure of doing perturbation theory around an $SU_3\times SU_3$ symmetry limit may be rather suspect.[3]

The purpose of this investigation is to make an approximate calculation of the $O(\epsilon^2)$ term in order to get an indication of its sign and magnitude. We find that the dominant corrections contribute in such a manner as to improve (through only marginally) the agreement with experiment.

In order to proceed, we first make an assumption about the form of the symmetry breaking. We make the usual ansatz[4] that the hadronic Hamiltonian is

$$H=H_0+\epsilon(u_0+cu_8),$$

where $H_0$ is invariant under $SU_3\times SU_3$ and the $u_i$ are elements of the representation $(3,3)+(3,3)$ with $\epsilon\ll1$ and $c+\sqrt{2}=\eta\ll1$.

In this model, we have the divergence equations

$$[Q_A{}^{\pi^+}(0),\partial^\mu A_\mu{}^{K^0}(0)]\equiv\Sigma_+{}^{\pi K}(0)=f(c)\partial^\mu V^{\mu K^+}(0), \quad (2a)$$

$$[Q_A{}^{K^0}(0),\partial^\mu A_\mu{}^{\pi^+}(0)]=\Sigma_+{}^{K\pi}(0)=[f(c)-1]\partial^\mu V_\mu{}^{K^+}(0), \quad (2b)$$

$$[Q_A{}^{\pi^-}(0),\partial^\mu A_\mu{}^{\bar{K}^0}(0)]=\Sigma_-{}^{\pi K}(0)=-f(c)\partial^\mu V_\mu{}^{K^-}(0), \quad (2c)$$

$$[Q_A{}^{\bar{K}^0},\partial^\mu A_\mu{}^{\pi^-}]=\Sigma_-{}^{K\pi}(0)=[1-f(c)]\partial^\mu V_\mu{}^{K^-}(0), \quad (2d)$$

where $f(c)=(2\sqrt{2}-c)/3c$. Also.

$$[Q^{\pi^+}(0),\Sigma_-{}^{\pi K}(0)]=\partial^\mu A_\mu{}^{\bar{K}^0}(0) \quad (3)$$

and

$$\Sigma_-{}^{K\pi}(0)-\Sigma_-{}^{\pi K}(0)=\partial^\mu V_\mu{}^{K^-}(0). \quad (4)$$

This last equation is, of course, true independently of the form of the symmetry breaking.

* Supported in part by the National Research Council of Canada.

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[1] R. Dashen and M. Weinstein, Phys. Rev. Letters **22**, 1337 (1969).

[2] $X_2$ Collaboration, Phys. Letters **29B**, 691 (1969); **29B**, 696 (1969).

[3] This procedure is discussed in detail in R. Dashen, Phys. Rev. **183**, 1245 (1969); R. Dashen and M. Weinstein, *ibid.* **183**, 1261 (1969); **188**, 2330 (1969). For an application to $K_{l4}$ decay, see S. P. de Alwis, Phys. Rev. D **1**, 2131 (1970).

[4] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1969).

Let us make the following definitions:

$$W_\mu{}^{\pi K}(p,q)=\int d^4x d^4y\, e^{-ipx+iqy}\langle 0|T\{A_\mu{}^{\pi^+}(x)\partial^\lambda A_\lambda{}^{K^0}(y)\Sigma_-{}^{\pi K^0}(0)\}|0\rangle, \tag{5a}$$

$$W^{\pi K}(p^2,q^2,k^2)=\int d^4x d^4y\, e^{-ipx+iqy}\langle 0|T\{\partial^\lambda A_\lambda{}^{\pi^+}(x)\partial^\mu A_\mu{}^{K^0}(y)\Sigma_-{}^{\pi K^0}(0)\}|0\rangle, \tag{5b}$$

$$f_\pi{}^2 m_\pi{}^4\Delta^\pi(p)=\int d^4x\, e^{ipx}\langle 0|\{\partial^\lambda A_\lambda{}^{\pi^+}(x)\partial^\mu A_\mu{}^{\pi^-}(0)\}|0\rangle, \tag{5c}$$

$$f_K{}^2 m_K{}^4\Delta^K(q)=\int d^4x\, e^{-iqx}\langle 0|T\{\partial^\lambda A_\lambda{}^{K^0}(x)\partial^\mu A_\mu{}^{\bar{K}^0}(0)\}|0\rangle, \tag{5d}$$

$$\Delta_S(k)=\int d^4x\, e^{ikx}\langle 0|T\{\partial^\lambda V_\lambda{}^K(x)\partial^\mu V_\mu{}^{\bar{K}}(0)\}|0\rangle$$

$$=\int\frac{\rho_S(x)}{x-k^2}dx, \tag{5e}$$

$$W^{\pi K}(p^2,q^2,k^2)=f_\pi m_\pi{}^2\Delta_\pi(p)f_K m_K{}^2\Delta_K(q)F^{\pi K}(p^2,q^2,k^2), \tag{6a}$$

$$W_\mu{}^{\pi K}(p,q)=-ip_\mu f_\pi\Delta_\pi(p)f_K m_K{}^2\Delta_K(q)F^{\pi K}(p^2,q^2,k^2)+f_K m_K{}^2\Delta^K(q)F_\mu{}^{\pi K}(p,q). \tag{6b}$$

From the current divergence equations (2a)–(4), we have, using the definitions (5a)–(5c), the Ward identity

$$p^\mu F_\mu{}^{\pi K}(p,q)+f_\pi(m_\pi{}^2-p^2)\Delta_\pi(p)F^{\pi K}(p^2,q^2,k^2) =(1/f_K m_K{}^2)\Delta^{K-1}(q)\{[f(c)]^2\Delta_S{}^K(k) +f_K{}^2M_K{}^2\Delta^K(q)\}. \tag{7}$$

Defining also the amputated $A_1$-$\kappa$-$K$ vertex $\Gamma_\lambda$ ($\kappa$ being the scalar $K\pi$ state) by

$$F_\mu{}^{\pi K}(p,q)=\Delta_{\mu\lambda}{}^{A_1}(p)\Delta_S{}^K(k)\Gamma_\lambda(p,q),$$

where $\Delta_{\mu\lambda}{}^{A_1}$ is the $A_1$ propagator, we have, taking $p^2=m_\pi{}^2$ and $q^2=m_K{}^2$ in Eq. (7),

$$F^{\pi K}(k^2)=-\frac{f_K}{f_\pi}m_K{}^2+C_A f_\pi{}^{-1}p^\mu\Gamma_\mu(p,q)\int\frac{\rho_S(x)}{x-k^2}dx, \tag{8a}$$

where

$$C_A=\int\frac{\rho_{A_1}(x)}{x}dx.$$

Similarly, we can derive an equation for $F^{K\pi}(k^2)$:

$$F^{K\pi}(k^2)=-\frac{f_\pi}{f_K}m_\pi{}^2+C_{KA}f_K{}^{-1}p^\mu\tilde{\Gamma}_\mu(p,q)\times\int\frac{\rho_S(x)}{x-k^2}dx, \tag{8b}$$

where $\tilde{\Gamma}_\mu$ is the amputated $K_A$-$\kappa$-$\pi$ vertex. Thus we have, using Eqs. (8a) and (8b) and (4),

$$f_S(k^2)=\langle\pi|\partial^\mu V_\mu{}^K|k\rangle=\left(\frac{f_K}{f_\pi}m_K{}^2-\frac{f_\pi}{f_K}m_\pi{}^2\right)+\frac{P(k^2)}{C_S}\int\frac{\rho_S(x)}{x-k^2}dx, \tag{9}$$

where $P(K^2)$ is a linear combination of $p^\mu\Gamma_\mu$ and $q^\mu\tilde{\Gamma}_\mu$ and

$$C_S=\int\frac{\rho_S(x)}{x}dx.$$

So far, we have made no assumptions about the smallness of the chiral symmetry breaking (only about the *form*) or equivalently a partially conserved axial-vector-current–type assumption, nor any smoothness assumptions about the amputated vertex functions. Thus Eq. (9) is correct to all orders in the symmetry breaking. Now since

$$f_S(0)=(m_K{}^2-m_\pi{}^2)f_+(0),$$

we have

$$P(0)=(m_K{}^2-m_\pi{}^2)f_+(0)-\left(\frac{f_K}{f_\pi}m_K{}^2-\frac{f_\pi}{f_K}m_\pi{}^2\right).$$

Also the theorem of Dashen and Weinstein gives[5]

$$f_S(m_K{}^2-m_\pi{}^2)=(m_K{}^2-m_\pi{}^2)f_K/f_\pi+O(\epsilon^3\eta).$$

[5] See Eq. (6) of Ref. 1. The parameter $\epsilon_1$ of Ref. 1 is our $\epsilon$ and $\epsilon_2$ is our $\eta\epsilon$.

This enables us to write

$$P'(0)=\left(\frac{f_K}{f_\pi}-f_+(0)\right)+O(\epsilon^2\eta).$$

Hence we may write Eq. (9) for small $t=k^2$ as

$$f_S(t)=\left(\frac{f_K}{f_\pi}m_K{}^2-\frac{f_\pi}{f_K}m_\pi{}^2\right)+C_S{}^{-1}\left[(m_K{}^2-m_\pi{}^2)f_+(0)-\frac{f_K}{f_\pi}m_K{}^2+\frac{f_\pi}{f_K}m_\pi{}^2+\left(\frac{f_K}{f_\pi}-f_+(0)\right)t+O(\epsilon t^2)\right]\times\int\frac{\rho_S(x)}{x-t}dx. \quad (9')$$

This gives us an approximate iterative method for evaluating the $O(\epsilon^2)$ corrections to the Dashen-Weinstein theorem. In particular, it enables us to investigate the possible dynamical enhancement of these $O(\epsilon^2)$ terms.

Let us first look at a simple $\kappa$-dominance model in which

$$\rho_S(x)=f_\kappa{}^2\delta(x-m_\kappa{}^2),$$

where $\kappa$ is a scalar $K\pi$ state; $f_\kappa$ is defined by

$$\langle 0|V_\mu|\kappa\rangle=f_\kappa p_\mu{}^\kappa.$$

Then, we have from (9′)

$$\frac{1}{f_+(0)}f_S{}^1(0)=\frac{\lambda_+}{m_\pi{}^2}(m_K{}^2-m_S{}^2)+\xi(0)=\left(\frac{f_K}{f_\pi f_+(0)}-1\right)\left(1-\frac{m_K{}^2-m_\pi{}^2}{m_\kappa{}^2}\right),$$

where we have neglected the small quantity

$$(f_K/f_\pi-f_\pi/f_K)m_\pi{}^2\sim O(\epsilon^2\eta).$$

Thus, for low values for the mass of the $\kappa$, the second-order $[O(\epsilon^2)]$ terms can be fairly significant, contributing about $-0.11$ for $m_\kappa\sim 700$ MeV, but they become negligible for $\kappa$ masses above 1000 MeV (as seems to be the case experimentally).

As an improvement on the simple $\kappa$-dominance model, let us use the model of Amatya *et al.*[6] in which the scalar two-particle continuum contribution to $\rho_s$ is evaluated using two-particle unitarity. We rewrite Eq. (9′) neglecting the small terms $(f_K/f_\pi-f_\pi/f_K)m_\pi{}^2$:

$$f_S(t)=\frac{f_K}{f_\pi}(m_K{}^2-m_\pi{}^2)+C_S{}^{-1}\left(f_+(0)-\frac{f_K}{f_\pi}\right)\times[(m_K{}^2-m_\pi{}^2)-t]\int\frac{\rho_S(t)}{x-t}dx, \quad (9'')$$

where we assume also that since $P(t)$ is essentially an amputated vertex, its momentum dependence is small so that we neglect the $O(t^2)$ terms in $P(t)$ even for large values of $t$.

Keeping the $K\pi$ contribution to $\rho_S(x)$, we have

$$\rho_S(x)\simeq\rho_S{}^{K\pi}(x)=\frac{3}{16\pi^2}\frac{p}{\sqrt{x}}|f_S(x)|^2, \quad (10)$$

with

$$p^2(x)=(1/4x)(x^2+m_K{}^4+m_\pi{}^4-2xm_K{}^2-2xm_\pi{}^2-2m_K{}^2m_\pi{}^2). \quad (11)$$

Now define a function $D(t)$ in the usual manner by

$$f_S(t)=\frac{f_K}{f_\pi}\frac{m_K{}^2-m_\pi{}^2}{D(t)}. \quad (12)$$

Then

$$\mathrm{Im}D(t)=-\frac{f_K}{f_\pi}[(m_K{}^2-m_\pi{}^2)-t](m_K{}^2-m_\pi{}^2)\times C_S{}^{-1}\left(f_+(0)-\frac{f_K}{f_\pi}\right)\frac{3}{16\pi^2}\frac{p(t)}{\sqrt{t}}. \quad (13)$$

Also since we must have[5]

$$D(m_K{}^2-m_\pi{}^2)=1,$$

we find

$$D(t)=1-(m_K{}^2-m_\pi{}^2)\times\left(f_+(0)-\frac{f_K}{f_\pi}\right)C_S{}^{-1}\frac{f_K}{f_\pi}[(m_K{}^2-m_\pi{}^2)-t]\times\frac{3}{16\pi^2}\int_{(m_K+m_\pi)^2}^{\Lambda}\frac{p(x)}{\sqrt{x}}\frac{dx}{x-t}, \quad (14)$$

where we have cut off the integral at $x=\Lambda$, since the two-particle approximation is not expected to be valid up to arbitrarily high energies.

Let us define

$$I_\Lambda(t)=\frac{3}{16\pi^2}\int_{(m_K+m_\pi)^2}^{\Lambda}\frac{p(x)}{\sqrt{x}}\frac{dx}{x-t}. \quad (15)$$

[6] A. Amatya, A. Pagnamenta, and B. Renner, Phys. Rev. **172**, 1755 (1968); A. Pagnamenta and R. Renner, *ibid.* **172**, 1761 (1968). This method has recently been applied in a modified form by J. Brehm, E. Golowich, and J. C. Prasad, Phys. Rev. Letters **23**, 666 (1969). Two more papers using this modified form of Brehm *et al.* are P. V. Collins, Cambridge Report No. DAMTP 70/3, 1970 (unpublished); D. A. Nutbrown, Phys. Rev. D **2**, 1140 (1970).

Evaluating the integral, we find for large $\Lambda$

$$\frac{32\pi^2}{3}I_\Lambda(t)$$

$$=\ln\frac{\Lambda}{m_K m_\pi}+\frac{m_K{}^2-m_\pi{}^2}{t}\ln\frac{m_\pi}{m_K}-2\frac{p(t)}{\sqrt{t}}\ln\frac{m_K{}^2+m_\pi{}^2-t-2(\sqrt{t})p(t)}{2m_K m_\pi}\quad\text{for}\quad t<(m_K-m_\pi)^2$$

$$=\ln\frac{\Lambda}{m_K m_\pi}+\frac{m_K{}^2-m_\pi{}^2}{t}\ln\frac{m_\pi}{m_K}+\frac{[-p^2(t)]^{1/2}}{\sqrt{t}}\left[\tfrac{1}{2}\pi-\tan^{-1}\frac{m_K{}^2-m_\pi{}^2}{4(\sqrt{t})p(t)}\right]\quad\text{for}\quad (m_K-m_\pi)^2<t<(m_K+m_\pi)^2 \tag{16}$$

$$=\ln\frac{\Lambda}{m_K m_\pi}+\frac{m_K{}^2-m_\pi{}^2}{t}\ln\frac{m_\pi}{m_K}-\frac{2p(t)}{\sqrt{t}}\left\{\ln\left[\frac{2(\sqrt{t})p(t)+t-m_K{}^2-m_\pi{}^2}{2m_K m_\pi}\right]\pm i\pi\right\}\quad\text{for}\quad t=\mathrm{Re}t\pm i\epsilon,\quad \mathrm{Re}t>(m_K+m_\pi)^2.$$

The normalization condition

$$D(0)=f_K/f_\pi f_+(0)$$

fixes

$$C_S=f_+(0)(f_K/f_\pi)(m_K{}^2-m_\pi{}^2)^2 I_\Lambda(0)\,, \tag{17}$$

and following Brehm *et al.* (Ref. 5), $\Lambda$ can be determined in terms of the mass of a scalar $K\pi$ resonance $\kappa$, by requiring that

$$\mathrm{Re}D(m_\kappa{}^2)=0\,. \tag{18}$$

Thus

$$(m_K{}^2-m_\pi{}^2)\frac{\lambda_+}{m_\pi{}^2}+\xi(0)=\left(\frac{f_K}{f_\pi f_+(0)}-1\right)\left(\frac{f_K}{f_\pi f_+(0)}\right)^{-1}\times\left(1-(m_K{}^2-m_\pi{}^2)\frac{I_\Lambda'(0)}{I_\Lambda(0)}\right), \tag{19}$$

where

$$\frac{32\pi^2}{3}I_\Lambda'(0)=\frac{m_K{}^4-m_\pi{}^4+4m_\pi{}^2m_K{}^2\ln(m_\pi/m_K)}{2(m_K{}^2-m_\pi{}^2)^3}\,,$$

$$\frac{32\pi^2}{3}I_\Lambda(0)=\ln\frac{\Lambda}{m_K m_\pi}+\frac{m_K{}^2+m_\pi{}^2}{m_K{}^2-m_\pi{}^2}\ln\frac{m_\pi}{m_K}-1\,,$$

where $\Lambda$ is given in terms of $m_\kappa{}^2$ by (18).[7]

The term $(m_K{}^2-m_\pi{}^2)I_\Lambda'(0)/I_\Lambda(0)$ $(=\Delta$, say) has a maximum value of 0.6 for $m_\kappa{}^2\simeq1000$ MeV. For larger values of $m_\kappa$, say $m_\kappa\simeq1500$, we have $\Delta\simeq0.3$, and for $m_\kappa\simeq700, \Delta\simeq0.28$. Thus, for the experimentally observed $\kappa$'s (i.e., in the region of 1000 MeV), this model predicts a rather significant enhancement of the second-order effects, giving for the right-hand side of (19) a value $\simeq0.08$. (The experimental value for the left-hand side, as we noted at the beginning, is $-0.16\pm0.4$.) Of course, the errors are too large for us to make a good quantitative comparison to our model, but it seems to us that the second-order terms do indeed contribute with the right sign.

I wish to thank Dr. Leaf Turner for reading the manuscript.

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[7] As by-products of this model, we can of course obtain the $\kappa$ width and the $K\pi$ phase shifts. These calculations are, however, tedious, of no direct interest to our present problem, and not particularly illuminating, in view of the present unclear experimental situation regarding the $\kappa$, and the nonexistence of any data on $K\pi$ scattering.