PHYSICAL REVIEW D VOLUME 2, NUMBER 7 1 OCTOBER 1970

A Partial Conservation of Axial-Vector Current Check on the Consistency among $\pm$-Parity Meson Couplings in the Quark Model

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(Received 18 May 1970)

The supermultiplet coupling constant $\bar{g}_1$ for the transition from $L^P=1^+$ meson states to those of $L^P=0^-$ in a relativistically boosted $SU(6)\times O(3)$ quark model developed previously by the author is evaluated by exploiting the connection between the $A_1\rho\pi$ and $\rho\pi\pi$ couplings in the model via partial conservation of axial-vector current and Weinberg sum rules. The new determination of $\bar{g}_1$ is in excellent agreement with its value obtained earlier from a direct fit to the experimental decay widths.

I. INTRODUCTION

SINCE the discovery of the Weinberg sum rules,[1] a considerable amount of effort has been devoted to studying the relations among $A_1\rho\pi$ and $\rho\pi\pi$ coupling constants, using (i) the methods of current algebra with pole dominance,[2] (ii) hard-pion techniques (via the Ward identity[3]), and (iii) chiral dynamics methods for the construction of phenomenological Lagrangians.[4] One common feature of most of these approaches is the introduction of a free parameter which essentially defines the ratio D/S of the D-wave and S-wave coupling in $A_1\rho\pi$. In the theory of Schwinger,[4] which is a rather specialized version of chiral dynamics, the ratio D/S vanishes exactly. However, this value is not favored by experiment.[5] A somewhat different approach to the construction of $A_1\rho\pi$ couplings is represented by the quark model in an $SU(6)\times O(3)$ framework,[6] in which A_1 is just *one* member of one ($^3P_1$) of the four nonets $^1P_1$ and $^3P_{0,1,2}$, and does not enjoy the special status it has in current algebra or chiral dynamics as the derivative of the pion field. The purpose of this paper is to show that if the partially conserved axial-vector current (PCAC) relation between the A_1 current and the π field is used in conjunction with one of the Weinberg sum rules within the general (phenomenological) framework of the $SU(6)\times O(3)$ quark model, it is possible to make an *independent* determination of the $A_1\rho\pi$ coupling constant, one which is in surprisingly good agreement with its empirical value determined earlier from a fit to the strong decay widths of the various $L^P=1^+$ mesons. Note that in the quark model the $A_1\rho\pi$ coupling constant, involving a $J^{PC}=1^{++}$ meson, is *geometrically* related to those of other positive-parity (0^{++}, 1^{+-}, and 2^{++}) mesons to PP and PV states. Thus the extra ingredient of PCAC seems to provide a consistent correlation among the couplings of (+) and (−) parity mesons within the quark model, a correlation whose predictive power goes much beyond those of current (or field) algebra or chiral dynamics. In other words, it now provides a consistent and unified framework, not only for the couplings of the 1^{++} mesons, but for those of 0^{++}, 1^{+-}, and 2^{++} states to PP and PV systems, so that essentially one universal parameter (say the $\rho\pi\pi$ coupling constant) describes transitions both among and within the $L^P=0^-$ and $L^P=1^+$ supermultiplets.

Calculations of positive-parity meson couplings to P and V mesons on the quark model have been made by various authors.[7] More recently, a relativistic formulation of higher meson (M_L) couplings to PV and PP states was made through a standard relativistic boosting of $SU(6)\times O(3)$ couplings and the introduction of a phenomenological form factor $\bar{f}_L$ defined by[8,9]

$$\bar{f}_L=2M\bar{g}_L\mu^{-L-1}\left(\frac{\mu M}{-P\cdot k}\right)^{L+1}\left(\frac{\mu}{m_\pi}\right)^{1/2}, \tag{1}$$

where M and μ are the masses of the decaying meson (four-momentum P_μ) and the radiation quantum P meson (four-momentum k_μ), respectively; $\bar{g}_L$ is a dimensionless parameter representing the coupling constant for the supermultiplet transition $L^P\to 0^-$, and the exponents $(L\pm1)$ are appropriate for $(L\pm1)$-wave couplings, respectively.[10] This coupling scheme

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[1] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).

[2] S. G. Brown and G. B. West, Phys. Rev. **168**, 1605 (1968), and earlier references in the paper; D. A. Geffen, Phys. Rev. Letters **19**, 770 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* **19**, 1067 (1967); R. Arnowitt, M. H. Friedman, and P. Nath, *ibid.* 1085 (1967).

[3] H. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).

[4] J. Schwinger, Phys. Letters **24B**, 475 (1967); J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967); B. W. Lee and H. T. Nieh, *ibid.* **166**, 1507 (1968).

[5] It not only predicts too small a width for $A_1\to\rho\pi$, but also gives a zero angular correlation (isotropy) for the $\rho\pi$ system, which seems to be ruled out by experiment.

[6] See, e.g., R. H. Dalitz, in *High-Energy Physics*, edited by B. DeWitt and M. Jacob (Gordon and Breach, New York, 1965).

[7] A. N. Mitra and P. P. Srivastava, Phys. Rev. **164**, 1803 (1967); J. Uretsky, in *Lectures in Theoretical Physics*, edited by H. H. Aly (Wiley-Interscience, New York, 1968).

[8] A. N. Mitra, Nuovo Cimento **61A**, 344 (1969).

[9] D. K. Choudhury and A. N. Mitra, Phys. Rev. D **1**, 351 (1970).

[10] The notation is the same as in Ref. 9 except that the form factor has now been written in an explicitly covariant form which is immediately suggestive of an off-shell extension. Actually, this form was used by the author in a recent extension of $\bar{B}B_LP$ couplings to $\bar{B}B_LV$ couplings through the extra ingredient of Schwinger's partial symmetry [J. Schwinger, Phys. Rev. Letters **18**, 923 (1967)], but in a less covariant notation [A. N. Mitra, Nuovo Cimento **64A**, 603 (1969)]. The magnitudes of all these forms of course agree exactly on the mass shell for the various particles.

was compared in detail with the experimental decay modes of various positive-parity mesons[9] with a good deal of success; it furnished the following value of the supermultiplet coupling constant $\bar{g}_1$:

$$\bar{g}_1^2/4\pi = 0.08 \pm 0.01. \tag{2}$$

II. EVALUATION OF $\bar{g}_1$ BY PCAC

We now make an independent determination of $\bar{g}_1$ in terms of the $g_{\rho\pi\pi}$ coupling constant using PCAC and the Weinberg sum rules. The argument goes as follows: The effective Lagrangian in momentum space for the $A_1 \to \rho\pi$ transition has, apart from the factor $(\mu M/-P\cdot k)^{L\pm1}$ of Eq. (1), the structure[9]

$$2\sqrt{2}M\bar{g}_1\bar{\pi}T_a[(k\cdot\rho)(k\cdot A^a)+\mu^2\rho\cdot A^a]\mu^{-2}, \tag{3}$$

where the isospin structure for the π and ρ fields has been written in the spherical representation, so that T^a represents the *actual* $I=1$ angular momentum, operating between the π and ρ states. From (3) it is of course easy to identify the D-wave and S-wave coupling terms in the sense that other authors[2,11] have used the terminology, but it is, strictly speaking, only an approximate description. In this approximation, according to our recipe [Eq. (1)], we must use the exponents $(L\pm1)$ for the two terms of (3), respectively, so that the full effective Lagrangian is

$$\mathcal{L}_1 = 2\sqrt{2}M\bar{g}_1\bar{\pi}T^a[(k\cdot\rho)(k\cdot A^a)(M/P\cdot k)^2+\rho\cdot A^a]. \tag{4}$$

For a more accurate description we must break up $(k\cdot\rho)(k\cdot A^a)$ covariantly into its D- and S-wave parts (using the rest frame of A_1) in the following manner:

$$(k\cdot\rho)(k\cdot A^a) = k_\mu k_\nu T_{\mu\nu}{}^a + \tfrac{1}{3}k_\mu k_\nu D_{\mu\nu}\rho\cdot A^a, \tag{5}$$

where

$$D_{\mu\nu} = \delta_{\mu\nu} + P_\mu P_\nu/M^2 \tag{6}$$

and

$$T_{\mu\nu}{}^a = \tfrac{1}{2}(\rho_\mu A_\nu{}^a + \rho_\nu A_\mu{}^a - \tfrac{2}{3}D_{\mu\nu}\rho\cdot A^a). \tag{7}$$

The two terms of (5) should now be associated with the exponents $(L\pm1)$, respectively, of Eq. (1), so that we now have the more accurate effective Lagrangian in momentum space (within, of course, the rules of our phenomenology),

$$\mathcal{L}_2 = 2\sqrt{2}M\bar{g}_1\bar{\pi}T_a[(M/P\cdot k)^2 k_\mu k_\nu T_{\mu\nu}{}^a + \rho\cdot A^a(1+\tfrac{1}{3}k_\mu k_\nu \mu^{-2}D_{\mu\nu})]. \tag{8}$$

Before proceeding, it is necessary to switch the roles of the A_1 and π fields from the decay process $A_1 \to \rho\pi$ to the virtual process in which A_1 plays the role of the radiation quantum (instead of π). This is a nontrivial step where the crossing property of our form factor (1) comes in an essential manner. Fortunately, the form factor, in spite of its phenomenological character, happens to be *crossing-symmetric* in the four-momenta of the A_1 and π mesons, so we feel that this step is probably not dangerous. We therefore rewrite the various momenta as

$$k_\mu \to -k_\mu \equiv q_\mu, \quad P_\mu \to -P_\mu = Q_\mu = p_\mu - q_\mu, \tag{9}$$

where only the ρ momentum p_μ and the π momentum q_μ are on the mass shell. The essential trick is now to regard $\mathcal{L}_1$ or $\mathcal{L}_2$ as providing a structure of the matrix element of the strong axial-vector current $J_\mu{}^A$ between π and ρ states, through the usual identification

$$\mathcal{L}_1(\text{or } \mathcal{L}_2) = J_\mu{}^{Aa}A_\mu{}^a, \tag{10}$$

so that we have from (4), (9), and (10) the expression[12]

$$\langle\pi(q)|J_\mu{}^{Aa}|\rho(p)\rangle = 2\sqrt{2}M\bar{g}_1\bar{\pi}T^a \times[q_\mu(q\cdot\rho)(M/q\cdot Q)^2+\rho_\mu], \tag{11}$$

and a similar expression from (8) and (10).

Equation (11) is still not ready for the application of the PCAC relations to the current $J_\mu{}^{Aa}$ since its units are not yet normalized to those of the standard isovector axial-vector current $\bar{A}_\mu{}^a$ whose matrix elements between quark states is defined as $\bar{Q}i\gamma_5\gamma_\mu\frac{1}{2}\tau^a Q$. Probably the simplest (and hopefully correct) way to normalize our current $J_\mu{}^{Aa}$ is to divide this operator by the quantity g_A defined by

$$(2k_0V)^{1/2}\langle 0|\bar{A}_\mu{}^a|A^b(k)\rangle = m_A{}^2 g_A{}^{-1}\delta_{ab}\mathcal{E}_\mu{}^{(A)}, \tag{12}$$

in the same way as the vector coupling constant g_ρ is defined by[13]

$$(2k_0V)^{1/2}\langle 0|\bar{V}_\mu{}^a|\rho^b(k)\rangle = m_\rho{}^2 g_\rho{}^{-1}\delta_{ab}\mathcal{E}_\mu{}^{(\rho)}. \tag{13}$$

There is one big difference, however, between g_ρ and g_A. While the ρ-meson universality[14] permits the evaluation of g_ρ via the identification $g_\rho = g_{\rho\pi\pi}$, there is no *a priori* reason to suppose there is a similar universality for g_A. However, the two Weinberg sum rules (which are consistent with each other for $m_A = \sqrt{2}m_\rho$) provide a determination for g_A which is what we use in this paper. Thus the equation

$$m_\rho{}^4/g_\rho{}^2 = m_A{}^4/g_A{}^2$$

gives

$$g_A = 2g_\rho \approx 2g_{\rho\pi\pi}. \tag{14}$$

We are now in a position to apply the PCAC relation to the current $g_A{}^{-1}J_\mu{}^A$, which reads

$$g_A{}^{-1}\partial_\mu J_\mu{}^{Aa}(x) = f_\pi m_\pi{}^2\phi^a(x) \to f_\pi J_\pi{}^a(x), \tag{15}$$

where $\phi^a(x)$ is the pion field and $J_\pi{}^a(x)$ is the corresponding pion current, related to the former in the soft-pion limit by $J_\pi = m_\pi{}^2\phi$; and f_π is given by[15]

$$f_\pi \approx 0.70\ \mathrm{m}_\pi. \tag{16}$$

[11] Fayyazuddin and Riazuddin, Phys. Rev. **171**, 1428 (1968).

[12] For simplicity we have suppressed the obvious factors for relativistic normalization for the ρ and π states.

[13] We are using the convention of T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[14] For a review, see J. J. Sakurai, in Proceedings of the International Conference on Electron and Photon Interactions at High Energies, University of Liverpool, 1969 (unpublished).

[15] This is because our normalization of $J_\mu{}^{Aa}$ corresponds to the *actual* isospin current $\bar{Q}i\gamma_\mu\gamma_5\frac{1}{2}\tau^a Q$.

Now from the usual $\rho\pi\pi$ coupling we can infer the matrix element of the pion current $J_\pi{}^a$ between ρ and π states as[12]

$$\langle\pi(q)|J_\pi{}^a|\rho(p)\rangle=2ig_{\rho\pi\pi}\bar{\pi}T^a q_\mu\rho_\mu. \quad (17)$$

Substitution of Eq. (11), corresponding to the Lagrangian $\mathfrak{L}_1$, and Eq. (17) into (15) gives in the soft-pion limit ($Q^2\approx 0$) the essential PCAC connection for the supermultiplet coupling constant $\bar{g}_1$, viz.,

$$g_A f_\pi g_{\rho\pi\pi}=\sqrt{2}m_A\bar{g}_1\{[2m_A/(m_\rho{}^2-\mu^2)]^2\tfrac{1}{2}(m_\rho{}^2-\mu^2)+1\}$$
$$\approx 2\bar{g}_1 m_\rho(4+1), \quad (18)$$

where in the last relation we have used the results $m_A\approx\sqrt{2}m_\rho$ and $\mu^2\ll m_\rho{}^2$. In the same way, if we use the current $J_\mu{}^A$ corresponding to the Lagrangian $\mathfrak{L}_2$, Eq. (8), we have instead of (18),

$$g_A f_\pi g_{\rho\pi\pi}\approx\sqrt{2}m_A\bar{g}_1\left[\frac{2m_A{}^2}{m_\rho{}^2}\left(1-\frac{m_\rho{}^2}{6m_A{}^2}\right)+\left(\frac{2}{3}+\frac{m_\rho{}^2}{24\mu^2}\right)\right]$$
$$\approx 2\bar{g}_1 m_\rho(3.67+1.93). \quad (19)$$

In Eqs. (18) and (19) we have shown the contributions of the D-wave and S-wave couplings separately for a clearer comparison between the roles of the two Lagrangians $\mathfrak{L}_1$ and $\mathfrak{L}_2$. Substitution of Eqs. (14) and (16) into (18) or (19) yields

$$\bar{g}_1{}^2/4\pi\approx 10^{-2}[0.84;0.67](g_{\rho\pi\pi}{}^2/4\pi)^2, \quad (20)$$

where the two numbers in (20) represent the effects of $\mathfrak{L}_1$ and $\mathfrak{L}_2$, respectively. As to the value of $g_{\rho\pi\pi}$, we must use the result that the quark model gives for it, viz.,[16]

$$\frac{g_{\rho\pi\pi}{}^2}{4\pi}=\frac{1}{4\pi}\left(\frac{2m_\rho}{\mu}f_q\right)^2\approx 3.6. \quad (21)$$

Substitution of (21) into (20) yields our final estimate of $\bar{g}_1$ as

$$\bar{g}_1{}^2/4\pi\approx[0.109;0.087], \quad (22)$$

to be compared with the empirical estimate 0.08 ± 0.01 found in Ref. 9. We consider the second number as a good support for our empirical algorithm in Eq. (1) for the association of the appropriate form factor with coupling in a particular wave.

[16] R. Van Royen and V. F. Weisskopf, Nuovo Cimento **50A**, 617 (1967). This precaution of using the quark model value for g_ρ rather than its experimental value is necessitated by the appearance of the quantity f_π in Eqs. (18) or (19), for which we have also taken the quark model value, viz., $f_\pi=(5/3)m_N G_{NN\pi}{}^{-1}\approx 0.70\ m_\pi$.

III. SUMMARY AND CONCLUSIONS

We have tried to present a somewhat unorthodox application of PCAC to relate the $A_1\rho\pi$ and $\rho\pi\pi$ coupling constants. Thus instead of dispersion relations subtracted or unsubtracted for the various form factors, we have a specific structure for them to start with. This structure, which was determined from a detailed study of the experimental decay widths of the various mesons, has now been subjected to a PCAC treatment after effecting the necessary "crossing" in the appropriate momenta. To an extent, our ansatz Eq. (1) yielded a PCAC result for $\bar{g}_1$ in good agreement with its direct determination from the various decay widths, giving us some confidence in the capacity of the form factor to allow a meaningful extrapolation off the mass shell, at least in the variable $(p-q)^2$. This is particularly encouraging in the context of applications involving off-shell extensions of the form factor. One such application, the electromagnetic mass difference of pseudoscalar mesons, is currently in progress. It may also be remarked that a very similar parametrization to Eq. (1) for meson-baryon couplings was found to give a similar fit to the decays of various baryon resonances in the same $SU(6)\times O(3)$ framework.[8,17] Applications of these B_LBP couplings which involve an extension of the form factor off the mass shell for B_L states are of course more easily compared with experiment. Some of these applications (e.g., $PB\to PB$, $PB\to VB$, $\gamma p\to PB$) are currently in progress and will be reported separately.

ACKNOWLEDGMENTS

This work was performed during my visit to the International Centre for Theoretical Physics, Trieste, which was made possible by the Swedish International Development Authority. I am grateful to Professor Abdus Salam and Professor P. Budini as well as to the International Atomic Energy Agency and UNESCO for warm hospitality at the Centre. I also acknowledge informative conversations with Professor R. Dashen and Dr. T-H. Chang.

[17] D. L. Katyal and A. N. Mitra, Phys. Rev. D **1**, 338 (1970).