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Energy Loss of High-Energy Particle Pairs in Matter*

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Problems associated with the passage of high-energy charged-particle pairs through matter (the Chudakoff effect) are investigated. It is shown that such an effect could not produce lightly ionized tracks which would be mistaken for quarks in cosmic-ray experiments. Treatments of polarization of the medium and Landau straggling are given.

I. INTRODUCTION

THE recent experiment by the Sydney group[1] in which evidence for the existence of quarks was deduced from the presence of lightly ionizing tracks in cloud chambers has sparked renewed interest in topics connected with the energy loss through ionization which occurs when charged particles pass through matter.[2,3] Because of the fundamental significance that the verification of the discovery of quarks would have for the physics of elementary particles, and because the Sydney experiments will surely be repeated by other groups, the investigation of any aspect of the behavior of high-energy particles in matter which would affect the interpretation of the results of such an experiment becomes extremely interesting.

One effect which could produce a lightly ionizing track in a cloud chamber was first investigated by Chudakoff.[4] It was pointed out that if two particles of opposite charge were to traverse a cloud chamber very close to each other (as would be the case, for example, if an electron-positron pair were photoproduced in the rim of the chamber), the effects of each charge would tend to be canceled by the other, so that it is possible that such a pair would leave a track with less ionization than that due to a single particle with unit charge. Such a track could, of course, be interpreted as being due to a fractionally charged quark. In Ref. 1 some simple arguments were presented which ruled out the interpretation of their results by this effect, but in the light of the comments made above, a more thorough study of the problem is clearly in order. It is the purpose of this paper to present the results of a study of the ionization loss problem as it applies to the passage of particle pairs through matter.

In Sec. II we present the basic working equations, and show the results of numerical calculations. We show that quarklike tracks can be produced by a pair only if their separation is less than $\approx 10^{-9}$ cm, so that the conclusions of the Sydney group on this point are strengthened. In Sec. III we discuss possible corrections to the results of Chudakoff due to polarization effects in the medium, and show that at sufficiently high energies, such effects can be neglected. Finally, in Sec. IV we discuss the problem of straggling for particle pairs. The net result of these calculations is that in future quark searches by cloud-chamber techniques, we can safely ignore the possibility that faintly ionized tracks are produced by pairs traversing the chamber.

II. CALCULATION OF RESTRICTED ENERGY LOSS

Consider a pair of particles of mass m and charge $\pm e$, separated by a distance l, passing by an atomic electron. Let r be the perpendicular distance from the bound electron to the one member of the pair, and r' the distance to the other. [The geometry is sketched in Fig. 1(a).] Then, as in Ref. 4, the momentum transferred to the atomic electron by the pair is just

$$\Delta\mathbf{p}=\frac{2e^2}{c}\left[\left(\frac{1}{r}-\frac{\cos\varphi}{r'}\right)\hat{r}+\frac{\sin\varphi}{r'}\hat{k}\right], \tag{1}$$

where φ is the angle between r and r', $\hat{r}$ is a unit vector in the direction of r, and $\hat{k}$ is a unit vector perpendicular to $\hat{r}$. The energy lost to this atomic electron is then given by

$$\epsilon(r,r')=\frac{(\Delta\mathbf{p})^2}{2m}=\frac{2e^4}{mc^2}\left(\frac{1}{r^2}+\frac{1}{r'^2}-\frac{2\cos\varphi}{rr'}\right). \tag{2}$$

If we now define the angle α as the angle between r and the line joining the constituents of the pair, and let $A=2e^4/mc^2$, we find

$$\epsilon(r,r')=\frac{A}{r^2}\frac{1}{1+(r/l)^2-2(r/l)\cos\alpha}, \tag{3}$$

so that the energy loss averaged over the angle α is just

$$\langle\epsilon\rangle=\frac{A}{r^2}\frac{1}{|1-(r/l)^2|}. \tag{4}$$

The energy loss per unit path length for the pair is then given by

$$\frac{dE}{dx}=2\pi NZ\int_{r_{\min}}^{r_{\max}}\langle\epsilon\rangle r\,dr, \tag{5}$$

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[1] C. B. A. McCusker and I. Cairns, Phys. Rev. Letters **23**, 658 (1969); I. Cairns, C. B. A. McCusker, L. S. Peak, and R. L. S. Woolcott, Phys. Rev. **186**, 1394 (1969).
[2] R. K. Adair and H. Kasha, Phys. Rev. Letters **23**, 1355 (1969).
[3] H. Frauenfelder, U. E. Kruse, and R. D. Sard, Phys. Rev. Letters **24**, 33 (1970).
[4] A. E. Chudakoff, Izvest. Akad. Nauk SSSR Ser. Fiz. **19**, 651 (1955) [Columbia Technical Translations **19**, 589 (1955)].

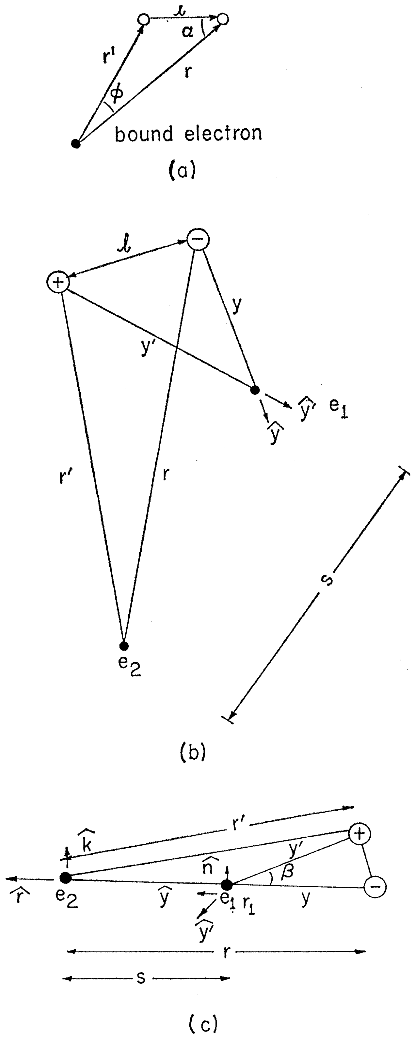

FIG. 1. (a) Diagram showing the geometry for the pair passing a single bound electron. The velocity of the pair is directed out of the page. (b) Geometry for the polarization problem. The pair velocity is directed out of the page. (c) Geometry under which polarization effects are maximized.

where Z is the atomic number of the atoms to which the pair is losing energy, and N is the number of such atoms per unit volume in the chamber.

The limits on the integral $r_{\min}$ and $r_{\max}$ can be determined in many ways.[5] If we were interested in the total energy loss, these limits would be determined by the values of r at which the classical impulse approximation used to derive Eq. (2) breaks down. In the usual treatment, $r_{\min}$ is determined by the condition that the energy loss $\langle\epsilon\rangle$ not exceed the maximum possible energy loss in a two-body collision, while $r_{\max}$ is determined by the condition that the time during which the pair interacts with the bound electron is much smaller than the time which characterizes the motion of the electron in the atom.

In a cloud chamber, however, the limits are set somewhat differently. There is some energy loss $E_{\min}$, below which no ionization will be detected in the chamber. From Eq. (4), we can determine $r_{\max}$ from this experimental lower limit on E. Similarly, for very large energy losses, the bound electron will be torn from the atom, and will have enough energy to ionize other atoms. Such energy losses are customarily excluded by experimenters when measuring the ionization on particle tracks, so that there is a maximum energy loss $E_{\max}$, from which we can determine a value for $r_{\min}$ in Eq. (5). The point is that, in our case, the upper and lower bounds on the integral are determined by experimental rather than theoretical criteria. We have used $E_{\min}=12$ eV and $E_{\max}$ between 0.5 and 2.0 keV in our calculations. It should be mentioned in passing that the value of $r_{\min}$ ($r_{\max}$) which we get from experimental considerations is much larger (smaller) than the classical limits discussed above, so that no violation of the basic working assumptions is made. Were this not the case, corrections to the simple theory would have to be made, and the problem would become more complicated.

For reference, we note that for a single particle traversing a medium, the ionization loss is given by

$$\frac{dE}{dx}=2\pi NZA\ln\left(\frac{\sqrt{E_{\max}}}{\sqrt{E_{\min}}}\right). \tag{6}$$

In Fig. 2 we show the results of integrating Eq. (5) between the experimental limits described above. We immediately notice several features of this result which are of interest. First, we note that the restricted energy loss (that is, the energy losses between experimentally imposed limits) can actually exceed twice the restricted ionization loss due to a single particle of unit charge, although, as expected, for large separations the energy

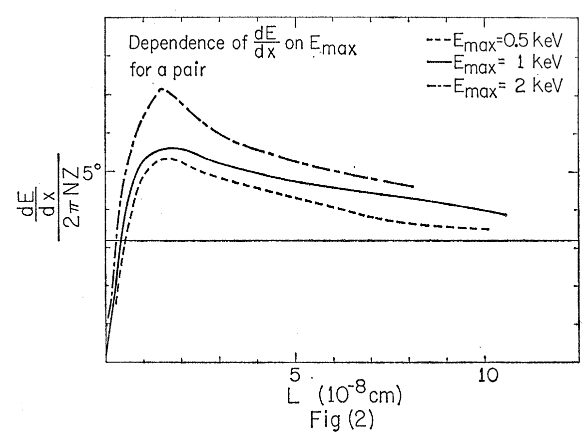

FIG. 2. Ionization loss by the pair as a function of separation for several different maximum energy cutoffs. The horizontal line represents twice the energy lost by a single particle for the case $E_{\max}=1$ keV.

[5] A treatment of the classical theory of ionization loss is given in many textbooks; see, for example, J. D. Jackson, *Classical Electrodynamics* (Interscience, New York, 1962), Chap. 13.

loss associated with the pair will approach that due to the sum of each of the constituents treated singly. More important, however, we note that the ionization rises very rapidly as a function of the separation. Thus, a pair will have an ionization loss equal to that of a single particle of unit charge when $l\approx 2\times 10^{-9}$ cm, and for a pair to imitate the track due to a quark of charge $\frac{2}{3}e$, the separation would have to be of order $l\approx 1\times 10^{-9}$ cm.

The implication of this result for the Sydney experiment can easily be seen if we recall that the minimum opening angle for a photoproduced electron-positron pair is just

$$\theta_{\min}=4mc^2/E_\gamma. \tag{7}$$

The tracks seen were 10 cm long, so in order to explain the light ionization as being due to a particle pair, the angle of production of the pair would have to be $\theta\approx 10^{-9}/10=10^{-10}$ rad, which would correspond to an energy of $E_\gamma\sim 10^{16}$ eV, an order of magnitude larger than the energy of the total shower in which the tracks occurred.

Thus, provided that there are no important corrections to be made to the energy-loss formula given in Eq. (4), there seems to be good evidence that lightly ionized tracks seen in future cloud-chamber quark searches need not be interpreted as being due to the presence of particle pairs. It is to the question of corrections to the initial results presented here that we now turn our attention.

III. POLARIZATION CORRECTIONS

One obvious correction which would have to be considered before the results in Sec. II could be accepted would be that due to polarization of the medium by the pair. That is, we would have to look at processes in which [see Fig. 1(b)] an electron e_1 in an atom near the pair is displaced, thereby setting up an additional electric field at the electron e_2, whose energy gain we are calculating. Since we shall mainly be interested in finding the kinematic regions where such effects can be safely neglected, we shall present here only an approximate treatment of the problem.

We shall find that the effects due to polarization are completely negligible at energies of the type discussed in Sec. II. The basic physical reason for this is that the time during which the pair is in the vicinity of an atom (i.e., during which the pair can cause a displacement of the bound electron) is very short compared to atomic times, so that the atoms do not have time to react and set up large induced fields during the time of collision. This result is very reasonable physically, and the reader who does not wish to work through the details of the derivation will find the result displayed in Eq. (20).

To begin our attack, let us consider the pair passing by an electron e_2 in an atom. We shall then carry out the following steps [see Fig. 1(b)]:

(1) Calculate the displacement of e_1 from equilibrium as a function of time during the time that the pair is near the atom containing e_1;

(2) calculate the additional field at e_2 due to the induced dipole moment caused by the displacement of e_1. This field must then be added to that due to the pair itself in calculating energy loss.

We take as our model of the atom a simple harmonic oscillator of frequency ω, and neglect both damping and motion of the nucleus. The equation governing the motion of e_1 is then

$$\ddot{\mathbf{x}}_1+\omega^2\mathbf{x}_1=e\boldsymbol{\varepsilon}_1(t)/m, \tag{8}$$

where $x_1(t)$ is the displacement of e_1 from equilibrium, and $\boldsymbol{\varepsilon}_1(t)$ is the (time-dependent) field due to the constituents of the pair. The equation can then be integrated directly to give

$$\mathbf{x}_1(t)=\frac{e}{m\omega}\left[\cos\omega t\int_{-\infty}^{t}\boldsymbol{\varepsilon}(t')(-\sin\omega t')dt' +\sin\omega t\int_{-\infty}^{t}\boldsymbol{\varepsilon}(t')\cos\omega t'dt'\right]\hat{x}_1, \tag{9}$$

where we have set $x_1=0$ at $t=-\infty$.

Recalling that the field at e_1 due to the positive component of the pair [see Fig. 1(b)] is

$$\boldsymbol{\varepsilon}_+(t)=\frac{q\gamma y'}{[(y')^2+\gamma^2c^2t^2]^{3/2}}\hat{y}', \tag{10}$$

and the field due to the negative component, $\boldsymbol{\varepsilon}_-(t)$, can be obtained from Eq. (10) by letting $q\to -q$ and $y'\to y$, we note that $\boldsymbol{\varepsilon}_+(t')$ is important only when

$$t'<y'/\gamma c=\tau_+, \tag{11}$$

and similarly for $\boldsymbol{\varepsilon}_-(t')$. For high-energy pairs, $\omega\tau_\pm\ll 1$. This means that we can replace $\cos\omega t'$ by 1 and $\sin\omega t'$ by $\omega t'$ in Eq. (9).

Further simplifications of the rather complicated geometry can be made if we recall that our main purpose is to find the kinematic regions where polarization effects can be neglected, so that if we make approximations which overestimate these effects, we shall not change our final conclusions. The induced field at the atom containing e_2 due to the displacement of e_1 is just

$$\boldsymbol{\varepsilon}_{\text{pol}}(t)=2e\mathbf{x}_1(t)/s^3, \tag{12}$$

where s is the distance between e_1 and e_2. Clearly, we will maximize polarization effects if we minimize s. This corresponds to replacing the geometry of Fig. 1(b) with that of Fig. 1(c), where all of the atoms whose induced fields affect e_1 are at a distance $s=r-y$ from e_1. For this configuration, we can put Eqs. (10) and (11) into Eq.

(9) and integrate to get

$$\mathbf{x}_1(t)=\frac{e^2}{m\omega\gamma^2c^3}\left\{y'\left[\frac{\omega}{(\tau_+{}^2+t^2)^{1/2}}+\frac{\omega t}{\tau_+{}^2}\left(\frac{t}{(\tau_+{}^2+t^2)^{1/2}}-1\right)\right]\hat{y}'-y\left[\frac{\omega}{(\tau_-{}^2+t^2)^{1/2}}+\omega t\left(\frac{t}{(\tau_-{}^2+t^2)^{1/2}}-1\right)\right]\hat{y}\right\}, \quad (13)$$

where all of the unit vectors are defined in Fig. 1(c). Equation (12) then gives the additional field at e_2 due to the induced dipole moment at e_1. The additional momentum transferred to e_2 owing to this new field will be

$$\begin{aligned}\Delta\mathbf{p}_{\mathrm{pol}}&=e\int\boldsymbol{\varepsilon}_{\mathrm{pol}}(t)dt\\&=\frac{2e^4}{ms^3\gamma^2c^3}\left[\frac{1}{2}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)+\sqrt{2}\right]\times[(y-y'\cos\beta)\hat{y}+y'\sin\beta\,\hat{n}]\\&=\frac{P'}{s^3}[l\cos\alpha\,\hat{y}+(r-s-l\sin\alpha)\hat{n}],\end{aligned} \quad (14)$$

where we have cut off the time integrals at $t=\pm\tau_\pm$ and have done some simplifying trigonometry to get to the last step.

The total momentum transferred to the electron e_2 will then be given by

$$\Delta\mathbf{p}=\frac{2e^2}{c}\left(\frac{1}{r}-\frac{\cos\varphi}{r'}\right)\hat{r}+\frac{\sin\varphi}{r'}\hat{k}-\sum_j\frac{P'}{s_j{}^3}[l\cos\alpha\,\hat{y}+(r-s-l\sin\alpha)\hat{k}], \quad (15)$$

where s_j is the distance from e_1 to the jth electron of the type e_1, $\sum_j$ represents a sum over all such electrons, and we have used the fact that $\hat{n}$ and $\hat{k}$ are collinear. If we take $s_j=aj$, where a is the projection of some average interatomic distance on the impact parameter plane, then the sum can be carried out explicitly to give

$$\begin{aligned}\sum_j\frac{P'}{(aj)^3}&[l\cos\alpha\,\hat{y}+(r-aj-l)\sin\alpha\,\hat{k}]\\&=(P'/a^3)\zeta(3)[l\cos\alpha\,\hat{y}+(r-l)\sin\alpha\,\hat{k}]-(P'/a^2)\zeta(2)\sin\alpha\,\hat{k}\\&=P[l\cos\alpha\,\hat{y}+(r-l)\sin\alpha\,\hat{k}]-P^\dagger\sin\alpha\,\hat{k},\end{aligned} \quad (16)$$

where ζ is the Reimann ζ function, and has been incorporated into the new constants P and $P^\dagger$.

The energy loss is then given by

$$\begin{aligned}\epsilon(r,r')=\frac{\Delta\mathbf{p}^2}{2m}&=A\left(\frac{1}{r^2}+\frac{1}{r'^2}-\frac{2\cos\varphi}{rr'}\right)-\frac{P}{mc^2}l\cos\alpha\times\left[\frac{2e^2}{r}-\frac{2e^2\cos\varphi}{r'}\right]-\frac{2e^2P}{mc^2}(r-l\sin\alpha)\frac{\sin\varphi}{r'}\\&+\frac{2e^2P^\dagger}{mc^2}\frac{\sin\varphi}{r'}+\mathcal{O}(P^2),\end{aligned} \quad (17)$$

where we have dropped higher-order terms in P. Carrying out the averages over the angle α then yields the corrected form of Eq. (4):

$$\begin{aligned}\langle\epsilon\rangle=\frac{A}{r^2}\frac{1}{|1-r^2/l^2|}-\frac{AP}{e^2}\frac{l^2}{r^2}\frac{1}{|1-r^2/l^2|}&-\frac{AP}{l^2}\left[\frac{r^2+l^2}{4r^2}(1-Q)+\frac{r^2}{(r^2+l^2)Q}+\frac{r^2+l^2}{4r^2}(1-Q)\right]\\&+\frac{AP^\dagger}{e^2}\frac{r}{(r^2+l^2)Q},\end{aligned} \quad (18)$$

where we have written

$$Q=\left[1-\frac{4r^2l^2}{(r^2+l^2)^2}\right]^{1/2}. \quad (19)$$

To get some feeling for the importance of the polarization corrections at high energies, assume an interatomic spacing of 10^{-7} cm. In this case, we find

$$\begin{aligned}\langle\epsilon\rangle=\frac{A}{r^2|1-r^2/l^2|}\Bigg\{1-\frac{3.75\times10^{10}}{\gamma_e{}^2}&\times\left[l^2+\frac{r^2+l^2}{4r}|1-r^2/l^2|(1-Q)+\frac{r^4|1-r^2/l^2|}{(r^2+l^2)Q}+\frac{(r^2+l^2)|1-r^2/l^2|(1-Q)}{4}\right]\\&+\frac{5.15\times10^3}{\gamma_e{}^2}\frac{r^2}{(r^2+l^2)Q}\Bigg\},\end{aligned} \quad (20)$$

where γ_e refers to the velocities of the electrons in the pair. Since we saw in Sec. II that it would take extremely high-energy electrons to simulate quark tracks (typically we would need $\gamma_e\gtrsim10^{12}$), it is clear that no corrections to our previous results need be made because of the effects of polarization.

IV. LANDAU STRAGGLING FOR PAIRS

There remains one more way in which a pair might masquerade as a fractionally charged particle, and that is the possibility that for some reason the straggling for pairs might be considerably broader than that for single particles, so that although the mean energy losses computed in Sec. II might be considerably in excess of those expected for such a particle, statistical fluctuations might still give a considerable fraction of lightly ionized tracks. It will be recalled that a good part of the controversy surrounding the results of Ref. 1 concerned the problem of statistical fluctuations for single particles.[3]

The problem of statistical fluctuations in ionization processes has been fully dealt with elsewhere.[6] If $f(x,\Delta)$ is the probability that a particle will have lost energy Δ in traversing a thickness x of material, and $\omega(\epsilon)d\epsilon$ is the probability of losing between ϵ and $\epsilon+d\epsilon$ of energy per unit path length, then we have

$$f(x,\Delta)=\int_{-\infty}^{\infty}dp \times\exp\left[ip\Delta-x\int_{\epsilon_{\min}}^{\epsilon_{\max}}\omega(\epsilon)(1-e^{-ip\epsilon})\right]d\epsilon. \quad (21)$$

To calculate $\omega(\epsilon)$ from the results in Sec. II, we simply recall that the classical cross section is given in terms of the impact parameter by

$$d\sigma=2\pi b|db|, \quad (22)$$

so that for a pair, whose energy loss per collision is given in terms of the impact parameter in Eq. (4), we have

$$\omega(\epsilon)=\frac{\pi NZA}{(1\mp 4A/\epsilon l^2)^{1/2}}\frac{1}{\epsilon^2}, \quad (23)$$

where N is the number of target particles per unit volume, Z is the number of electrons on each such target, and the minus (plus) sign is to be taken when r is less than (greater than) the separation l. As in Sec. II, the limits of the integral in the exponent will be taken from experiment. Because of the rather complicated dependence of the energy loss on impact parameter, it is easier to change variables in the integral over energy in Eq. (21), giving us

$$I(p)=\int_{\epsilon_{\min}}^{\epsilon_{\max}}\omega(\epsilon)[1-e^{-ip\epsilon}]d\epsilon =\int_{r_{\max}}^{r_{\min}}\omega(\epsilon(r))[1-e^{-ip\epsilon(r)}]D(r)dr, \quad (24)$$

where we have written the Jacobean

$$D(r)=A\left[\frac{1}{r^3(r^2/l^2-1)}\mp\frac{2r_1}{l^2r^2(r^2/l^2-1)^2}\right], \quad (25)$$

where the minus (plus) sign is to be taken when r is less than (greater than) the separation. If we define new variables by

$$\xi=\pi ANZx$$

and (26)

$$\omega(\epsilon)=\pi ANZ\omega'(\epsilon),$$

then Eq. (21) becomes

$$f(\xi,\Delta)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dp \times\exp\left[ip\Delta-\xi\int_{r_{\max}}^{r_{\min}}\omega'(\epsilon)(1-e^{-ip\epsilon})D(r)dr\right]. \quad (27)$$

We could now proceed to evaluate these integrals numerically and plot out results. In fact, this is what was done in the results which are discussed below. We can, however, gain some insight into the straggling problem if we note that in Eq. (29), the parameter ξ is typically a very large number, so that we will be looking for $I(p)$ for small values of p. If we expand the exponential in $I(p)$, we find

$$I(p)\approx ip\int\epsilon\omega'(\epsilon)D(r)dr+\tfrac{1}{2}p^2\int\epsilon^2\omega'(\epsilon)D(r)dr+\cdots =ipS+p^2T+\cdots, \quad (28)$$

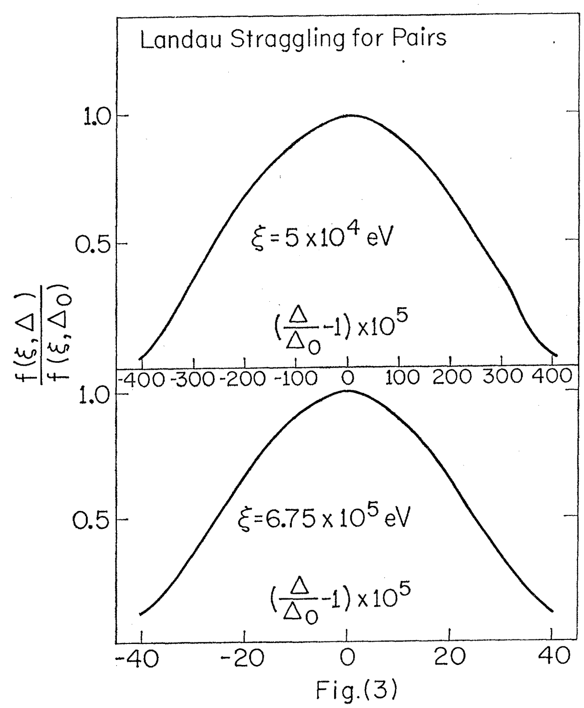

FIG. 3. The normalized energy distributions for pairs traversing different thicknesses of material. The value $\xi=6.75\times10^5$ eV corresponds to a 10-cm track in hydrogen at STP, and $\xi=5\times10^4$ eV to a track less than 1 cm long.

[6] L. Landau, J. Phys. (USSR) **8**, 201 (1944); I. Blunck and S. Liesegang, Z. Physik **128**, 500 (1950).

so that

$$f(x,\Delta)\approx\int dp\, e^{ip(\Delta-\xi S)}e^{-\xi p^2 T}$$

$$=\left(\frac{\pi}{\xi T}\right)^{1/2} e^{-(\Delta-\Delta_0)^2/4\xi T}, \quad (29)$$

where we have defined

$$\Delta_0=\xi S. \quad (30)$$

Thus, without doing any explicit calculations, we see that for large ξ we expect the distribution of energies of particles emerging from a slab of thickness x to follow a normal curve centered around Δ_0.[7] In Fig. 3 we show the results of explicit calculations of Eq. (27) for $f(x,\Delta)$, and we see that for the two cases considered ($\xi=6.75\times10^5$ eV, corresponding to a 10-cm track in hydrogen at STP), this expectation is indeed realized.

We also note that the original problem which we posed, the question of whether straggling might lead a pair to produce a faint cloud-chamber track, can now be answered in the negative, since the probability of finding $\Delta<\frac{1}{2}\Delta_0$ is very small indeed, and, since high-energy pairs are not produced often, we can rule out this possibility.

[7] It is amusing to note that if we were calculating the straggling for a single particle, where $\omega(\epsilon)=A/\epsilon^2$, we would find $\Delta_0=\xi/\epsilon_{\min}$ and $T=\epsilon_{\max}$. For the case where $\epsilon_{\max}$ is restricted only by the classical upper bound discussed in Sec. II, the spread of energies will be maximized, while for the case of the restricted energy-loss problem considered in this paper, $\epsilon_{\max}$ will be less than its highest allowable value. This means that for the restricted problem, we would expect a narrower spread of energies, a circumstance remarked upon in Ref. 3.

V. CONCLUSION

The net result of the foregoing study of the ionization loss of charged pairs in matter is that in future quark searches, the production of faintly ionizing tracks by such an effect can be safely ruled out, and that, in the absence of other explanations of such tracks, they will have to be interpreted as evidence for the existence of fractionally charged particles.

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Two-Pion Decay Mode of the ω and ρ-ω Mixing*

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Recent experiments reporting the 2π decay mode of the ω meson and its interference with that mode of the ρ meson are analyzed in terms of the propagator method for mixing of particle states. Both vector mixing and mass mixing are included, and the energy dependence of the ρ width is explicitly taken into account. It is shown that the effect as observed in the reaction $e^+e^-\to\pi^+\pi^-$ can be accounted for in terms of available parameters. The consequent determination of the parameters makes possible an analysis of strong-interaction phenomena of the same type, giving information on the relative phases of ω and ρ production amplitudes. This is applied to the ρ-ω interference observed in $\pi^+p\to\pi^+\pi^-\Delta^{++}$. Other experiments are suggested.

I. INTRODUCTION

DIRECT evidence for the 2π decay mode of the ω meson produced in the reaction

$$e^+e^-\to\pi^+\pi^- \quad (1)$$

has recently been reported.[1] There are also reports of evidence for this decay mode in other processes, in particular, in the reaction[2]

$$\pi^+p\to\pi^+\pi^-\Delta^{++}. \quad (2)$$

In both cases, it is the interference between the 2π mode of the ρ meson and the 2π mode of the ω that is observed. Therefore, the process is sensitive to the phase of the mixing of the "bare" ρ and ω states of definite G parity.

Reaction (1) is particularly suitable for determining the parameters of this mixing since, in addition to these parameters, only known quantities are needed to describe the interference. This is a direct consequence of the absence of any strong-interaction effect in reaction (1) other than the coupling of ρ and ω to the 2π state. It lends a special significance to the reaction because, once the mixing parameters are determined in this way, they can be used to analyze other processes, such as reaction (2), to obtain valuable information concerning the phases of the amplitudes for production of the vector mesons in strong interactions.

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