Highly Inelastic Neutrino-Nucleon Scattering*

ROBERT V. BUDNY
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 22 December 1969)

Lower bounds for cross sections for neutrino-nucleon scattering summed over final hadron states at large energy and momentum transfer are estimated assuming the usual current-current coupling, Regge behavior, and current-algebra sum rules. The cross sections are estimated using an added assumption of pointlike baryon constituents in nucleons. The cross sections compare well with CERN bubble-chamber data.

I. INTRODUCTION

THE structure functions which determine the cross section for inelastic neutrino-nucleon scattering summed over final hadron states are determined, according to unitarity, by the absorptive part of the amplitude for forward scattering of the Cabibbo current by the nucleon. Bjorken[1] has recently expressed the high-energy neutrino-proton cross section in terms of the "deep-inelastic" limits of these structure functions as $-q^2$, $\nu\equiv P\cdot q\rightarrow\infty$. Here it is shown that these limits can be estimated assuming the current-current coupling with strangeness conservation, Regge theory, current-algebra sum rules, saturation assumptions, and pointlike baryon constituents. The resulting predictions agree surprisingly well with the data if the chiral $U(6)\times U(6)$ algebra[2] (quark model) is assumed.

In Sec. II the asymptotic behavior of the structure functions as $-q^2$, ν, and $\rho\equiv\nu/(-q^2)\rightarrow\infty$ is computed using the assumptions that the behavior of the amplitude for forward scattering of the Cabibbo current with q^2 very negative (spacelike) and fixed as $\nu\rightarrow\infty$ is given by Regge theory, and that the structure functions obey an integral spectral representation. These assumptions are given indirect support by their success in highly inelastic electron-proton scattering.[3,4]

In Sec. III, the limits of the structure functions as $-q^2$, ν, $\rho\rightarrow\infty$ are estimated. Adler[5] derived a sum rule for one of the structure functions using the equal-time current algebra of the time components of the Cabibbo current. Bjorken[6] and Gross and Llewellyn Smith[7] have used the quark model and field algebra equal-time commutators of space components of the Cabibbo current and the Bjorken-Johnson-Low (BJL) expansion[8] of the forward current scattering matrix element to derive asymptotic sum rules for the other structure functions. The asymptotic values of the structure functions are assumed to saturate the quark model sum rules.

These assumptions are sufficient to estimate a lower bound on the nucleon-averaged cross section. One further assumption provides an estimate of the cross section. In the "parton" model,[7,9] the nucleon is assumed to contain pointlike constituents which scatter weak and electromagnetic currents incoherently. If it is assumed that baryon partons such as pointlike quarks are responsible for scattering the weak current, then a relation among the structure functions follows allowing all the limits of the structure functions for neutrino or antineutrino scattering on neutrons or protons to be estimated. The resulting cross sections coincide with the lower bound estimated without this assumption. The relation is inconsistent with Pomeranchon dominance in the Regge limit.

In Sec. IV the asymptotic form of the structure functions is extrapolated, allowing the neutrino and antineutrino cross sections to be estimated. They are compared with available data. The data do not determine whether the Pomeranchon dominates.

In Sec. V it is concluded that the Pomeranchon does not appear to be important for existing data. Parton models that are consistent with the assumptions are discussed.

Many of the assumptions are speculative. For instance, it is not known at what energy the current-current coupling begins to fail, nor whether Regge behavior and duality are even approximately correct in weak interactions. The close agreement between the estimated cross section and the measured data suggests that the assumptions are approximately correct.

II. ASYMPTOTIC FORM OF $F_i(\rho)$

The matrix element for the spin average of the forward scattering of the Cabibbo current J_μ carrying

---

* Work supported in part by the U. S. Atomic Energy Commission.

[1] J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).

[2] R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters **13**, 678 (1964).

[3] R. A. Brandt, Phys. Rev. Letters **22**, 1149 (1969).

[4] R. A. Brandt, Phys. Rev. **187**, 2192 (1969).

[5] S. L. Adler, Phys. Rev. **143**, 1144 (1966).

[6] J. D. Bjorken, Phys. Rev. **163**, 1767 (1967).

[7] D. J. Gross and C. H. Llewellyn Smith, Nucl. Phys. **B14**, 337 (1969).

[8] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966); K. Johnson and F. Low, Progr. Theoret. Phys. (Kyoto), Suppl. No. **37–38**, 74 (1966). Problems in the naive application of this expansion are discussed in R. A. Brandt and J. Sucher, Phys. Rev. Letters **20**, 1131 (1968); R. Jackiw and G. Preparata, *ibid.* **22**, 975 (1969); and S. L. Adler and W.-K. Tung, *ibid.* **22**, 978 (1969). In Ref. 7, Gross and Llewellyn Smith argue that the naive application in the deep-inelastic limit might be justified.

[9] J. D. Bjorken and E. A. Paschos, Phys. Rev. **185**, 1975 (1969).

four-momentum q by a proton of four-momentum P is

$$M_{\mu\nu}=\frac{iP_0}{M}\int d^4x\, e^{+iq\cdot x}\langle P|T(J_\mu(x)J_\nu^\dagger(0))|P\rangle$$
$$=g_{\mu\nu}T_1(q^2,\nu)+P_\mu P_\nu T_2(q^2,\nu)+\epsilon_{\mu\nu\alpha\beta}P_\alpha q_\beta T_3(q^2,\nu)$$
$$+P_\mu q_\nu T_4(q^2,\nu)+q_\mu P_\nu T_5(q^2,\nu)+q_\mu q_\nu T_6(q^2,\nu)\,, \quad (1)$$

where $\nu=P\cdot q$, $M=$the nucleon mass, and the T_i's are Lorentz scalars. It is conventional to define the structure functions W_i by

$$\frac{\mathrm{Im}T_1}{\pi}=-W_1,\quad \frac{\mathrm{Im}T_2}{\pi}=\frac{W_2}{M^2},\quad \frac{\mathrm{Im}T_3}{\pi}=\frac{-i}{2M^2}W_3.$$

The matrix element given by

$$H_{\text{weak}}{}^{\text{eff}}=\frac{G}{2\sqrt{2}}(J_\mu j_\mu{}^\dagger+j_\mu{}^\dagger J_\mu)_s$$

for neutrino-proton scattering summed over final hadron states is $\langle j_\mu(0)j_\nu{}^\dagger(0)\rangle[M_{\mu\nu}]$, where $j_\mu(x)=\bar{l}(x)\gamma_\mu(1+\gamma_5)\nu(x)$ is the leptonic current and $[M_{\mu\nu}]$ is the discontinuity in ν of $M_{\mu\nu}$. In the limit $m_l=m_\nu=0$, only W_1, W_2, and W_3 contribute to the matrix element, resulting in the cross section

$$\frac{d^2\sigma^{(\bar{\nu},\nu)}}{d|q^2|d\nu}=\frac{E'}{E}\frac{G^2}{2\pi M}\Big[2W_1{}^{(\bar{\nu},\nu)}\sin^2(\tfrac{1}{2}\theta)+W_2{}^{(\bar{\nu},\nu)}\cos^2(\tfrac{1}{2}\theta)$$
$$\pm\frac{(E+E')}{M}W_3{}^{(\bar{\nu},\nu)}\sin^2(\tfrac{1}{2}\theta)\Big]\,, \quad (2)$$

where E and E' are the neutrino and final lepton energies in the laboratory frame and θ is their scattering angle. The square of the momentum transferred to the nucleon is $q^2=-4EE'\sin^2(\frac{1}{2}\theta)$. Crossing gives $W_i{}^{\nu p}(q^2,\nu)=-W_i{}^{\bar{\nu}p}(q^2,-\nu)$. The $W_i(q^2,\nu)$ can be measured using (2).

Bjorken[1] has shown that the W_i are expected to have finite limits

$$W_1\rightarrow(1/\mathrm{M})F_1(\rho),$$
$$W_j\rightarrow(M/\nu)F_j(\rho),\quad j=2,3 \quad (3)$$

as $-q^2$, $\nu\rightarrow\infty$ and $\rho\equiv\nu/(-q^2)$ remains fixed. This will be referred to as the B limit. For large $-q^2$ and ν, (3) leads to

$$\frac{d^2\sigma^{(\bar{\nu},\nu)}}{dxd\rho}=\frac{G^2ME}{2\pi\rho^2}\Big[\frac{x^2}{2\rho}F_1{}^{(\bar{\nu},\nu)}(\rho)+\Big(1-x-\frac{Mx}{4\rho E}\Big)$$
$$\times F_2{}^{(\bar{\nu},\nu)}(\rho)\pm\frac{x}{2\rho}(1-\tfrac{1}{2}x)F_3{}^{(\bar{\nu},\nu)}(\rho)\Big]\,, \quad (4)$$

where $x=\nu/ME$. Since (1) is positive semidefinite, the F_i's are constrained by

$$0\leq\tfrac{1}{2}|F_3|\leq F_1\leq\rho F_2. \quad (5)$$

Recently, Brandt[3] has shown that in the analogous case of electron-proton scattering, the behavior of the electroproduction limits $F_1{}^{\text{em}}(\rho)$ and $F_2{}^{\text{em}}(\rho)$ as $\rho\rightarrow\infty$ can be determined assuming (3), that the Regge behavior of $W_1{}^{\text{em}}$, $W_2{}^{\text{em}}$ as $\nu\rightarrow\infty$, q^2 fixed, is dominated by the Pomeranchon, and the Deser-Gilbert-Sudarshan (DGS) spectral representation[10] for $W_1{}^{\text{em}}$ and $W_2{}^{\text{em}}$. The method can be easily extended to derive the asymptotic behavior in the neutrino-production case.

The W_i are assumed to satisfy the DGS representation with W_1 requiring one subtraction:

$$W_1=\int_0^\infty da\int_{-1}^1 db\,[\nu\sigma_1(a,b)+\bar{\sigma}(a,b)]$$
$$\times\delta(q^2+2b\nu+b^2-a)\epsilon(\nu+b)\,, \quad (6\text{a})$$
$$W_j=\int_0^\infty da\int_{-1}^1 db\,\sigma_j(a,b)$$
$$\times\delta(q^2+2b\nu+b^2-a)\epsilon(\nu+b)\,,\quad j=2,3. \quad (6\text{b})$$

Also the W_i's are assumed to have the Regge behavior as $\nu\rightarrow\infty$, q^2 fixed. This will be referred to as the R limit. Specifically,

$$W_1\underset{R}{\rightarrow}R_1(q^2)\nu^{\alpha_1}, \quad (7\text{a})$$

$$W_2\underset{R}{\rightarrow}R_2(q^2)\nu^{\alpha_2-2}, \quad (7\text{b})$$

$$W_3\underset{R}{\rightarrow}R_3(q^2)\nu^{\alpha_3-1}, \quad (7\text{c})$$

where α is the $t=0$ intercept of the Regge singularities dominating current-proton scattering. In the case of W_1, the Regge behavior is given by Eq. (6a) to be

$$W_1\underset{R}{\rightarrow}\Big[\lim_{\nu\rightarrow\infty;\,q^2\text{ fixed}}\frac{1}{2}\int da\,\sigma_1\Big(a,\frac{a-q^2}{2\nu}\Big)\Big].$$

$\sigma_1(a,b)$ is assumed to vanish rapidly for large a, say $a>a_0$. When $\nu\gg-q^2\gg a_0$, the integrand becomes $\sigma_1(a,-q^2/2\nu)\rightarrow\sigma_1(a,0)$. In order to get (7a), $\sigma_1(a,b)$ must be singular as $b\rightarrow0$, i.e., $\sigma_1(a,b)=\sigma_1(a)b^{-\alpha_1}+\eta_1(a,b)$, where $\eta_1(a,b)$ is less singular as $b\rightarrow0$ than $b^{-\alpha_1}$. This gives

$$W_1\underset{R}{\rightarrow}\tfrac{1}{2}(2\rho)^{\alpha_1}\int_0^\infty da\,\sigma_1(a)$$
$$(\nu\gg-q^2\gg a_0,\text{ i.e., }\rho\rightarrow\infty). \quad (8\text{a})$$

Equation (6a) also gives

$$W_1\underset{B}{\rightarrow}\tfrac{1}{2}(2\rho)^{\alpha_1}\int da\,\sigma_1(a)\quad(\rho\rightarrow\infty). \quad (8\text{b})$$

[10] J. M. Cornwall and R. E. Norton, Phys. Rev. **173**, 1637 (1968).

These results hold for $W_1^{\bar{\nu}p}$ and $W_1^{\nu p}$ with $\sigma_1^{\bar{\nu}p}(a)$ and $\sigma_1^{\nu p}(a)$, respectively. The above procedure gives

$$\nu W_2 \longrightarrow \tfrac{1}{2}(2\rho)^{\alpha_2-1}\int da\, \sigma_2(a)\,, \tag{8c}$$

$$\nu W_3 \longrightarrow \tfrac{1}{2}(2\rho)^{\alpha_3}\int da\, \sigma_3(a) \tag{8d}$$

in both the R limit for large $-q^2$ and the B limit as $\rho\longrightarrow\infty$.

It is convenient to define $W_i^{(\pm)}(q^2,\nu)=W_i^{\bar{\nu}}(q^2,\nu)\pm W_i^{\nu}(q^2,\nu)$ since the $W_i^{(+)}$ $(W_i^{(-)})$ are odd (even) under $\nu\longrightarrow-\nu$. $F_i^{(\pm)}(\rho)$ and $\sigma_i^{(\pm)}(a)$ will be defined similarly. The procedure used by Brandt[3] to estimate the values of the limits in the electroproduction case $F_i^{\mathrm{em}(+)}(\rho)$, $i=1$, 2, does not work here since there are no data analogous to the total photoproduction cross section.

An alternative procedure for estimating these limits is to use partial conservation of axial-vector current (PCAC) to determine $\nu W_2^{(\pm)}(0,\nu)$.[11] If $\Delta S=0$,

$$\nu W_2 \xrightarrow[q^2\to 0]{} (F_\pi^2/\pi)\sigma(\pi p)\,,$$

where $\sigma(\pi p)$ is the total πp cross section at high energy. Using $F_\pi=0.9M_\pi$, $\sigma(\pi^-p)=25$ mb, and $\sigma(\pi^+p)=23$ mb gives $\nu W_2^{(+)}(0,\nu)=2M/\pi$ and $\nu W_2^{(-)}(0,\nu)=0.1M/\pi$. The DGS spectral representation (6b) gives the same formal limit of νW_2 in the B limit as $\rho\longrightarrow\infty$ and in the R limit as $-q^2\longrightarrow 0$; however, the assumption (3) does not support the interchange of limit and integration in the latter case. If the interchange is justifiable then (6b) and PCAC imply

$$\begin{aligned} F_2^{(+)}(\rho) &\longrightarrow 2/\pi \quad (\rho\longrightarrow\infty)\,,\\ F_2^{(-)}(\rho) &\longrightarrow 0.1/\pi \quad (\rho\longrightarrow\infty)\,. \end{aligned} \tag{9}$$

Equations (5) and (9) can be used to estimate lower bounds on cross sections as will be discussed in Sec. IV. However it does not seem possible to justify the extrapolation from $q^2=0$ to the B limit in view of the fact that scale invariance in electroproduction does not occur for $-q^2<0.7$ GeV^2. Also the vector contribution to $\nu W_2(q^2,\nu)$ may not be included in (9) since its spectral function $\sigma_j^V(a,b)$ may contain a factor q^2 as in electroproduction.[3] This contribution adds a term $\sigma_j^V(a)a$ to the integrands in (8) but does not alter the equality of the R limits for large $-q^2$ to the B limits for large ρ. This contribution vanishes in the R limit as $-q^2\longrightarrow 0$, invalidating (9). Another procedure which appears more reasonable is to use Regge behavior and saturation of current-algebra sum rules. This approach is discussed next.

[11] J. D. Bjorken and E. A. Paschos, Phys. Rev. D 1, 1450 (1970). I am grateful to Dr. Paschos for pointing out the relevance of this to me.

III. LIMITS OF $F_i(\rho)$ AS $\rho\longrightarrow\infty$

Several current-algebra sum rules for the structure functions have been derived. These include Adler's "forward" sum rule[5]

$$\int_{-q^2/2}^{\infty} d\nu\, W_2^{(-)}(q^2,\nu)=2MK\,, \tag{10}$$

derived from $[J_0(0,\mathbf{x}),J_0^\dagger(0)]$, where $K=2I_3\cos^2\theta_C+(\tfrac{3}{2}+I_3)\sin^2\theta_C$, I_3 is the isospin of the target nucleon, and θ_C is the Cabibbo angle. Bjorken used the BJL expansion to derive a "backward" sum rule[6]

$$\lim_{-q^2\to\infty}\left[-q^2\int_{-q^2/2}^{\infty}\frac{d\nu}{\nu^2}W_1^{(-)}(q^2,\nu)\right]=\frac{2K}{M}, \tag{11}$$

using the space-space equal-time commentators of the quark model $[U(6)\times U(6)$ algebra$]$.

Gross and Llewellyn Smith[7] have recently derived sum rules involving $F_i^{(+)}(\rho)$, using the BJL expansion. For the quark model, these are

$$\begin{aligned}\int_{1/2}^{\infty}\frac{d\rho}{\rho^2}\left[\left(F_2^{(+)}-\frac{1}{\rho}F_1^{(+)}\right)\hat{P}_i\hat{P}_j+\frac{1}{\rho}F_1^{(+)}(\hat{P}_i\hat{P}_j-\delta_{ij})\right]\\ =\lim_{|\mathbf{p}|\to\infty}\frac{iM}{|\mathbf{p}|^2}\int d^3x\,\langle P|[\dot{J}_i(0,\mathbf{x}),J_j^+(0)]|P\rangle_{\mathrm{av}}\\ =A(\hat{P}_i\hat{P}_j-\delta_{ij})\,,\end{aligned} \tag{12a}$$

where A is unknown, and

$$\frac{1}{2}\int_{1/2}^{\infty}\frac{d\rho}{\rho^2}F_3^{(+)}=-6+(3-2I_3)\sin^2\theta_C\,. \tag{12b}$$

The sum rules (10)–(12) can be used to determine combinations of $F_i(\rho)$ if the asymptotic behavior in ν and ρ given by (8) is assumed to approximate the structure functions to threshold ($\rho=\tfrac{1}{2}$). This is the case if $\eta_i(a,b)=0$ in the spectral functions. In the electroproduction case, $F_2^{\mathrm{em}}(\rho)$ approaches its asymptotic value very rapidly[12] and there is evidence that at least one of the $F_i^{\nu}(\rho)$ does also.[13]

The values of α_i in (7) must be chosen. If the usual Regge trajectories are assured to dominate in Cabibbo current-nucleon scattering, then $F_1^{(-)}$ and $F_2^{(-)}$ have $\alpha_1^{(-)}\simeq\alpha_2^{(-)}\simeq\tfrac{1}{2}$, corresponding to the ρ, A_1 trajectories since $I=1$ is exchanged in the cross channel. $F_3^{(+)}$ has $\alpha_3^{(+)}\simeq\tfrac{1}{2}$ corresponding to the ω, ϕ trajectories, since negative G parity and $I=0$ is exchanged in the cross channel. Regge behavior for $F_2^{(-)}$ is inconsistent with the PCAC prediction (9). These assumptions and (10)–

[12] E. Bloom, M. Breidenbach, D. Coward, H. De Staebler, J. Drees, J. Freidman, H. Kendall, L. Mo, and R. Taylor, Phys. Rev. Letters 23, 935 (1969).

[13] I. Budagov, D. Cundy, C. Franzinetti, W. Fretter, H. Hopkins, C. Manfredotti, G. Myatt, F. Nezrick, M. Nikolic, T. Novey, R. Palmer, J. Pattison, D. Perkins, C. Ramm, B. Roe, R. Stump, W. Venus, H. Wachsmuth, H. Yoshiki, Phys. Letters 30B, 364 (1969).

(12) lead to

$$F_1^{(-)}(\rho)=K(1-\alpha_1^{(-)})(2\rho)^{\alpha_1^{(-)}}\simeq I_3(2\rho)^{1/2}, \quad (13a)$$

$$F_2^{(-)}(\rho)=2K(1-\alpha_2^{(-)})(2\rho)^{\alpha_2^{(-)}-1}\simeq 2I_3(2\rho)^{-1/2}, \quad (13b)$$

$$F_3^{(+)}(\rho)=-(6-3\sin^2\theta_C+2I_3\sin^2\theta_C)(1-\alpha_3^{(+)})(2\rho)^{\alpha_3+} \simeq -3(2\rho)^{1/2}, \quad (13c)$$

since θ_C is small. $\sin^2\theta_C=0$ will be assumed in the following. This corresponds to no contribution from the strangeness changing currents and implies

$$F_i^{\bar\nu p}(\rho)=F_i^{\nu n}(\rho) \quad \text{and} \quad F_i^{\nu p}(\rho)=F_i^{\bar\nu n}(\rho), \quad i=1,2,3. \quad (14)$$

These relate the useful nucleon average of the structure function to the $(+)$ combination. The sum rule (12a) implies

$$F_1^{(+)}(\rho)=\rho F_2^{(+)}(\rho). \quad (15)$$

This has recently been derived without assuming current algebra.[14]

Given (13c), the inequalities in (5) estimate the following lower bound on $F_1^{(+)}$ and $F_2^{(+)}$ and therefore lower bounds on the nucleon-averaged cross sections of ν and $\bar\nu$ given by (4):

$$\rho F_2^{(+)}(\rho)=F_1^{(+)}(\rho)\geq \tfrac{3}{2}(2\rho)^{1/2}. \quad (16)$$

The values of $F_i(\rho)$ are incorrect near threshold ($\rho=\frac{1}{2}$) where they should be zero. Since the sum rules weight the small-ρ region of integration, the constants determined by the saturation assumptions are only approximations that could easily be incorrect by a factor of 2. If F_i rapidly and smoothly approach the assumed asymptotic ρ behavior, then (13) is an underestimate. A correction for the threshold behavior can be estimated by assuming that $F_i(\rho)$ has an expansion in decreasing powers of ρ. This is consistant with the B and R limits in Sec. II and has the advantage of admitting a zero at the threshold. If the leading term given by (8) and one additional term are assumed and the zero at threshold imposed, then the sum rules imply

$$\begin{aligned} &F_1^{(-)}=\tfrac{3}{2}I_3(2\rho)^{1/2}(1-1/2\rho), \\ &F_2^{(-)}=3I_3(2\rho)^{-1/2}(1-1/2\rho), \\ &F_3^{(+)}=-\tfrac{9}{2}(2\rho)^{1/2}(1-1/2\rho) \quad \text{(polynomial correction).} \end{aligned} \quad (17)$$

Brandt has proposed[4] that the structure functions depend only on a power of $(2\rho-1)$. This behavior and the assumptions leading to (13) give

$$\begin{aligned} &F_1^{(-)}=(4/\pi)I_3(2\rho-1)^{1/2}, \\ &F_2^{(-)}=(4/\pi)I_3(2\rho-1)^{-1/2}, \\ &F_3^{(+)}=-(12/\pi)(2\rho-1)^{1/2} \quad \text{(Brandt's correction).} \end{aligned}$$

These are similar to (17), although $F_1^{(-)}\neq\rho F_2^{(-)}$.

To estimate additional F_i's, more must be assumed. The left-hand side of the inequality (16) can be determined assuming the following consequence of a class of parton models. If the relevant partons are baryons as in the usual pointlike quark model instead of antibaryons or mesons, then at large E the cross section for scattering antineutrons in the backward direction is zero due to angular momentum conservation.[7] When E is much greater than the parton mass, this corresponds to $x=1$ as indicated by Fig. 1. Equation (4) then implies for large E

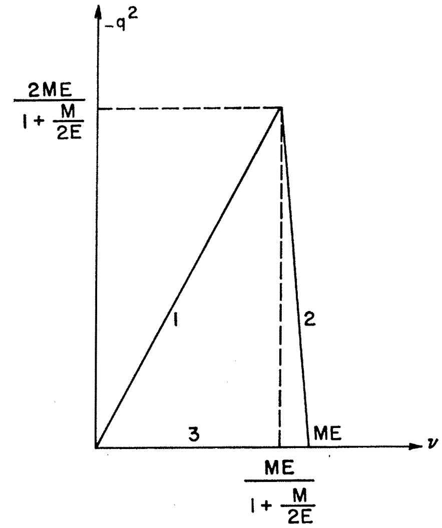

FIG. 1. Physical region for neutrino scattering with a particle of mass M. The boundaries are quasi-elastic scattering 1, backward scattering 2, and forward scattering 3.

$$F_1^{\bar\nu}=-\tfrac{1}{2}F_3^{\bar\nu} \quad \text{(light baryon constituents)} \quad (18)$$

for protons and neutrons. This gives $F_1^{(\pm)}=-\frac{1}{2}F_3^{(\pm)}$ and therefore with (13c),

$$\begin{aligned} &F_i^{\bar\nu p}=F_i^{\nu n}=2F_i^{\bar\nu n}=2F_i^{\nu p}, \quad i=1,2,3 \\ &F_1^{\bar\nu p}=\rho F_2^{\bar\nu p}=-\tfrac{1}{2}F_3^{\bar\nu p}=(2\rho)^{1/2}. \end{aligned} \quad (19)$$

These correspond to the minimum allowed by (16).

Usual Regge theory suggests that the Pomeranchon and P' trajectories should contribute to $F_1^{(+)}$ and $F_2^{(+)}$ since $I=0$ in the cross channel. The relation $F_1^{(+)}=-\frac{1}{2}F_3^{(+)}\sim\rho^{1/2}$ indicates that only the P' is consistent with the above assumptions. This problem will be discussed in Sec. IV and V. The value of the unknown A can be estimated by (12a) and (19).

IV. PREDICTED CROSS SECTIONS AND DATA

The values of the structure functions in (16) give the following lower bounds for the nucleon average deep-inelastic cross section for large ρ:

$$\left\langle \frac{d^2}{dx\,d\rho}\begin{pmatrix}\sigma^{\bar\nu}\\ \sigma^{\nu}\end{pmatrix}\right\rangle \equiv \frac{1}{2}\frac{d^2}{dx\,d\rho}\begin{pmatrix}\sigma^{\bar\nu p}+\sigma^{\bar\nu n}\\ \sigma^{\nu p}+\sigma^{\nu n}\end{pmatrix} \geq \frac{G^2ME}{\pi}(2\rho)^{-5/2}3\begin{pmatrix}(x-1)^2-Mx/4\rho E\\ 1-Mx/4\rho E\end{pmatrix}. \quad (20a)$$

[14] R. Chanda, D. Majumdar, and S. Sen, Syracuse University Report No. NYO-3399-217, SU-1206-217 (unpublished). The field algebra for the equal-time commutators modifies (11) and (15).

The equality results from the equality in (16) or from (19). Integration on x and ρ when $E\gg M$ over the region in Fig. 1 gives

$$\left\langle\frac{d}{d\rho}\begin{pmatrix}\sigma^{\bar\nu}\\ \sigma^{\nu}\end{pmatrix}\right\rangle\geq\frac{G^2ME}{\pi}(2\rho)^{-5/2}3\begin{pmatrix}\frac{1}{3}\\ 1\end{pmatrix}, \tag{20b}$$

$$\left\langle\begin{pmatrix}\sigma^{\bar\nu}\\ \sigma^{\nu}\end{pmatrix}\right\rangle\geq\frac{G^2ME}{\pi}\begin{pmatrix}\frac{1}{3}\\ 1\end{pmatrix}. \tag{20c}$$

In the differential cross section (4), $F_{1,3}$ are weighted by ρ^{-3} and F_2 is weighted by ρ^{-2} so extrapolation of the large-ρ behavior appears risky. In the sum rules (10), (11), and (12b), the F_i are weighted by an additional factor of ρ so the extrapolation errors are partially canceled. If the polynomially corrected F_i's (17) are used, (20c) is multiplied by the factor 0.6.

Experiments in deep-inelastic ν-p scattering appear feasible in the near future. The added assumption (18) which led to (19) predicts that $d\sigma^{\nu p}/dxd\rho$ is independant of x at large ρ, E, and that

$$\begin{aligned}\frac{d\sigma^{\bar\nu p}}{d|q^2|}=2\frac{d\sigma^{\bar\nu n}}{d|q^2|}&=\frac{G^2}{\pi}\left[2-\frac{16}{3}\left(\frac{2ME}{-q^2}\right)^{-1/2}+4\left(\frac{2ME}{-q^2}\right)^{-1}\right.\\&\qquad\left.-\frac{2}{3}\left(\frac{2ME}{-q^2}\right)^{-2}\right],\end{aligned} \tag{21}$$

$$\frac{d\sigma^{\nu p}}{d|q^2|}=\frac{1}{2}\frac{d\sigma^{\nu n}}{d|q^2|}=\frac{G^2}{\pi}\left[1-\left(\frac{2ME}{-q^2}\right)^{-1/2}\right],$$

$$\sigma^{\bar\nu p}=2\sigma^{\bar\nu n}=(4/9)G^2ME/\pi\,,$$

$$\sigma^{\nu p}=\tfrac{1}{2}\sigma^{\nu n}=\tfrac{2}{3}G^2ME/\pi\,.$$

They agree, as they must for consistency, with the limit derived by Adler[5] from (10):

$$\lim_{E\to\infty}\left(\frac{d\sigma^{\bar\nu p}}{d|q^2|}-\frac{d\sigma^{\nu p}}{d|q^2|}\right)=\frac{G^2K}{\pi}. \tag{22}$$

The limit of $F_2^{(+)}(\rho)$ derived from PCAC, Eq. (9), can be used with (5) to estimate an *upper* bound for the cross section:

$$\left\langle\frac{d^2}{dxd\rho}\begin{pmatrix}\sigma^{\bar\nu}\\ \sigma^{\nu}\end{pmatrix}\right\rangle\leq\frac{G^2ME}{2\pi^2\rho^2}\begin{pmatrix}1\\ 1\end{pmatrix}, \tag{23a}$$

$$\left\langle\begin{pmatrix}\sigma^{\bar\nu}\\ \sigma^{\nu}\end{pmatrix}\right\rangle\leq\frac{G^2ME}{\pi^2}\begin{pmatrix}1\\ 1\end{pmatrix}, \tag{23b}$$

when $E\gg M$. The extrapolation of the large-ρ behavior to threshold used in deriving (23b) is more risky than that used in deriving (20c), since there is no compensating extrapolation in this case. If $F_2^{(+)}$ has the same behavior near threshold as F_2^{em}, then (23b) overestimates the upper bounds by a factor of 2. The estimates (20c) and (23b) are clearly conflicting.

There is a considerable amount of data on high-energy ν scattering and a small amount on $\bar\nu$ scattering. The data[13] indicate that for E between 1 and 12 GeV

$$\langle\sigma^{\nu}\rangle_{\mathrm{exp}}=(G^2ME/\pi)(0.6\pm0.15)\,, \tag{24}$$

which compares favorably with the second equality in (20c). If the "corrected" F_i's, Eq. (17), are used, the agreement is exact. The data[15] on $\bar\nu$ are consistent with $\langle\sigma^{\bar\nu}\rangle=\frac{1}{3}\langle\sigma^{\nu}\rangle$, in agreement with the equality in (20c); however, there are very few events at high $-q^2$ and ν. This suggests that the assumption of light baryon partons and consequently (18) may be correct for E between 1 and 12 GeV and that extrapolation of PCAC to the B limit resulting in (23) is not justified.

The data[16] for $\langle d\sigma^{\nu}/d\rho\rangle$ with $\rho>2$ in the categories $E>4$; $E>2$, $x<\frac{1}{2}$; and $E>2$, $x>\frac{1}{2}$ are plotted in Figs. 2–4 along with the best fits with $C\rho^{-5/2}$ and $C\rho^{-2}$. The values of χ^2 are given in Table I. The ρ^{-2} behavior would result from Pomeranchon dominance of $F_1^{(+)}$ and $F_2^{(+)}$. There also are data on the x dependence of the neutrino cross section since the ratio $E'/E=1-x$ is measurable. The data[13] are consistent with the independance predicted by (20a). Drell, Levy, and Yan[17] also predict the x dependance given in (20a).

Although the agreement of the equality in (20c) with the data suggests that (18) is approximately correct, it is not clear how significantly the Pomeranchon has affected the available data where $\nu\leq12$ GeV^2 and $\rho\leq56$, since the data are fitted acceptably with and without the Pomeranchon. In deep-inelastic electron-proton scattering, F_2^{em} might have the same asymptotic behavior in ρ as $F_2^{(+)}$ in the region where the current-current coupling is valid, since an isospin rotation relates the isovector component of J_μ^{em} with the vector component of J_μ. The electron data[12] for $\theta=6°$, assuming $F_1^{\mathrm{em}}=\rho F_2^{\mathrm{em}}$, indicate that F_2^{em} is approximately

TABLE I. Best values of C and the values of χ^2 and the confidence levels for the power-law fits to the data with $\rho>2$ in Figs. 2–4.

	$C\rho^{-2}$			$C\rho^{-2.5}$		
Data class	C	χ^2	P	C	χ^2	P
$E>4$	89	0.69	1	207	4.9	0.3
$E>2$, $x<\frac{1}{2}$	135	2.04	0.7	332	0.77	0.8
$E>2$, $x>\frac{1}{2}$	140	0.77	0.8	324	2.5	0.7

[15] E. C. M. Young, CERN Report No. 67-12 (unpublished).
[16] The $E>4$ data appeared in Ref. 13. The $E>2$ data will be published by the same authors elsewhere.
[17] S. D. Drell, D. J. Levy, and T.-M. Yan, Phys. Rev. **187**, 2159 (1969); T.-M. Yan and S. D. Drell, Phys. Rev. D **1**, 2402 (1970).

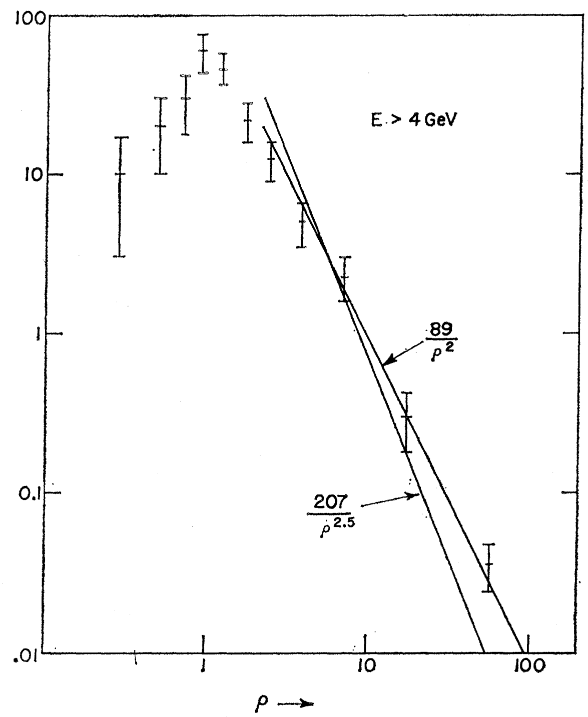

FIG. 2. Number of events versus ρ with $E>4$. The lines are the best fits to the five data points above $\rho=2$.

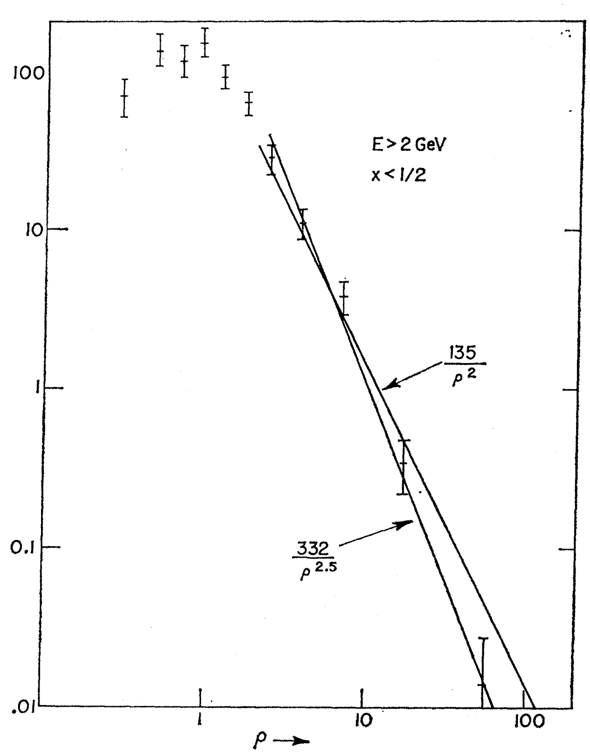

FIG. 3. Number of events versus ρ with $E>2$ and $x<\frac{1}{2}$. The lines are the best fits to the five data points above $\rho=2$.

constant for $2<\rho<4$ and *decreases* for $5<\rho<20$. Pomeranchon dominance, scale invariance, and a nonpathological DGS representation predict that F_2^{em} should remain flat.[3] It is debatable whether the decrease is characteristic of the deep-inelastic (B) limit or results from the experimental values of $-q^2$ being too small. Since the decrease is indicated even by the data with $-q^2\geq 1$ GeV^2, the former case is reasonable. This decrease starting at $\rho=4$ suggests that only the neutrino data with $\rho>4$ in each of Figs 2–4 can represent the asymptotic ρ behavior. The data at $\rho=56$ may not be in the B-limit region since $-q^2$ is small. This leaves three data points in the B limit. Even five data points

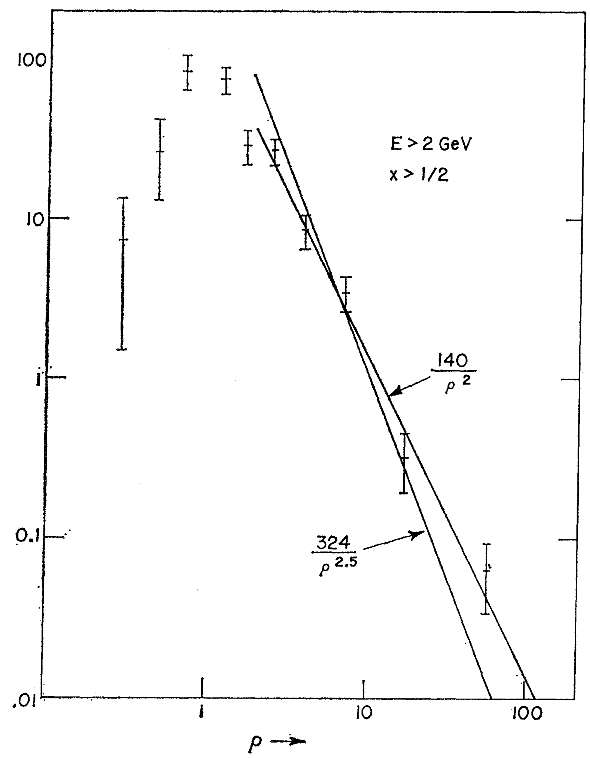

FIG. 4. Number of events versus ρ with $E>2$ and $x>\frac{1}{2}$. The lines are the best fits to the five data points above $\rho=2$.

were insufficient since the χ^2 for both fits in Table I are large.

The decrease of F_2^{em} is fitted by $0.6\rho^{-0.4}$ and by $0.5\rho^{-0.3}$. The assumptions in Sec. III led to $F_2^{(+)}\sim\rho^{-0.5}$. Clearly the corresponding value for α between 0.6 and 0.7 suggested by the electron data would not alter the predictions in Sec. IV significantly. The total cross section is proportional to $(1-\alpha)/(2-\alpha)$ so this would decrease (20c). Also the fit to the ρ dependence would be improved; however, it is not evident what leads to such a value for α.

In deep-inelastic electron-proton scattering the cross section analogous to (4) should hold above some momentum transfer $(-q^2)_{\min}=c$ where the Bjorken limit is assumed to hold. If F_2^{em} has the asymptotic behavior $R\rho^{\alpha-1}$ for $\rho>\rho_0$, the cross section at large E is

$$\sigma^{ep}{}_{-q^2>c}\geq(4\pi e^2R/c)\begin{cases}\ln(2ME/c\rho_0)\,, & \alpha=1\\ \rho_0{}^{1-\alpha}/(1-\alpha)\,, & 1>\alpha>0\,.\end{cases}\tag{25}$$

The $\ln E$ behavior is distinctive and could provide a means of determining if and when $\alpha=1$.

V. CONCLUSIONS

Scale invariance, quark sum rules, and duality (Regge asymptotics and saturation of sum rules) led to estimates of lower bounds for the nucleon-averaged cross section (20c). The data for $\langle\sigma^\nu\rangle$ agree sufficiently well with the lower bound to suggest that the relation $F_1{}^{(+)}=-\frac{1}{2}F_3{}^{(+)}$ corresponding to the equality for $\langle\sigma^\nu\rangle$ is approximately valid. This relation contradicts Pomeranchon dominance of $F_1{}^{(+)}$. Unless the agreement with the lower bound is accidental, the Pomeranchon is not dominating the present data. If the Pomeranchon begins to dominate at higher ν causing $F_1{}^{(+)}>-\frac{1}{2}F_3{}^{(+)}$, the value of $(d/dE)\langle\sigma^\nu\rangle$ will increase. This would indicate a violation of at least one of the assumptions such as scale invariance. Nondominance of the Pomeranchon is an unexpected consequence, but does not appear impossible in view of the evidence that the Pomeranchon has special properties such as distinctive behavior in finite-energy sum rules which is not shared by canonical Regge poles.[18]

The very tenuous agreement of the data for $\langle\sigma^{\bar\nu}\rangle$ with the lower bound estimated in (20c) also suggests that the relation $F_1{}^{(+)}=-\frac{1}{2}F_3{}^{(+)}$ is approximately valid. There are no data to suggest that $F_1{}^{(-)}=-\frac{1}{2}F_3{}^{(-)}$ as well; however, both relations follow from (18), which is a consequence of models of nucleons containing only light, pointlike, baryon constituents. An example is the naive quark model. Since the current algebra from which the sum rules were derived agrees with this model, and since duality appears to be connected with the quark model,[18] the assumptions leading to the equality in (20c) appear to be consistent. The assumption (18) results in the conclusion $\sigma^{\bar\nu}\neq\sigma^\nu$ in (21), which is striking.

The deep-inelastic electroproduction data indicate that the Pomeranchon may not be dominating there also; however, the Pomeranchon does seem to dominate photoproduction since the cross section at large ν is quite constant. This suggests that the Pomeranchon's residue decreases rapidly with $-q^2$, whereas the residue of ordinary Regge poles decreases gradually. This conclusion is orthogonal to Harari's "diffractive" model,[19] where the Pomeranchon dominates at large $-q^2$ for arbitrary ν.

If the Pomeranchon becomes important at values of ν above those measured, then the simplest assumption is that relation (18) breaks down with F_1 and F_2 becoming larger. In the Bjorken-Paschos[9] version of the parton model, the nucleon is considered to have a three-quark core and a cloud composed of a large number of quark-antiquark pairs.[20] The breakdown of (18) could result from the cloud becoming an important scatterer at high ν. Perhaps the Pomeranchon scatters only on the cloud. If the partons are bare mesons and a bare nucleon, the breakdown of (18) would occur if the bare mesons become important scatterers at high ν; however, this would contradict the results of Drell, Levy, and Yan.[17] Pomeranchon dominance would provide a natural explanation of the PCAC implication of constant $F_2{}^{(+)}$ at large ρ [Eq. (9)].

ACKNOWLEDGMENTS

I thank Richard Brandt for much encouragement, George Snow for very helpful criticism, and Ronald Peierls for the hospitality extended by the Brookhaven National Laboratory, where part of the work was completed.

[18] H. Harari, BNL Report No. BNL 50212 (C-58) (unpublished).

[19] H. Harari, Phy. Rev. Letters **24**, 286 (1970).

[20] Bjorken and Paschos argue that a large number of quark-antiquark pairs are needed in their model to explain the apparent smallness of the average charge per quark; however, they assume no correlation between the momentum distribution and charge of the quarks. It is conceivable that if this assumption is relaxed, the average charge per quark need not be small and that the usual three-quark model can explain the data, in which case (18) holds for sufficiently high E.