Compositeness of External Particle and Absence of Fixed Poles in Photoproduction Processes*

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We present a new argument, based on the study of the potential-theory model, which claims that the right-signature fixed poles are absent in the photoproduction processes if the external particles lie on Regge trajectories. Our result is in complete agreement with the conclusions of other authors who have studied relativistic models of photoproduction.

I. INTRODUCTION

NONLINEAR unitarity relations are known to rule out the possibility of having right-signature fixed J poles in the strong-interaction amplitudes. But in the electromagnetic or weak processes of the hadrons like $W+A\to B+C$ (where W represents either a photon or a weakly interacting particle; A, B, and C are hadrons), if one works to the lowest order in the electromagnetic or weak-coupling constant, then linear unitarity relations are satisfied. So it is possible to have fixed poles of right signature in these processes.[1] However, it is commonly believed that the amplitude for $W+A\to B+C$ will have a fixed pole at $J=0$ (when A is a boson) or $J=\frac{1}{2}$ (when A is a fermion) if the particle A is "elementary," but that the fixed pole will not be present if A is a composite object lying on a Regge trajectory.[2–5]

In this paper we study the nonrelativistic potential scattering model of the photoproduction process, working to the lowest order in the electric charge e. Specifically we consider the process $\gamma+A\to B+C$, with A being a bound state of B and C.[6] That is, we assume that B and C interact through some central potential and produce A as a bound state which lies on a Regge trajectory. Particular attention is paid to investigating whether or not the compositeness of A would rule out the existence of a right-signature fixed pole in this process.

Our results are in complete agreement with the conclusions of other authors[2–5] who have studied relativistic models of photoproduction. Loosely speaking, it has been found that if A is assumed to be a bound state of B and C (typically in the ladder approximation), then certain fixed poles in $\gamma+A\to B+C$ which would normally occur in perturbation theory are not present.[2,4]

The paper is organized as follows. In Sec. II, the nonrelativistic potential scattering model of $\gamma+A\to B+C$ is discussed. In Sec. III, the low-energy limit of the scattering amplitude is derived to see whether our model has the same general features as that in the relativistic theory of photoproduction. In Sec. IV, a few relevant points are discussed.

II. NONRELATIVISTIC POTENTIAL SCATTERING MODEL

Let us consider the process

$$\gamma+A\to B+C\,, \tag{2.1}$$

where γ denotes a photon and A is a bound state of B and C. Also, B carries electric charge e, and C is neutral. Particular attention is paid to examining whether or not the composite character of A may rule out the possibility of having a fixed pole at $J=0$ in this process. Since the spin of the particles just complicates the calculation, for simplicity we assume that particles A, B, and C be spinless. The process is described by the total Hamiltonian H_t with

$$H_t=H_0+H_{\rm st}+H_{\rm em}\,,$$

where H_0, $H_{\rm st}$, and $H_{\rm em}$ denote free Hamiltonian, strong-interaction Hamiltonian, and electromagnetic-interaction Hamiltonian, respectively.

We consider the problem in the c.m. system and denote by $\mathbf{p}$ and $\mathbf{k}$ the momentum of B and γ in this system, respectively. Let $\mathbf{r}$ be the relative coordinate between B and C. Then we have in this c.m. system

$$H_0=p^2/2\mu=-\nabla^2/2\mu\,,$$
$$H_{\rm st}=H_{\rm st}(r)\,,$$
$$H_{\rm em}=-\frac{e}{m_B}\frac{1}{(2\pi)^{3/2}}e^{i\mathbf{k}\cdot\mathbf{r}}\boldsymbol{\varepsilon}\cdot\mathbf{p}\,,$$

where m_B is the mass of particle B, μ is the reduced mass, and $\boldsymbol{\varepsilon}$ is the photon polarization vector. To the lowest order of electromagnetic interaction, the S matrix describing the process now reads, in the c.m. system,

$$S_{fi}=\langle B,C;\,\text{out}|H_{\rm em}|A\rangle$$
$$=\frac{1}{(2\pi)^{3/2}}\int\langle\psi_p^-(r)|\frac{ie}{m_B}e^{i\mathbf{k}\cdot\mathbf{r}}\boldsymbol{\varepsilon}\cdot\boldsymbol{\nabla}|\phi_A(r)\rangle d^3r\,, \tag{2.2}$$

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where $\phi_A(r)$ is the bound-state wave function of A and $\psi_p^-(r)$ is the outgoing state. $\phi_A(r)$ satisfies the equation

$$H\phi_A = E_A\phi_A\,,$$

where $H=H_0+H_{\rm st}$ and E_A is the binding energy of A. The outgoing state $\psi_p^-(r)$ is given by the expression[7]

$$\psi_p^-(r)=\chi_p+\int \frac{d^3p'\chi_{p'}(r)}{E(p)-i\eta-E(p')}\langle\chi_{p'}|T|\chi_p\rangle\,, \quad (2.3)$$

where χ_p is the free-particle state in the c.m. system; i.e.,

$$\chi_p=(2\pi)^{-3/2}e^{i\mathbf{p}\cdot\mathbf{r}}\,,$$

T is the T matrix of the strong interaction, and $E(p)=p^2/2\mu$. η is a very small number.

To see whether the right-signature fixed poles exist in the scattering amplitude S_{fi}, we expand this amplitude into partial waves. First we notice that the outgoing wave function $\psi_p^-(r)$ has the following partial-wave expansion[7]:

$$\psi_p^-(r)=\sum_{l,m} Y_l^{m*}(\hat{p})Y_l^m(\hat{r})\psi_{p,l}^-(r)\,, \quad (2.4)$$

with

$$\psi_{p,l}^-(r)=i^l(\tfrac{1}{2}\pi)^{1/2} \times\left[j_l(pr)+\int\frac{p'^2 j_l(p'r)T_l(p';p)}{E(p)-E(p')-i\eta}d'p\right], \quad (2.5)$$

where $j_l(pr)$ is the spherical Bessel function, and $T_l(p';p)$ is the lth partial wave of the T matrix for the strong interaction. We assume that the "strong" process

$$B+C\rightarrow B+C \quad (2.6)$$

Reggeizes in the usual way, so that we can analytically continue T_l into the complex l plane. We note that fixed poles of the right signature cannot be present in the analytically continued strong-interaction amplitude T_l, for it satisfies a nonlinear unitarity relation. The analytically continued j_l is an entire function in the complex l plane. So we see that there would be no fixed singularities of angular momentum l in the analytically continued partial-wave amplitude $\psi_{p,l}^-(r)$.

On the other hand, the bound-state wave function $\phi_A(r)$ is in the S state because particle A is assumed to be spinless. So we have

$$\nabla\phi_A(r)=\hat{r}\partial\phi_A/\partial r\,. \quad (2.7)$$

With Eqs. (2.4) and (2.7), we find that the partial-wave expansion of S_{fi} takes the form

$$S_{fi}=(\tfrac{4}{3}\pi)^{1/2}\frac{ie}{m_B}\sum_{l_1,m_1,l,m,\mu} Y_l^m(\hat{p})Y_l^{m_1*}(\hat{k}) \times\int Y_l^{m*}(\hat{r})Y_{l_1}^{m_1}(\hat{r})Y_1^{-\mu}(\hat{r})d\Omega \times\int\psi_{p,l}^{-*}(r)j_{l_1}(kr)\frac{\partial\phi_A}{\partial r}\epsilon_\mu r^2dr\,.$$

With the help of the 3-j symbols, we get

$$S_{fi}=\frac{ie}{m_B}\sum_{l,m,l_1,m_1,\mu} Y_l^m(\hat{p})Y_{l_1}^{m_1*}(\hat{k})(2l+1)(2l_1+1) \times\begin{pmatrix} l & l_1 & 1\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} l & l_1 & 1\\ -m & m_1 & -\mu\end{pmatrix} \times\int\psi_{p,l}^{-*}(r)j_{l_1}(kr)\frac{\partial\phi_A}{\partial r}\epsilon_\mu r^2dr\,. \quad (2.8)$$

We choose $\hat{k}$ to be in the z-axis direction and $\hat{p}$ to be in the xz plane. We consider right-handed polarized photons. After using the explicit formulas for the 3-j symbols and with some algebra, we get the following expression for the partial-wave expansion of the scattering amplitude:

$$S_{fi}=\frac{ie\sin\theta}{4\pi\sqrt{2}m_B}\Big[\sum_{l=0}P_l'(\cos\theta)f_l+\sum_{l=1}P_l'(\cos\theta)h_l\Big]\,, \quad (2.9)$$

where

$$f_l=\int_0^\infty \psi_{p,l}^{-*}(r)j_{l+1}(kr)\frac{\partial\phi_A}{\partial r}r^2dr \quad (2.10)$$

and

$$h_l=\int_0^\infty \psi_{p,l}^{-*}(r)j_{l-1}(kr)\frac{\partial\phi_A}{\partial r}r^2dr\,. \quad (2.11)$$

It is easily seen that f_l and h_l can be analytically continued into the complex l plane by simply using the analytically continued $\psi_{p,l}^-$ and spherical Bessel function in Eqs. (2.10) and (2.11).

After using the Watson-Sommerfeld transformation to transform the sums in Eq. (2.9) into integrals in the complex l plane, we see that if there are fixed poles in S_{fi}, they must come from the analytically continued f_l or h_l. To see whether there are fixed poles in the continued f_l or h_l, we recall that there are no fixed poles in the continued $\psi_{p,l}^-$ and j_l. So the only possibility for fixed singularities to arise in f_l or h_l should be due to the divergence of the integrals in (2.10) or (2.11). However, it is assumed that particle A is a bound state of B and C. So the wave function $\phi_A(r)$ decreases exponentially as r

[7] M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964), pp. 231, 234, and 235.

becomes very large; i.e.,

$$\phi_A(r) \xrightarrow[r\to\infty]{} \text{const}\, \frac{\exp[-(-2\mu E_A)^{1/2}r]}{r}, \quad (2.12)$$

where the binding energy E_A is negative.

On the other hand, for all energy E above the threshold, the asymptotic behavior of $\psi_{p,l}^{-*}(r)$ has the following form[7]:

$$\psi_{p,l}^{-*}(r) \xrightarrow[r\to\infty]{} (-i)^l\left(\frac{2}{\pi}\right)^{1/2} \times\left[\frac{\sin(pr-\frac{1}{2}\pi l)}{pr}+\frac{\pi p^2}{dE/dp}\frac{e^{ipr}}{pr}e^{-i\pi l/2}T_l(p)\right]. \quad (2.13)$$

Also, we have

$$j_l(kr) \xrightarrow[r\to\infty]{} \frac{\sin(pr-\frac{1}{2}\pi l)}{pr}. \quad (2.14$$

With Eqs. (2.12)–(2.14), it is clear that since the wave function $\phi_A(r)$ decreases exponentially as $r\to\infty$, the integrals in (2.10) and (2.11) do not diverge for any fixed l at all energy E above the threshold. As a result, we see that there are no fixed poles in the scattering amplitude. It should be stressed that it is the composite character of A which rules out the presence of fixed poles.

III. LOW-ENERGY LIMIT OF SCATTERING AMPLITUDE

In order to check whether or not the nonrelativistic model is gauge invariant and has the same general features as that in the relativistic theory of photoproduction, we study the low-energy limit of the scattering amplitude. The scattering amplitude S_{fi} is known to be singular as $k\to 0$. To calculate the singular part, first we notice that $\psi_{p,l}^{-}(r)$ has the following asymptotic behavior[7]:

$$\psi_{p,l}^{-}(r) \xrightarrow[r\to\infty]{} i^l\left(\frac{2}{\pi}\right)^{1/2} \times\left[\frac{\sin(pr-\frac{1}{2}\pi l)}{pr}+\frac{\pi p^2}{dE/dp}\frac{e^{-ipr}}{pr}e^{i\pi l/2}T_l(p)\right]. \quad (3.1)$$

As $k\to 0$, from energy conservation we have

$$E(p) \xrightarrow[k\to 0]{} E_A.$$

Since the binding energy E_A is negative, the momentum p of the outgoing particle becomes imaginary as $k\to 0$. For imaginary p, we set $p=i|p|$. The analytic continuation of the asymptotic form of $\psi_{p,l}^{-*}$ for imaginary p is

$$\psi_{p,l}^{-*}(r) \xrightarrow[r\to\infty]{} (-i)^l\left(\frac{2}{\pi}\right)^{1/2} \times\left[\frac{\exp(-|p|r-\frac{1}{2}i\pi l)-\exp(|p|r+\frac{1}{2}i\pi l)}{2i^2|p|r} +\frac{\pi p^2}{dE/dp}\frac{e^{-|p|r}}{|p|r}e^{-i\pi l/2}T_l(p)\right]. \quad (3.2)$$

To calculate the singular part of S_{fi} as $k\to 0$, we first note that T_l has a pole of the bound state A at $l=0$; that is,

$$T_0(p) \xrightarrow[E(p)\to E_A]{} \frac{\text{residue}}{E(p)-E_A}.$$

However, this pole will not be present in S_{fi} because it is equivalent to a zero-zero transition as $k\to 0$. From angular momentum conservation, this is forbidden. So we see from the asymptotic form (3.2) of $\psi_{p,l}^{-*}(r)$ and (2.12) that only the plane-wave contribution to the asymptotic form of $\psi_{p,l}^{-*}(r)$ would make S_{fi} singular as $k\to 0$. Therefore, to calculate the singular part of S_{fi}, we only need to replace ψ_p^- by the plane-wave state χ_p; i.e.,

$$S_{fi} \xrightarrow[k\to 0]{} \frac{ie}{(2\pi)^{3/2}m_B}\int\langle\chi_p(r)|e^{i\mathbf{k}\cdot\mathbf{r}}\boldsymbol{\varepsilon}\cdot\nabla|\phi_A(r)\rangle d^3r. \quad (3.3)$$

Since χ_p is an eigenstate of ∇, and $\boldsymbol{\varepsilon}\cdot\mathbf{k}=0$, it follows that

$$S_{fi} \xrightarrow[k\to 0]{} \frac{-e}{(2\pi)^{3/2}m_B}\boldsymbol{\varepsilon}\cdot\mathbf{p}\langle\chi_{p-k}|\phi_A\rangle. \quad (3.4)$$

On the other hand, we have

$$\begin{aligned}\langle\chi_{p-k}|\phi_A\rangle &= (1/E_A)\langle\chi_{p-k}|H|\phi_A\rangle\\ &=[E(p-k)/E_A]\langle\chi_{p-k}|\phi_A\rangle\\ &\quad+(1/E_A)\langle\chi_{p-k}|H_{\text{st}}|\phi_A\rangle\end{aligned}$$

or

$$\langle\chi_{p-k}|\phi_A\rangle=[1/E_A-E(p-k)]\langle\chi_{p-k}|H_{\text{st}}|\phi_A\rangle.$$

So we get

$$\langle\chi_{p-k}|\phi_A\rangle \xrightarrow[k\to 0]{} (-1/k)\langle\chi_p|H_{\text{st}}|\phi_A\rangle. \quad (3.5)$$

However, $\langle\chi_p|H_{\text{st}}|\phi_A\rangle$ is just the $A-B-C$ coupling constant g_{ABC}. This can be easily seen as follows: The T matrix for the strong interaction is given by

$$T=H_{\text{st}}+H_{\text{st}}\frac{1}{E-H+i\eta}H_{\text{st}}.$$

So we have

$$\langle\chi_p|T|\chi_p\rangle=\langle\chi_p|H_{\text{st}}|\chi_p\rangle+\langle\chi_p|H_{\text{st}}\frac{1}{E-H+i\eta}H_{\text{st}}|\chi_p\rangle.$$

As $E(p)\to E_A$, we only need to consider the pole terms of the bound state A; i.e.,

$$\langle\chi_p|T|\chi_p\rangle \xrightarrow[E(p)\to E_A]{} \frac{|\langle\chi_p|H_{\mathrm{st}}|\phi_A\rangle|^2}{E(p)-E_A}.$$

Therefore, we get

$$\langle\chi_p|H_{\mathrm{st}}|\phi_A\rangle = g_{ABC}. \tag{3.6}$$

Combining Eqs. (3.4)–(3.6) yields

$$S_{fi} \xrightarrow[k\to 0]{} \frac{e}{(2\pi)^{3/2}} \frac{\boldsymbol{\varepsilon}\cdot\mathbf{p}}{m_B} g_{ABC} \frac{1}{k}. \tag{3.7}$$

On the other hand, the low-energy limit of the scattering amplitude in the relativistic theory of the photoproduction process $\gamma+A \to B+C$ can be obtained by using Low's approach.[8,9] The process is described by the matrix element

$$\langle B,C|J_\mu{}^{\mathrm{em}}(x)|A\rangle,$$

where $J_\mu{}^{\mathrm{em}}$ is the electromagnetic current. The corresponding process with no photon, i.e., $A \to B+C$, is just the $A-B-C$ vertex. The external line insertions of the photon are just the Born diagrams in Fig. 1. The sum of these two diagrams is divergenceless and gauge invariant; thus by using Low's argument, it follows that as the photon energy k tends to zero, these two diagrams describe the photoproduction amplitude up to terms of order k. Let S_R be this amplitude. We have

$$S_R = (2\pi)^{-3/2} \frac{e\epsilon_\mu(2p-k)_\mu}{(p-k)^2+m_B{}^2} \bar{g}_{ABC} + \frac{(2\pi)^{-3/2} e\epsilon_\mu(2p_A+k)_\mu \bar{g}_{ABC}}{(p_A+k)^2+M_A{}^2}, \tag{3.8}$$

where $\bar{g}_{ABC}$ is the vertex function for the $A-B-C$ vertex; as $k\to 0$, $\bar{g}_{ABC}$ becomes the $A-B-C$ coupling constant g_{ABC}. That is,

$$\bar{g}_{ABC} \xrightarrow[k\to 0]{} g_{ABC}. \tag{3.9}$$

Since the sum of these two diagrams is gauge invariant, we can choose to work in the radiation gauge. In this gauge,

$$\epsilon_\mu = (\boldsymbol{\varepsilon},0).$$

Using the Lorentz condition $\mathbf{k}\cdot\boldsymbol{\varepsilon}=0$, it is easily seen that in the c.m. system, the second term in Eq. (3.8) does not contribute because

$$(2p_A+k)_\mu \cdot \epsilon_\mu = 2\mathbf{p}_A\cdot\boldsymbol{\varepsilon} = -2\mathbf{k}\cdot\boldsymbol{\varepsilon} = 0.$$

So we find in the c.m. system that

$$S_R \xrightarrow[k\to 0]{} \frac{e}{(2\pi)^{3/2}} \frac{\boldsymbol{\varepsilon}\cdot\mathbf{p}}{p_0 k - \mathbf{p}\cdot\mathbf{k}} g_{ABC}.$$

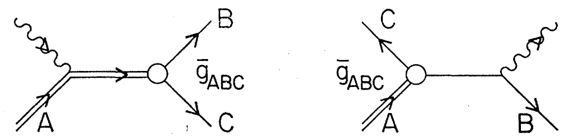

FIG. 1. Born-approximation diagrams (external line photon insertions).

Taking the nonrelativistic limit $p_0 \to m_B$, $p \ll m_B$, we get

$$S_R \xrightarrow[k\to 0]{} \frac{e}{(2\pi)^{3/2}} \frac{\boldsymbol{\varepsilon}\cdot\mathbf{p}}{m_B} g_{ABC} \frac{1}{k}. \tag{3.10}$$

Comparing (3.7) and (3.10), we see immediately that the results are exactly the same. It then follows that our nonrelativistic model not only is gauge invariant but also satisfies the same kinematic condition as the relativistic theory of photoproduction.

IV. DISCUSSION

We have shown in our nonrelativistic potential-scattering model that the composite character of the A particle removes the fixed pole. However, if one assumes an "elementary" A with a basic $A-B-C$ coupling, then the usual Born diagrams for $\gamma+A \to B+C$ yield a fixed pole which persists in the nonrelativistic limit. It is amusing to note that the presence or absence of the fixed pole is apparently dependent on the "elementarity" of an external particle, whereas one usually thinks of J-plane properties as being associated with internal or exchanged particles.

Our results are in complete agreement with the conclusions of other authors who have studied relativistic models of photoproduction. It should be stressed that it is the composite character of particle A in our calculation, not the nonrelativistic kinematics, which removes the fixed pole. Our model is seen to be gauge invariant and has the same general features as relativistic photoproduction. Since there is no obvious reason to believe that relativity plays any special role, as far as the right-signature fixed pole is concerned, our model may indicate the actual state of affairs.

Recently it was pointed out by Dashen and Lee[6] that high-energy backward pion photoproduction is the best place to look for the right-signature fixed poles. The experiment done by Tompkins *et al.*[10] confirms that the fixed poles are absent in the backward pion photoproduction. This absence of fixed poles may very well be due to the composite nature of the nucleon.

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