

Energy and momentum distributions of hadrons produced in inelastic muon-proton scattering*

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We present inclusive distributions for final-state hadrons produced in inelastic muon-proton scattering. Over the total energy range $2 < W < 4.7$ GeV and the momentum-transfer range $0.3 < Q^2 < 4.5$ GeV², the fractional momentum and energy distributions approximately scale. Distributions in transverse momentum display an interesting two-component behavior. They show no dependence on the virtual-photon "mass squared" Q^2 , and have average values typical of other hadron-initiated reactions. A comparison of our distributions with those seen in e^+e^- annihilation and neutrino-nucleon scattering shows agreement, in support of quark-parton fragmentation ideas. We further break these distributions down by event topology.

I. INTRODUCTION

In this paper, we present inclusive hadron distributions seen in a deeply inelastic muon-proton scattering experiment at the Stanford Linear Accelerator Center. Data were taken using a 14-GeV positive muon beam incident on a liquid hydrogen target. All charged final-state particles emerging from the target were detected in a 2-m-long streamer chamber. Charge identification and momentum analysis were provided by a 16-kG magnet; however, mass identification was generally not possible. The trigger for the experiment was provided by a scattered μ^+ traversing four banks of scintillator hodoscopes interspersed with 1.5 m of lead. The experimental apparatus is shown in Fig. 1. A detailed description of the experiment can be found in Ref. 1, which presents topological cross sections and average charged-hadron multiplicities.

The data presented in this paper are based on ~4000 inelastic events covering a kinematic range: $0.3 \leq Q^2 \leq 4.5$ GeV² and $2.0 \leq W \leq 4.7$ GeV. Here, Q^2 = negative square of the four-momentum transferred to the hadronic system, and W = center-of-mass energy of this system. Because of the lack of mass identification, most of the distributions presented will be for negative particles only, where pions are the dominant particle type. To minimize the effect of final-state protons in the positively charged data sample, we shall often restrict the W range to $2.8 \leq W \leq 4.7$ GeV, as discussed in the text. Inclusive data for negative particles covering a W range below that of this experiment can be found in Ref. 2. None of the distributions presented have been radiatively cor-

rected. These corrections are expected to be small for muons in our Q^2 and W range.

Our interest in looking at final states in deeply inelastic scattering is particularly motivated by the accessibility of final states to interpretation in terms of a simple quark-parton model. In such a model, as symbolically shown in Fig. 2, the final parton state is easily described: In the lab system, one parton absorbs all of the incident photon momentum; the remainder have low energy and will form the proton "debris." The struck parton is therefore strongly isolated in phase space from the remaining ones. This system must then somehow evolve into the hadrons we detect. Our failure so far to understand this process has made it difficult to put this model on a firmer footing. A detailed study of the final-state particles may help us to enhance the much needed understanding of this mechanism.

II. PROCEDURE

The inclusive hadron distribution describes the density in phase space for final-state hadrons. In the present experiment, since the "beam" particle is a virtual photon, the center-of-mass energy and projectile "mass" are continuous variables. The final-state particle density will thus be presented for bins of W and Q^2 defined by the initial photon-proton system. We define

$$E \frac{dN(W, Q^2)}{d^3p} = \frac{1}{\sigma_{\text{tot}}(W, Q^2)} E \frac{d\sigma(W, Q^2)}{d^3p},$$

where the cross sections are integrated over a range ΔQ^2 and ΔW with weighted average values given by Q^2 and W .

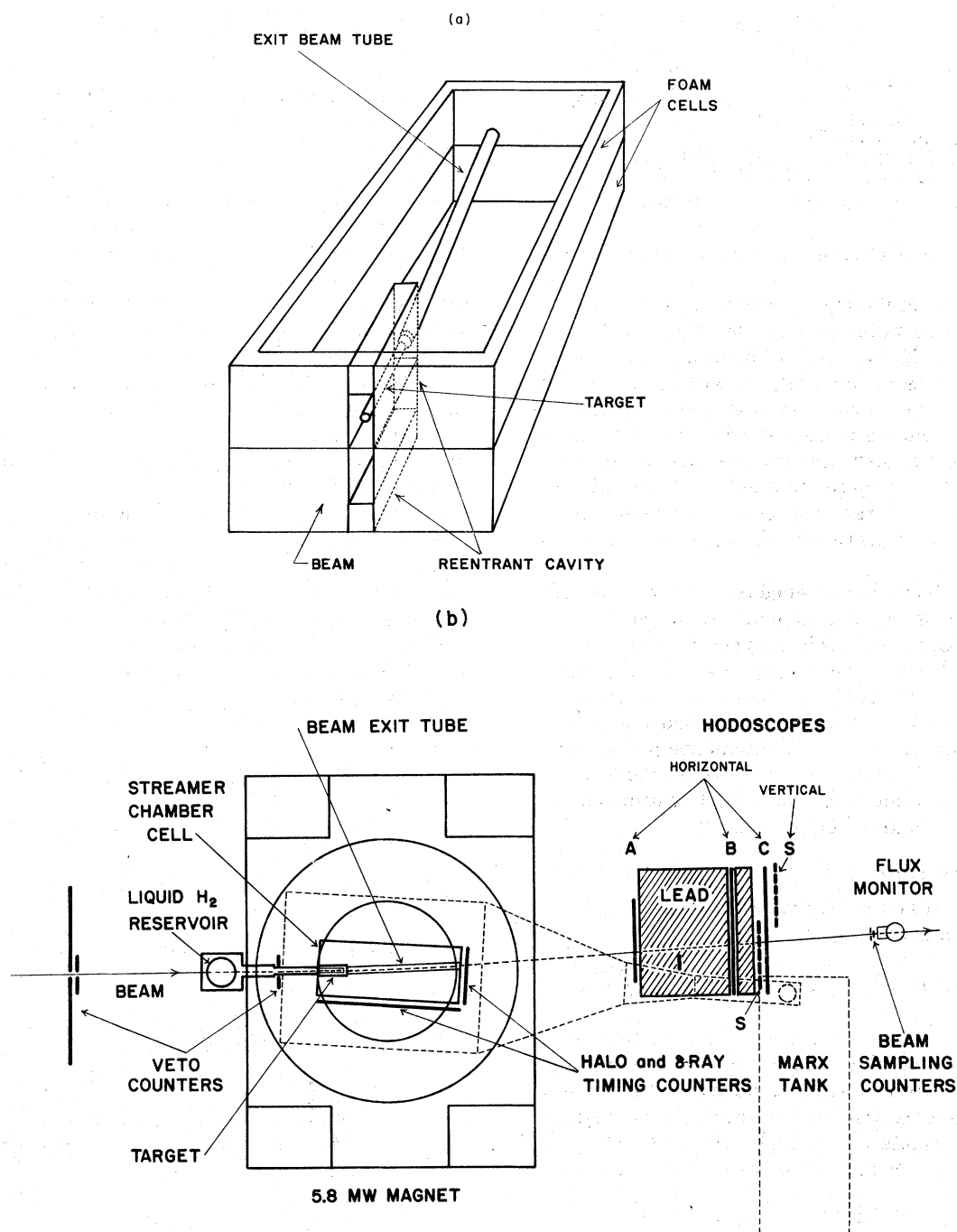


FIG. 1. Plan view of the detection apparatus. The hodoscopes and lead wall have openings for the unscattered beam and the streamer-chamber pulsing system. Also shown is a perspective view of the $2 \times 0.8 \times 0.6$ -m³ streamer chamber.

As is customary for hadronic reactions, we will display the distributions in terms of longitudinal and transverse variables, as defined with respect to the virtual-photon-proton collision axis. Since the muon scatter plane defines a direction in space

to which the virtual-photon polarization is correlated, the hadron distributions can also depend on the azimuthal angle ϕ of the hadron momentum vector with respect to the muon scatter plane. We find little dependence on this angle. For example,

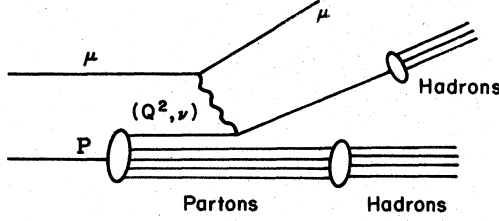


FIG. 2. Parton diagram for deeply inelastic process.

looking only at the high-momentum tracks in each event, the resulting particle density is uniform in azimuth to 10% (see Sec. VI below). Since the dependence on ϕ is so small, we will integrate all distributions over this variable. When calculating energies from the measured momenta, we assume that all rest masses are m_π . Keeping the limitation of this assumption in mind, we draw inferences from our fractional energy distributions mainly for fast particles which are predominantly pions.

All distributions are weighted for the muon and hadron detection efficiencies.¹ These weights are typically 2 and 1.15, respectively. For forward-produced hadrons (large momentum along incident photon direction) the hadron weight is typically 1.05; for very backward hadrons the weights are large. Distributions are presented only in the phase-space region where the hadron detection efficiency can be reliably estimated, i.e., to an accuracy better than 5%.

III. LONGITUDINAL-MOMENTUM DISTRIBUTIONS

Let us define the "structure function" for π^- production according to

$$F^{\pi^-}(x_F, W, Q^2) = \int E \frac{dN^{\pi^-}(W, Q^2)}{d^3p} \frac{dp_{\perp}^2 d\phi}{2\pi}.$$

It is the projection of the invariant inclusive π^- distribution onto the longitudinal-momentum variable. The distribution in longitudinal momentum is conveniently displayed in terms of the Feynman scaling variable x_F .³ It is defined as $p_{\parallel}^*/p_{\parallel\max}^*$, in the γ_p -proton center-of-mass system, where

$$p_{\parallel\max}^* \approx \frac{W^2 - m_p^2}{2W}.$$

This function is shown in Fig. 3. To study kinematical trends, we break up the entire W range above 2.8 GeV into two Q^2 ranges:

$$\left. \begin{array}{l} 0.5 \leq Q^2 \leq 1.2 \text{ GeV}^2 \\ 1.2 \leq Q^2 \leq 4.5 \text{ GeV}^2 \end{array} \right\}, \quad 2.8 \leq W \leq 4.7 \text{ GeV}.$$

Alternatively, we subdivide the full Q^2 range above 0.5 GeV² into two regimes:

$$\left. \begin{array}{l} 2.8 \leq W \leq 3.6 \text{ GeV} \\ 3.6 \leq W \leq 4.7 \text{ GeV} \end{array} \right\}, \quad 0.5 \leq Q^2 \leq 4.5 \text{ GeV}^2.$$

Also shown are the analogous distributions measured in π^-p and π^+p reactions at comparable center-of-mass energies,⁴ and the distribution seen in μp scattering at much higher W .⁵

We observe the following principal features:

1. Our data vary little if at all with W and Q^2 in the momentum region $-0.2 \leq x_F \leq 0.6$. For $x_F < -0.2$, a rise with W may be indicated; however, the statistical errors here are large. The changes at large positive x_F may be mainly reflecting the changes in the elastic ρ^0 polarization.⁶
2. For negative x_F , our distributions look similar to $\pi^-p \rightarrow \pi^- + X$.
3. The maximum value of $F^{\pi^-}(x_F, W, Q^2)$ occurs at $x_F \approx 0.05$, a feature equally observed for both pion-initiated reactions.
4. The data from Fermilab tend to lie ~20 to 50% above the data at our energies, except at very large x_F where errors are large. The former data correspond to very large values of the scaling variable $\omega' = (W^2 + Q^2)/Q^2$, where $F^{\pi^-}(x_F, W, Q^2) \approx F^{\pi^+}(x_F, W, Q^2)$.⁵ In Sec. V we discuss in detail the ω' dependence; here, we only note that a parton model lets us expect

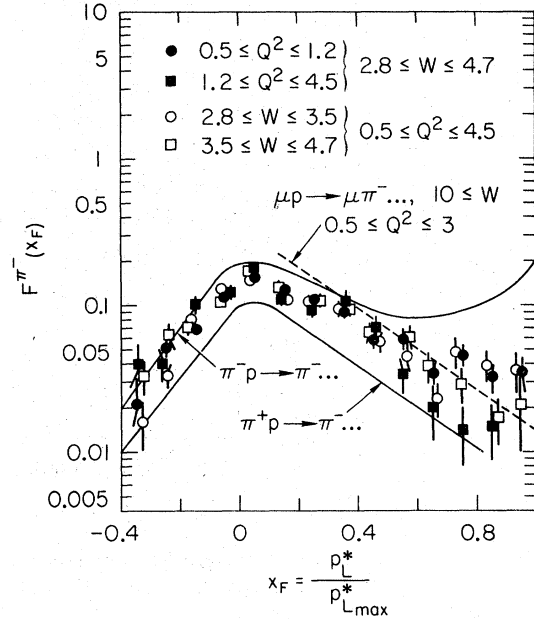


FIG. 3. Structure function defined as $(1/\pi)(E/P_{\max}^*) \times (1/\sigma)(d\sigma/dx_F)$, for negative hadrons, assumed to be pions. Also shown are analogous data for pion-initiated reactions (Ref. 4) (error bars, not shown, are small) and higher-energy muon production (Ref. 5) (error bars, not shown, are comparable to those for our points).

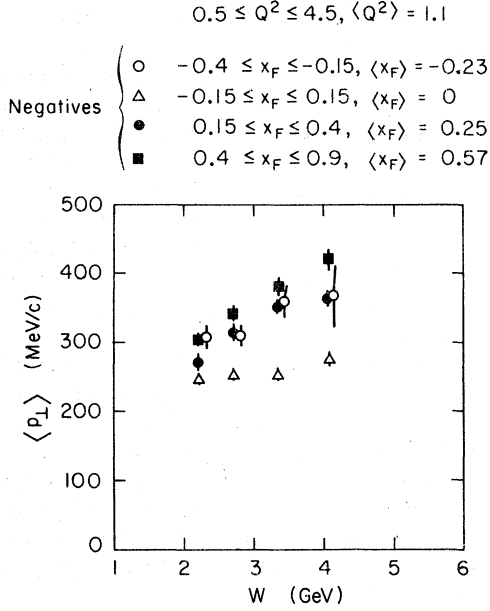


FIG. 4. Average transverse momentum for negative hadrons vs W for given regions in the scaling variable $x_F = p_\perp^*/p_{\parallel \max}^*$. Axis used to define $p_\perp$ is the virtual-photon-proton collision axis.

$$F^{\pi^-}(x_F, W, Q^2) + F^{\pi^+}(x_F, W, Q^2)$$

to be independent of ω' . Since we see a charge ratio ≈ 2 (cf. Sec. V) at our values of ω' , while expecting an approach to unity for $\omega' \rightarrow \infty$, we foresee an increase of $F^{\pi^-}(x_F, W, Q^2)$ by about 50% as ω' gets large. The Fermilab results are qualitatively in agreement with these expectations.

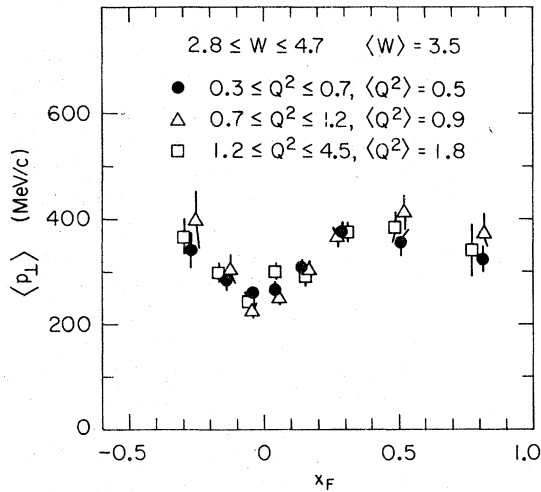


FIG. 5. Average transverse momentum for negative hadrons vs x_F for three bins in Q^2 . All data with W between 2.8 and 4.7 GeV are summed over.

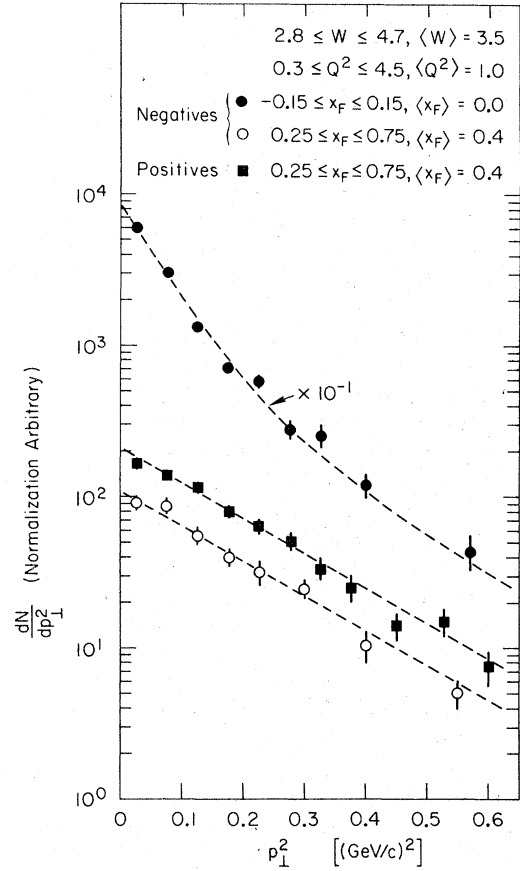


FIG. 6. Detailed $p_\perp^2$ distribution for negatives in two bins in x_F . Data for positives, also shown, are for large x_F only. Relative normalization between points is correct; thus the h^+/h^- ratio is ≈ 2 at all $p_\perp$ in the large- x_F kinematic region. (Small- x_F data have been divided by 10.) Curves shown are given by $N_0(e^{-1.5 \cdot 3 p_\perp^2} + 0.10 e^{-5 \cdot 3 p_\perp^2})$ for small x_F , and $N_1 e^{-5 \cdot 3 p_\perp^2}$ for large x_F .

To complement this discussion, we will plot the final-state distributions in terms of the related variable, $z \equiv E^h/\nu$, in Sec. V below. In the photon-fragmentation region, $z = x_F$ in the limit where m_p and $p_\perp$ can be ignored. The use of the z variable has the advantage that it is Lorentz-invariant, and that proton contamination at large values of z is smaller than for large x_F values. We thus will discuss distributions for positive particles in terms of z only. The whole region $x_F < 0$ is mapped into $z \approx 0$ at high energy.

IV. TRANSVERSE-MOMENTUM DISTRIBUTIONS

The average transverse momentum for negative final-state hadrons can depend on all the variables W, Q^2, x_F . To display the dependence on these variables we show, in Fig. 4, $\langle p_\perp \rangle = \int p_\perp (dN/dp_\perp^2 dp_\perp^2)$ vs W in four x_F bins; in Fig. 5, $\langle p_\perp \rangle$ vs x_F in three Q^2 bins. Here are the main fea-

tures:

(1) $\langle p_\perp \rangle$ is an increasing function of W . However, for small x_F values, the increase with W is much slower than at large x_F : In this regime $\langle p_\perp \rangle$ may be close to constant.

(2) $\langle p_\perp \rangle$ does not depend on Q^2 , for any value of x_F . This disagrees by about 2.5 standard deviations with the results quoted in Ref. 7, which show an increase of $\langle p_\perp \rangle$ with Q^2 at large x_F .

(3) $\langle p_\perp \rangle$ has a minimum for $x_F \approx 0$. It is approximately symmetric around zero in the x_F regime easily accessible to this experiment; we did not include the region $x_F < -0.4$ since the π^- detection efficiency becomes small here, and varies strongly with $p_\perp$.

To understand the source of the dip in $\langle p_\perp \rangle$ vs x_F , we plot, in Fig. 6, the detailed distribution $dN/dp_\perp^2$, for both small and large values of x_F . The range in x_F for each bin is chosen to correspond to a region over which $\langle p_\perp \rangle$ is roughly constant (see Fig. 5).

The $p_\perp^2$ distribution for small x_F has been fitted to a curve of the form

$$\frac{d\sigma}{dp_\perp^2} = N_0(e^{-B_1 p_\perp^2} + \alpha e^{-B_2 p_\perp^2}).$$

We find $B_1 = 15.5 \pm 0.5$, $B_2 = 5.3 \pm 1.0$, $\alpha = 0.10 \pm 0.02$, with a χ^2 parameter of 1.1 per degree of freedom. The distribution at large x_F can be described by one exponential in $p_\perp^2$ with a slope of 5.3 ($=B_2$ above). We also show the distribution for *positive* hadrons at large x_F , since the proton contamination can be assumed to be small here. These data are seen to be well represented by a similar exponential of slope B_2 . The minimum for $\langle p_\perp \rangle$ vs x_F can therefore, for negatives, be attributed to the part of the distribution with a steep slope.

These features can be compared to data available from the reaction $\pi^+ p \rightarrow \pi^- + X$.⁸ Here, again, the π^- distribution can be fitted in terms of two exponentials, but the fraction of π^- 's falling into the less steep exponential is 0.2, as compared to 0.38 ± 0.07 for our muon data (integrated over all x_F).

Reference 8 also contains an analysis of inclusive ρ^0 production. The inclusive ρ^0 cross section is large, $\rho^0/\pi^- \approx 0.2$, and the $\langle p_\perp \rangle(x_F)$ for the ρ^0 's shows no dip in the central region. Its value is consistent with the large x_F value of $\langle p_\perp \rangle$ for π^- seen in our experiment as extrapolated to a center-of-mass energy ≈ 5.7 GeV. These data suggest that the dip in $\langle p_\perp \rangle$ at small $|x_F|$, for negative pions (Fig. 5), may be largely due to the production of mesonic resonances whose subsequent decay yields the π 's.

These π^- tend to populate mainly the region of small x_F , since the parent momentum must be shared by two or more particles. For the low-mass resonances, ρ , ω , η , the resulting $p_\perp$ distri-

bution for pions is steeper than the parent distribution if the parent is produced with an $e^{-5.3p_\perp^2}$ distribution. The increase with W of the π^- spectrum resulting from the decay of these mesons is also much slower than the increase of the parent distribution, which would explain the different W dependence of $\langle p_\perp \rangle$ for large and small $|x_F|$. If this picture were true, "direct" π^- and, by inference, π^+ production, at our energies, might look like $e^{-5.3p_\perp^2}$ irrespective of x_F . We note that a large inclusive ρ^0 cross section has also been seen in photoproduction.⁹

For an alternative interpretation, we note that quark-counting rules can be translated into the prediction of typical power behavior of the transverse-momentum dependence of secondary hadron production. In this framework, π production from the $q\bar{q}$ sea may explain the steep central component of our observed $p_\perp$ dependence whereas the $e^{-5.3p_\perp^2}$ component remains identified with π production off valence quarks.¹⁰

V. ENERGY DISTRIBUTIONS FOR HADRONS

A. Energy-distribution functions for pions

The structure of an interaction yielding a number of final-state particles can also be usefully described in terms of the values for the variable $z \equiv E^h/\nu$; this simply indicates how much of the available energy any particular hadron (or hadron resonance) receives.¹¹ Thus, a diffractively produced system, such as in $\gamma N \rightarrow \rho^0 N$, would be expected to have $z_\rho \approx 1$ (i.e., $z_{\pi^+} + z_{\pi^-} \approx 1$).

The resulting distribution function

$$F_{\mu p}^{(h)} = \frac{1}{\sigma_{\text{tot}}} z \frac{d\sigma^{(h)}}{dz},$$

where $\sigma^{(h)}$ is the inclusive cross section for the production of h -type particles, is frequently called a hadronic "structure function," although it is not related to the well-known nucleon structure functions. Clearly, these functions are interpretable in terms of models such as diffractive or parton-scattering pictures. Thus, elastic ρ^0 production could lead to a distribution $F_{\mu p}^{(\rho)}(z)$ displaying a spike at $z_\rho \approx 1$. In Fig. 7, we show plots of these functions for both negative [7(a)] and positive [7(b)] particles; it is evident that breaking the data up into three bins of the scaling variable ω' (and thereby of Q^2 , for given W) does *not* reveal any marked dependence of the energy distribution function on the mass-squared of the incident photon. This observation is parallel to what we saw in the x_F distribution in Sec. III. Note that, to exclude data samples where π^+ and p are both large, $F_{\mu p}^{(h^*)}$ is plotted for $z > 0.3$ only.

To compare the fragmentation of spacelike and

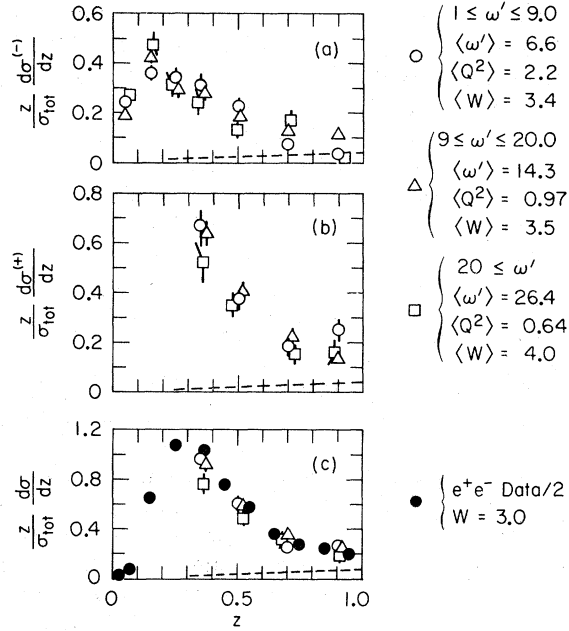


FIG. 7. Inclusive distributions in the fractional energy variable z for positive and negative hadrons. Also shown is the analogous distribution for hadrons seen in e^+e^- annihilation. Dashed curve is elastic ρ^0 contribution.

timelike photons, we plot, in Fig. 7(c), our full data sample ($F_{\mu p}^{(h+)} + F_{\mu p}^{(h-)}$) together with the corresponding energy distribution function from e^+e^- annihilation at $\sqrt{s} = 3$ GeV. While our comparison is limited to z values above 0.3, the close similarity of the $F_{\mu p}$ and $F_{e^+e^-}$ functions appears striking.¹² Note that, for proper normalization of the comparison, we had to plot $\frac{1}{2}F_{e^+e^-}$, since there are *two* leading partons in the annihilation process

(vs one in electroproduction).

B. Quark-parton-model formulation of inclusive distributions

In the quark-parton model, the distribution functions for hadrons which carry a finite fraction of the incident virtual photon's momentum have been described by a set of functions which give the probability for each type of parton to absorb the virtual photon multiplied by another set describing the hadron distributions resulting from each parton type. For the positively (or negatively) charged hadrons, we integrate over transverse momenta to find the inclusive distribution,

$$\frac{z}{\sigma(\omega')} \frac{d\sigma^+}{dz} = \sum_i z P_i(\omega') D_i^{h+}(z),$$

where the sum runs over quark and antiquark types i .¹³ $P_i(\omega')$, the probability of finding a quark of type i as the scattered quark, can be obtained from parton-model fits to $\nu W_2(\omega')$. For this purpose, we use the modified Kuti-Weisskopf quark-parton distributions used in Ref. 13, and the notation u, d, s ($\bar{u}, \bar{d}, \bar{s}$) for the three quark types and their antiparticles. The number of independent "quark-fragmentation functions" D_i^{h+} or D_i^{h-} can be reduced considerably. C invariance implies $D_u^{h+} = D_u^{h-}$, $D_d^{h+} = D_d^{h-}$, $D_s^{h+} = D_s^{h-}$, $D_u^{h-} = D_u^{h+}$; if we assume that only pions are produced, isospin symmetry gives $D_u^{h+} = D_d^{h+}$, $D_u^{h-} = D_d^{h-}$.

C. Extraction of quark-fragmentation functions

We can now use our data to extract these quark-fragmentation functions. We assume that only nonstrange quarks are scattered by the virtual photon, a rather good assumption in the quark-parton model of the proton, and also that pions

TABLE I. Measured values of energy-distribution functions $F_{\mu p}^{(hi)}$, defined in text, for bins in the scaling variable ω' . Data for positives are only given for $z > 0.3$ because of a large proton contamination at smaller z . $F_{\mu p}^{(hi)}$ is the sum of the distributions for positive and negative particles.

ω'	Distribution function	z range						
		0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.6	0.6 to 0.8	0.8 to 1
$\omega' \leq 9$	$F_{\mu p}^{(h+)}(\omega', z)$	0.24 ± 0.02	0.36 ± 0.03	0.34 ± 0.04	0.31 ± 0.05	0.23 ± 0.03	0.074 ± 0.02	0.035 ± 0.015
$\langle \omega' \rangle = 6.6$	$F_{\mu p}^{(h+)}(\omega', z)$				0.66 ± 0.07	0.37 ± 0.04	0.18 ± 0.04	0.25 ± 0.05
	$F_{\mu p}^{(h)}(\omega', z)$				0.97 ± 0.09	0.60 ± 0.05	0.25 ± 0.04	0.29 ± 0.05
$9 \leq \omega' \leq 20$	$F_{\mu p}^{(h+)}(\omega', z)$	0.19 ± 0.01	0.43 ± 0.03	0.29 ± 0.03	0.28 ± 0.03	0.19 ± 0.02	0.14 ± 0.02	0.12 ± 0.02
$\langle \omega' \rangle = 14.3$	$F_{\mu p}^{(h+)}(\omega', z)$				0.64 ± 0.05	0.41 ± 0.03	0.23 ± 0.03	0.14 ± 0.02
	$F_{\mu p}^{(h)}(\omega', z)$				0.92 ± 0.06	0.60 ± 0.04	0.37 ± 0.04	0.26 ± 0.03
$20 \leq \omega'$	$F_{\mu p}^{(h+)}(\omega', z)$	0.27 ± 0.03	0.48 ± 0.05	0.31 ± 0.05	0.24 ± 0.05	0.13 ± 0.03	0.17 ± 0.04	0.015 ± 0.015
$\langle \omega' \rangle = 26.4$	$F_{\mu p}^{(h+)}(\omega', z)$				0.52 ± 0.08	0.35 ± 0.05	0.15 ± 0.04	0.16 ± 0.05
	$F_{\mu p}^{(h)}(\omega', z)$				0.76 ± 0.09	0.48 ± 0.06	0.32 ± 0.06	0.18 ± 0.05

dominate the final-state hadron distributions. The effect of the latter assumption is examined in the next section of the paper. We now write the inclusive distributions in terms of only two quark-fragmentation functions:

$$F_{\mu p}^{(h+)}(\omega', z) = z \{ P_u(\omega') D_u^{h+}(z) + [1 - P_u(\omega')] D_u^{h-}(z) \},$$

$$F_{\mu p}^{(h-)}(\omega', z) = z \{ P_u(\omega') D_u^{h-}(z) + [1 - P_u(\omega')] D_u^{h+}(z) \}.$$

These fragmentation functions have not been calculable in any model as yet; to test their fundamental significance, distributions measured in several different processes, or in the same process but in different kinematic regions, should be compared.

To test these ideas, let us look again at Figs. 7(a), 7(b), and 7(c). The dashed curves shown in Fig. 7 give the average "elastic" ρ^0 contribution to the inclusive cross section; we single it out because it is not clear whether it should be included or not when we make parton-model calculations and comparisons. Note that the distribution for each charge is expected to scale (i.e., be a function of z only) only at fixed ω' . However, the sum over both positives and negatives is expected to scale and be independent of ω' , W , and Q^2 . It is equal to

$$F_{\mu p}^{(h)} = z [D_u^{h+}(z) + D_u^{h-}(z)].$$

The comparison in Fig. 7(c) (and Table I) therefore tests the quark-hadron fragmentation idea without the necessity of knowing the exact quark distributions in the proton. The data for all three bins in ω' are in excellent agreement. Also shown are data for final-state hadrons (summed

over both charges) produced in e^+e^- annihilation at a center-of-mass energy = 3.0 GeV,¹⁴ just below the "new resonance" region. A parton model for this distribution gives

$$\frac{1}{2} F_{e^+e^-}^{(h)} = \frac{1}{2} \left(\frac{z}{\sigma_{\text{tot}}} \frac{d\sigma^h}{dz} \right)$$

$$= \frac{1}{2} z \sum_{i,h} [P_i D_i^h(z) + P_{\bar{i}} D_{\bar{i}}^h(z)],$$

where P_i is proportional to the quark charge squared, and $z = E_{\text{had}}/E_{\text{had}}^{\text{max}}$. Ignoring a small possible difference due to the production of strange quarks, this should be equal to $z[D_u^{h+}(z) + D_u^{h-}(z)]$. The agreement between the fragmentation functions seen in these two processes is excellent.

For the modified Kuti-Weisskopf quark-parton distributions in the target proton, $P_u(\omega')$ is equal to 0.80 for $\langle\omega'\rangle = 6.6$, 0.77 for $\langle\omega'\rangle = 14.3$, and 0.73 for $\langle\omega'\rangle = 26.4$. Note that the approximate independence of ω' for $F_{\mu p}^{(h-)}$ and $F_{\mu p}^{(h+)}$ is due in this model to the small variation of $P_u(\omega')$ for our data. Using these values of $P_u(\omega')$ we tabulate in Table II the extracted values for $zD_u^{h-}(z)$ and $zD_u^{h+}(z)$ for the first two ω' bins. The third bin has too large a statistical error and low Q^2 values, and is therefore not given. The two bins give values of $zD_u^{h+}(z)$ in excellent agreement. The values for $zD_u^{h-}(z)$ are typically 1.5 to 2 standard deviations apart at each z value. The extracted value of this function is much more sensitive than $zD_u^{h+}(z)$ to the value of $P_u(\omega')$. Averaging the two ω' bins, we obtain a charge ratio,

$$\frac{D_u^{h+}(0.4 \leq z \leq 0.8)}{D_u^{h-}(0.4 \leq z \leq 0.8)} = 3.2 \pm 0.6,$$

in reasonable agreement with the result of Ref. 13.

TABLE II. Extracted quark-fragmentation functions using data of Table I and the modified Kuti-Weisskopf quark distributions for the proton. In the calculation all particles are assumed to be pions. The limitations of this assumption are discussed in the text. Values calculated for the two ω' bins are expected to agree in the quark-parton model.

		z range			
		0.3 to 0.4	0.4 to 0.6	0.6 to 0.8	0.8 to 1.0
$\langle\omega'\rangle = 6.6$	$zD_u^{h-}(z)$	0.19 ± 0.06	0.18 ± 0.04	0.04 ± 0.03	-0.04 ± 0.03
	$zD_u^{h+}(z)$	0.78 ± 0.09	0.42 ± 0.05	0.22 ± 0.05	0.32 ± 0.07
$\langle\omega'\rangle = 14.3$	$zD_u^{h-}(z)$	0.13 ± 0.04	0.10 ± 0.03	0.09 ± 0.03	0.11 ± 0.03
	$zD_u^{h+}(z)$	0.79 ± 0.07	0.50 ± 0.05	0.27 ± 0.05	0.15 ± 0.03
Average of above	$zD_u^{h-}(z)$	0.15 ± 0.04	0.13 ± 0.03	0.07 ± 0.02	0.04 ± 0.02
	$zD_u^{h+}(z)$	0.79 ± 0.06	0.46 ± 0.04	0.25 ± 0.04	0.18 ± 0.03
Ratio: $D_u^{h-}(z)/D_u^{h+}(z)$		0.19 ± 0.05	0.28 ± 0.06	0.38 ± 0.08	0.22 ± 0.10

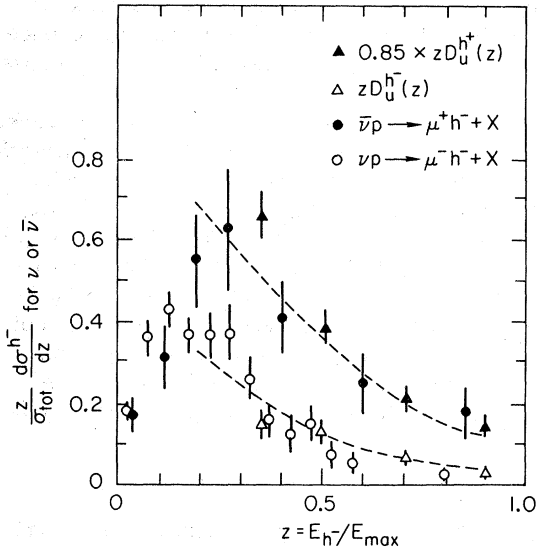


FIG. 8. Inclusive distributions for negative hadrons seen in neutrino and antineutrino scattering compared to the extracted quark-fragmentation functions using our data. Dashed curves are a prediction of L. M. Sehgal [Nucl. Phys. B90, 471 (1975)] using data of Ref. 13. See Ref. 17 for details.

D. Inclusion of particle types other than pions

We now attempt to correct for the approximate assumption that all final-state hadrons are pions. Using the data of Ref. 15, we estimate the proton and K^+ content of our h^+ sample.¹⁶ The result is

$$\frac{p}{\pi^+} = 0.2 \pm 0.1, \quad \frac{K^+}{\pi^+} = 0.2 \pm 0.1.$$

The calculation of $zD_u^{h^+}(z)$ in the last section is very insensitive to the above as long as we understand h^+ to include all positive particles, not just π^+ . Note, however, that the fractional proton component falls with W . If this trend continues up to Fermilab energies, the asymptotic value of $zD_u^{h^+}(z)$ is expected to be $\sim 15\%$ smaller than the value measured in this experiment. Based on these numbers, we have recalculated $zD_u^{h^-}(z)$, using for $P_u(\omega')$ the values previously quoted; we find that the presence of protons and kaons increases our value of $D_u^{h^-}(z)$ by about 25%, or one standard deviation, for all z values in Table II. These corrections for positive as well as negative hadrons change the charge ratio for pions to

$$\frac{D_u^{\pi^+}(0.4 \leq z \leq 0.8)}{D_u^{\pi^-}(0.4 \leq z \leq 0.8)} \approx 1.8 \pm 0.6.$$

This is considerably smaller than the charge ratio for the full sample of charged hadrons.

Our values of $zD_u^{h^-}(z)$ and $zD_u^{h^+}(z)$ should also be able to describe the corresponding distributions

observable in ν and $\bar{\nu}$ scattering. The relevant comparison is shown in Fig. 8, where we include a 15% reduction of $zD_u^{h^+}(z)$ to approximately subtract out the proton component.¹⁷ A similar distribution seems to describe the fractional momentum distribution of high- $p_\perp$ charged hadrons seen in p - p collisions yielding a large-transverse-momentum π^0 .¹⁸ This common distribution, for νp , $\bar{\nu} p$, $e^+ e^-$, μp , and certain hadronic interactions may point to a common mechanism in all these highly inelastic processes, as realized in the quark-parton model. We note that the existence of a large forward charge ratio as observed in these data, points in the same direction.¹³

E. Correlations of event topology and charge ratio

Figure 9 shows the distribution in the variable z for the fastest particle for each event, yielding one entry each for the quantity $z_{\max}$. We divided the distribution into contributions from positive and negative particles, assigning pion rest masses to all of them. For $z_{\max} \geq 0.4$, the distribution for positives is uniformly a factor of 2 larger than that for negatives, irrespective of z . Recall that the transverse momentum distribution for high- z positives and negatives, which we showed in Fig. 6, is similarly structureless, also giving a constant charge ratio of 2 for all $p_\perp$ values. The low- $z_{\max}$ region has a large excess for positives, stemming most probably from proton misidentification.

How does this remarkable regularity come about? To study this question, we break these

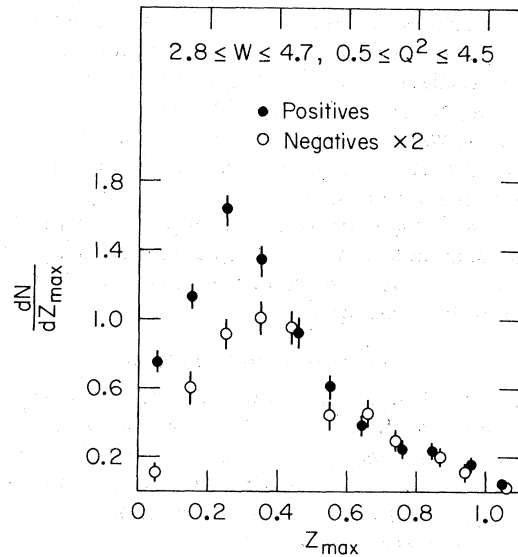


FIG. 9. Distribution in fractional energy for the fastest track in each event. Data for negatives have been multiplied by a factor of 2. All tracks are assumed to be pions when calculating energies. Points with $z_{\max} > 1$ are spillover due to finite resolution.

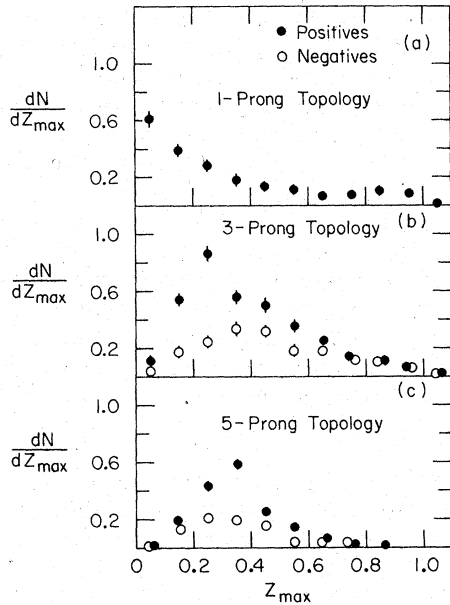


FIG. 10. Distribution in fractional energy for the fastest track in each event broken down by event topology. Normalization is such that integral over sum of all curves is equal to 1.

distributions down into contributing parts according to the event topology defined by the number of charged final-state hadrons. This is shown in Fig. 10, where we see that the nearly constant charge ratio is not reproduced for each topology. For $z_{\max} \geq 0.7$, the ratio for three-prong events alone is ≈ 1 . The overall charge ratio of two for all topologies is due to a collusion between the one-prong contribution, which is equal to the positive charged three-prong contribution in this $z_{\max}$ range. For $0.4 \leq z_{\max} \leq 0.7$, on the other hand, the one-prong contribution makes up a much smaller part of the total, and the charge ratio of two reflects a similar value for both three-prong and five-prong events. An attempt to interpret this phenomenon in terms of individual contributing diagrams remains to be undertaken, and should yield telling insights.

VI. AZIMUTHAL DISTRIBUTIONS

As a final topic, we show the inclusive distribution in the angle ϕ , defined for each hadron as the angle between the muon scatter plane and the plane containing the virtual photon and hadron. The most general form for this distribution, assuming one-photon exchange, is

$$\frac{dN}{d\phi} = N_0(1 + \alpha \cos\phi + \beta \cos 2\phi),$$

where α, β, N_0 are functions of z . The $\cos 2\phi$ term comes from a correlation of the hadron di-

rection with the transverse photon polarization, while the $\cos\phi$ term comes from an interference between transverse and longitudinal photon cross sections.

Figure 11(b) shows the ϕ distribution for tracks with $z > 0.4$. These are in the photon-fragmentation region and most likely show a correlation with the photon polarization. The best fit for $dN/d\phi$ gives $\alpha = 0.02 \pm 0.05$ and $\beta = 0.09 \pm 0.05$. Figure 11(a) shows the ϕ distribution for tracks with small z . This distribution is consistent with being flat in ϕ . We thus see that polarization effects are not strongly indicated by the ϕ distributions. Our procedure to integrate over the ϕ variable, as indicated in Sec. II, therefore appears to be justified.

VII. CONCLUSION

In the foregoing, we have presented the distributions of hadrons emerging from inelastic muon-proton collisions according to several energy and momentum variables. While doing this, we variously binned the data in terms of the hadronic energy W and virtual-photon mass-squared Q^2 , which together make up the scaling variable ω' .

We find the distribution in the fractional longitudinal-momentum variable x_F largely analogous so that observed in purely hadronic interactions, but with the express absence of a leading-particle

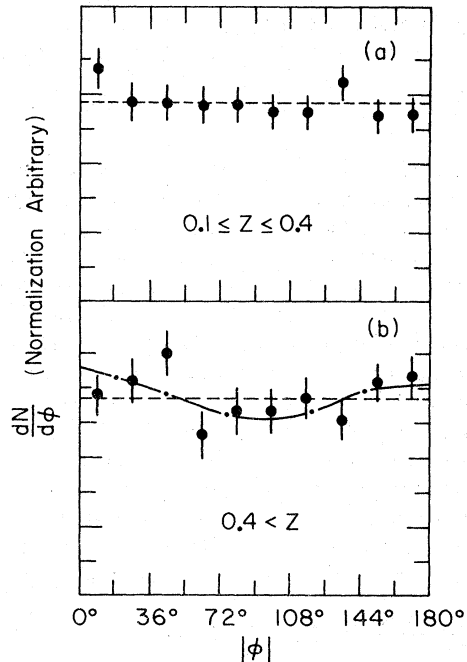


FIG. 11. ϕ distribution for tracks with small z (0.1 to 0.4) and large z ($0.4 < z$). Dashed curves shown are best fits using $dN/d\phi = \text{constant}$. Dashed-dotted curve is $dN/d\phi = N_0(1 + 0.02 \cos\phi + 0.09 \cos 2\phi)$.

effect. Longitudinal-momentum conservation and the observed¹ similarity of mean charged-hadron multiplicities make this result plausible.

The transverse-momentum distribution appears to show an interesting two-component behavior, with a universal slope describing successfully the $p_{\perp}$ dependence of all but the small- $p_{\perp}$, low- x_F regime. This regime may well be governed by the decay of heavier mesons; alternately, the production from the quark-antiquark sea may populate this region. Remarkably, no Q^2 trend is observed for the average transverse momentum.

Distributions in the fractional-energy variable z permit a linking of our data to quark-fragmentation functions which are also accessible from ν ,

$\bar{\nu}$ -nucleon scattering and from e^+e^- annihilation. The fact that our results are closely compatible with those data, and exhibit scaling behavior, makes a strong case for a parton-model approach to the three processes.

The leading-particle distribution in z shows a charge ratio h^+/h^- of almost exactly 2 for the entire range $0.4 \leq z \leq 0.9$. This ratio is seen to be due to an intricate conspiracy of the individual prong cross sections. The same ratio is also exhibited by the entire $p_{\perp}$ dependence of higher- x_F hadrons; it is the subject of an upcoming communication from this collaboration.¹⁹ Parallel data from muon-neutron collisions are also in preparation.

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