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Quantum corrections to classical confinement*

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We study two models involving charged scalar fields in one space dimension, which have baglike properties at the classical level. Following conventional renormalization procedures, we find that either the first model becomes a free theory or else its spectrum becomes unbounded below depending on how one renormalizes. The second model also becomes a free theory with a particular choice of renormalization. In no case are we able to retain the classical confinement, although we can restore interactions by adding suitable counterterms to the Hamiltonian.

I. INTRODUCTION

In this note we discuss the quantum properties of two models which seem to possess baglike behavior at the classical level. The first of these models has been suggested by Rebbi[1]; the second was the subject of a previous article.[2]

We shall find in the case of model I that, depending on the method of renormalization, we are faced with two alternatives: Either the model reduces to a free field theory, or else the energy spectrum is unbounded below. Model II does not suffer from the second of these diseases, but any practical renormalization scheme seems to lead to a free field theory. For either model, straightforward extensions to renormalizable interacting field theories of conventional polynomial type can be made.

For later use, we present in the remainder of this section a brief review of normal ordering, following a slightly less general treatment given by Coleman.[3] In Sec. II we shall discuss model I, and in Sec. III, we discuss model II. In Sec. IV we offer a few conclusions.

Both of the models involve a charged scalar field in one space dimension and one time dimension. The restriction to 1+1 dimensions is not essential for the existence of baglike properties in the classical theory; however, it does greatly facilitate our study of the quantum theory, for the principal reason that once the Hamiltonian has been normal-ordered, no further ultraviolet infinities can arise.[3]

Let $\pi(x,t)$ be the momentum conjugate to $\phi(x,t)$:

$$[\phi(x,t),\pi(y,t)]=i\delta(x-y)\,. \tag{1}$$

Given an arbitrary symmetric positive-definite matrix, $G(x,y)$, we define

$$\phi^{(\pm)}(x)=\tfrac{1}{2}\left[\phi(x)\pm i\int dy\,G(x,y)\pi(y)\right], \tag{2}$$

$$\pi^{(\pm)}(x)=\tfrac{1}{2}\left[\pi(x)\mp i\int dy\,G^{-1}(x,y)\phi(y)\right]. \tag{3}$$

Normal ordering with respect to G means moving all (−) operators to the right, and is denoted, as usual, by : :.

It is then trivial to show that

$$\phi(x)\phi(y)=:\phi(x)\phi(y):+\tfrac{1}{2}G(x,y)\,, \tag{4}$$

$$\pi(x)\pi(y)=:\pi(x)\pi(y):+\tfrac{1}{2}G^{-1}(x,y)\,. \tag{5}$$

The most familiar case is normal ordering with respect to a mass μ, in which case

$$G^{-1}=(-\nabla^2+\mu^2)^{1/2}\,. \tag{6}$$

The generalization to a charged field is straightforward. If

$$\phi^\dagger(x)\phi(x)=:\phi^\dagger(x)\phi(x):+\Delta(x) \tag{7}$$

[here $\Delta(x)$ is just shorthand for $\frac{1}{2}G(x,x)$], then we have the general normal-ordering formula:

$$F(\phi^\dagger\phi)=:\exp\left(\Delta\frac{\partial}{\partial\phi^\dagger}\frac{\partial}{\partial\phi}\right)F(\phi^\dagger\phi):. \tag{8}$$

One can use this to derive a result which will be important below, namely

$$\exp(\alpha\phi^\dagger\phi)=\frac{1}{1-\alpha\Delta}:\exp\left(\frac{\alpha\phi^\dagger\phi}{1-\alpha\Delta}\right):. \tag{9}$$

II. GAUSSIAN MODEL

Rebbi has suggested a model involving a charged scalar field with the self-interaction

$$\mathcal{H}_I=B(1-e^{-\phi^\dagger\phi/\epsilon})\,. \tag{10}$$

The motivation is as follows: It is known[4] that classically the MIT bag can be obtained from a theory in which a field ϕ has a mass M outside a region R, and is massless inside R, provided one adds a constant energy density B inside R. The bag equations are recovered in the limit $M \to \infty$. Now when $\phi^\dagger\phi \ll \epsilon$, $\mathcal{H}_I \simeq (B/\epsilon)\phi^\dagger\phi$, whereas when $\phi^\dagger\phi \gg \epsilon$, $\mathcal{H}_I \simeq B$. The inside region is thus defined by the requirement that $\phi^\dagger\phi \gg \epsilon$, the outside region is defined by $\phi^\dagger\phi \ll \epsilon$, and there will be a transition region when $\phi^\dagger\phi \sim \epsilon$. Furthermore, the bag should be obtained as $\epsilon \to 0$, in which limit the outside mass (B/ϵ) goes to infinity.

We turn now to the quantum theory. The full Hamiltonian density is

$$\mathcal{H} = \pi^\dagger(x)\pi(x) + \nabla\phi^\dagger(x)\nabla\phi(x) + B_0(1 - e^{-\phi^\dagger(x)\phi(x)/\epsilon_0}) + \gamma_0 , \tag{11}$$

where B_0, ϵ_0, and γ_0 are unrenormalized parameters. With the aid of Eq. (9), we can display all the infinities explicitly by normal ordering with respect to an arbitrary mass μ:

$$\begin{aligned}\mathcal{H}(x) = {} & N_\mu[\pi^\dagger(x)\pi(x) + \nabla\phi^\dagger(x)\nabla\phi(x)] \\ & + \frac{1}{2\pi}\Lambda^2 + \frac{1}{4\pi}\mu^2 + \gamma_0 \\ & + B_0 - \frac{B_0\epsilon_0}{\epsilon_0 + \Delta_\mu} N_\mu\left[\exp\left(-\frac{\phi^\dagger(x)\phi(x)}{\epsilon_0 + \Delta_\mu}\right)\right] .\end{aligned} \tag{12}$$

Here Λ is a cutoff introduced via

$$\Delta_\mu = \frac{1}{4\pi}\int_{-\Lambda}^{\Lambda} dk \, \frac{1}{(k^2 + \mu^2)^{1/2}} \simeq \frac{1}{2\pi}\ln\frac{2\Lambda}{\mu} , \tag{13}$$

and N_μ means normal-ordered with respect to μ.

We have basically two choices in renormalizing $\mathcal{H}$: (i) $\epsilon_0 + \Delta_\mu \to \infty$ as $\Lambda \to \infty$ or (ii) $\epsilon_0 + \Delta_\mu$ remains finite as $\Lambda \to \infty$.

Case (i). There are still several choices for the renormalization program which differ in detail among themselves. However, the basic conclusions do not depend on these details, so for definiteness let us choose

$$\epsilon_0 = \kappa\Delta_\mu \tag{14}$$

where κ is a new, finite parameter. Then we have

$$\begin{aligned}\mathcal{H} = {} & N_\mu(\pi^\dagger\pi + \nabla\phi^\dagger\nabla\phi) \\ & + \left\{\frac{1}{2\pi}\Lambda^2 + \frac{1}{4\pi}\mu^2 + \gamma_0 + B_0\left(\frac{1}{\kappa+1}\right)\right\} \\ & + \frac{B_0\kappa}{(\kappa+1)^2}\left[\frac{1}{\Delta_\mu}N_\mu(\phi^\dagger\phi) + \left(\frac{-1}{2(\kappa+1)\Delta_\mu^2}\right)N_\mu(\phi^\dagger\phi)^2 + O\left(\frac{1}{\Delta_\mu^3}\right)\right] .\end{aligned} \tag{15}$$

γ_0 is chosen to make the terms in curly brackets vanish. In order to make the coefficient of $N_\mu(\phi^\dagger\phi)$ finite, we need to choose

$$B_0 = \beta\Delta_\mu , \quad \beta \text{ finite.} \tag{16}$$

However, the terms proportional to $(\phi^\dagger\phi)^2$ and still higher powers of $(\phi^\dagger\phi)$ then all tend to zero, leaving us with a free field theory.

We remark that by introducing counterterms we can have nontrivial structure in the theory. For example, if we add a term $M_0^2\phi^\dagger\phi$ to $\mathcal{H}$, we can let $B_0 = \beta'\Delta_\mu^2$, and use the freedom implicit in M_0^2 to absorb the resulting infinity in the $\phi^\dagger\phi$ term. This process can be continued indefinitely by adding more and more counterterms, but clearly the theory is then governed not by the original interaction but by the counterterms.

Case (ii). We let $\epsilon_0 + \Delta_\mu = \epsilon_R$, a finite parameter. We define

$$B_R = B_0\epsilon_0/\epsilon_R$$

and, with a suitable choice for γ_0, (12) takes the form

$$\mathcal{H} = N_\mu(\pi^\dagger\pi + \nabla\phi^\dagger\nabla\phi) + B_R(1 - N_\mu e^{-\phi^\dagger\phi/\epsilon_R}) . \tag{12$'$}$$

Clearly, all the infinities have been absorbed.

We investigate the spectrum of $\mathcal{H}$ by evaluating ${}_m\langle 0|\mathcal{H}|0\rangle_m$, where $|0\rangle_m$ is the vacuum appropriate to a free field of mass m. This matrix element is most easily evaluated by using the re-normal-ordering formulas which follow from (4), (5), and (9):

$$N_\mu(\pi^\dagger\pi + \nabla\phi^\dagger\nabla\phi) = N_m(\pi^\dagger\pi + \nabla\phi^\dagger\nabla\phi) + \frac{1}{4\pi}(m^2 - \mu^2) ,$$

and

$$N_\mu(e^{-\phi^\dagger\phi/\epsilon_R}) = \frac{\epsilon_R}{\epsilon_R + (\Delta_m - \Delta_\mu)} N_m \times \left[\exp\left(-\frac{\phi^\dagger\phi}{\epsilon_R + \Delta_m - \Delta_\mu}\right)\right] ,$$

from which we have

$${}_m\langle 0|\mathcal{H}|0\rangle_m = \frac{1}{4\pi}(m^2 - \mu^2) + \frac{B_R}{2\pi}\left(\frac{\ln(\mu/m)}{\epsilon_R + (1/2\pi)\ln(\mu/m)}\right) . \tag{17}$$

Here we have noted that

$$\Delta_m - \Delta_\mu = \frac{1}{2\pi}\ln(\mu/m) . \tag{18}$$

It is clear that we can make ${}_m\langle 0|\mathcal{H}|0\rangle_m$ arbitrarily negative by choosing m appropriately (except for the case $\epsilon_R = 0$, which is meaningless anyway). Hence the spectrum is unbounded below.

III. θ-FUNCTION MODEL

This model is defined by the Hamiltonian density

$$\mathcal{H}=\mathcal{H}_0+B-(B-M^2\phi^\dagger\phi)\theta(B-M^2\phi^\dagger\phi)\,, \tag{19}$$

where

$$\mathcal{H}_0=\pi^\dagger\pi+\nabla\phi^\dagger\nabla\phi\,.$$

We see that when $(\phi^\dagger\phi)<B/M^2$, the interaction is $M^2\phi^\dagger\phi$. When $\phi^\dagger\phi>B/M^2$, the interaction is the constant B, so that the inside region is defined by $\phi^\dagger\phi>B/M^2$ and the outside by $\phi^\dagger\phi<B/M^2$. Contrary to model I, the transition is sharp, occurring at $\phi^\dagger\phi=B/M^2$, which defines the boundary. Also, in distinction to model I, exact classical solutions are known which exhibit the confined baglike behavior in the limit of large M^2. These have been discussed in Ref. 2.

The interaction is easier to handle if we write it in terms of a Fourier transform:

$$-(B-M^2\phi^\dagger\phi)\theta(B-M^2\phi^\dagger\phi)=\frac{M^2}{2\pi}\int_{-\infty}^{\infty}\frac{d\lambda}{(\lambda+i\epsilon)^2}e^{-i\lambda B/M^2}e^{i\lambda\phi^\dagger\phi}\,. \tag{20}$$

We can normal order with respect to an arbitrary reference mass μ, using Eq. (9):

$$-(B-M^2\phi^\dagger\phi)\theta(B-M^2\phi^\dagger\phi)=\frac{M^2}{2\pi}\int_{-\infty}^{+\infty}\frac{d\lambda}{(\lambda+i\epsilon)^2}\frac{e^{-i\lambda B/M^2}}{1-i\lambda\Delta}:\exp\left(\frac{i\lambda\phi^\dagger\phi}{1-i\lambda\Delta}\right):. \tag{21}$$

Here

$$\Delta=\tfrac{1}{2}(-\nabla^2+\mu^2)^{-1/2}(x,x).$$

In order to develop a convenient renormalization procedure, we expand the Hamiltonian in a power series in $\phi^\dagger\phi$. After some routine manipulations, we find that

$$\mathcal{H}=:\mathcal{H}_0:+\sum_{n=1}^{\infty}c_n:(\phi^\dagger\phi)^n:\,, \tag{22}$$

where

$$c_n=\frac{(-1)^{n+1}M^2}{(2\pi i)\Delta^{n-1}n!}\int_{-\infty}^{\infty}dy\frac{(y+i\epsilon)^{n-2}}{(y+i)^{n+1}}e^{-iy\gamma} \tag{23}$$

with

$$\gamma\equiv B/M^2\Delta\,,$$

and where

$${}_\mu\langle 0|\mathcal{H}|0\rangle_\mu=\frac{1}{2\pi}\Lambda^2+\frac{1}{4\pi}\mu^2+M^2\Delta(1-e^{-\gamma}) \tag{24}$$

has been subtracted from $\mathcal{H}$.

Explicitly, we have from (23)

$$c_1=M^2[1-(1+\gamma)e^{-\gamma}]\,, \tag{25}$$

and, for $n\geq 2$,

$$c_n=\frac{(-1)^{n+1}M^2}{(n!)^2\Delta^{n-1}}\left(\frac{d}{d\gamma}\right)^{n-2}(\gamma^n e^{-\gamma})\,. \tag{26}$$

As a first condition on our renormalization scheme, we require that the effective mass squared, c_1, be finite. We see that this can be achieved by choosing M^2 and γ finite. From (23a), we then must choose

$$B=\beta\Delta$$

with β finite. This choice is the same as we made in case (i) of model I, Eq. (16). It is now immediately evident from (26) that all coefficients c_n, $n>1$, tend to zero as Δ tends to infinity, and we are left with a free theory.

As a consistency check, we note that if we minimize ${}_\mu\langle 0|\mathcal{H}|0\rangle_\mu$, Eq. (24), with respect to μ [using $\partial\Delta/\partial\mu=-1/2\pi\mu$, cf. (13)], we find

$$\mu^2{}_{\min}=M^2[1-(1+\gamma)e^{-\gamma}]$$

in agreement with (25).

As for model I, the model of this section can also be made to describe interacting particles by the adjunction to the Hamiltonian of suitable counterterms. Unlike model I, however, there does not seem to be a simple alternative renormalization scheme analogous to the second case described in Sec. II.

IV. CONCLUSIONS

The results of this investigation are tentative at best. We have sought to study models of quark confinement, with the particular aim of seeing how quantum corrections influence classical intuition. Within the context of a pair of one-dimensional models and of *conventional renormalization procedures*, we have found a variety of ways in which the property of confinement can elude one on the quantum level.[5]

Do there remain alternative renormalization procedures which might lead to results distinct from the ones we have found? In this connection, we emphasize that our adherence to convention consisted in keeping the cutoff momentum Λ larger than any other parameter, i.e., letting $\Lambda\to\infty$ first in any discussion of functional dependence on parameters. It was an early and trivial observation that the classical bag limit of our models could be regained by letting Δ_μ [Eq. (13)] $\to 0$, i.e., by setting $\mu\to\infty$, Λ fixed. We have failed, so far, to find, a convergent expansion about this limit, but, in our view, the question remains open.

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[1]C. Rebbi, private communication. We are indebted to Dr. Rebbi for a useful discussion, in particular with regard to case (ii) of Sec. II.

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