Electromagnetic decay widths in a relativistic charmonium model

J. F. Gunion*
University of Pittsburgh, Pittsburgh, Pennsylvania 15260
and University of California, Davis, California 95616

L. F. Li†
Department of Physics, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 17 July 1975)

We consider a relativistic model of quark confinement incorporating both a linear large-distance potential and a Coulomb-like short-distance potential. Special emphasis is placed upon predictions for leptonic decay widths and the $2s$–$1p$ radiative transition of the charmonium model. We find that relativistic enhancement of the former force one to a situation in which the $2s-1p$ transition width is large.

I. INTRODUCTION

The recent discovery[1] of a series of new particles has focused renewed interest on the nature of the quark binding mechanism responsible for hadronic bound states. Asymptotically free non-Abelian gauge theories, which have been particularly successful in explaining the scaling phenomena in deep-inelastic electron scattering, strongly suggest that quarks interact very weakly at short distance but very strongly at large distance in such a way that quarks cannot escape.[2,3] On the basis of this picture of quark confinement, several nonrelativistic potential models[4] have been proposed in which the new resonances are closely related bound states. In these models the potential rises linearly at large distances, as suggested by the strong-coupling approximation to non-Abelian gauge theories[3] and by solvable 1-dimensional examples thereof.[5] The s-wave spectrum of such models appears to be in very satisfactory agreement with an interpretation of the new particles as ground and excited states of a linear potential. Though this general confinement picture seems appealing, a number of expected decay modes have not yet been observed.[6] However, in order to make reliable predictions one must include relativistic and quantum effects in the analysis. In a previous paper[7] we have examined the relativistic effects, within a particular framework, for the case in which only a linear potential is present. We found the relativistic effects on the particle spectrum of charm-anticharm states to be small. However, one does not necessarily expect the linear potential to be valid at small distance. In fact, in non-Abelian gauge theories, at small distance, the quark interactions can be well approximated by the lowest-order, Coulomb-like, interaction of the form $1/r$, in analogy with the usual gauge theory of electromagnetic interactions.[8] While the strength of this short-range Coulomb-like potential is not known, meaningful studies of relativistic effects must allow for its presence; it plays a critical role in the calculation of various decay widths. In this paper we incorporate a $1/r$ potential into our relativistic calculations with particular emphasis on its effect upon the leptonic decay widths and the electric dipole transition between the $2S$ and $1P$ states of the charmonium model. Both are crucial tests of this class of model.

To elaborate, we recall[9] that a $1/r$ potential in a relativistic wave equation gives rise to a mild singularity of the wave function at the origin. Thus the nonrelativistic approximation,[10] in which the leptonic width of a resonance is related to the wave function at the origin, $\psi(0)$, is no longer strictly valid. We expect in any case that the $1/r$ potential is shielded at very small r by virtual pair vacuum polarization modifications to the underlying gluon propagators. Indeed, in asymptotically free color gauge theories summation of vacuum polarization and other related effects can be accomplished to all orders, with the result that the singularity in $\psi(r)$ is made only logarithmic, or perhaps even eliminated, for color singlet states. Thus we do not expect the wave function to grow significantly once such effects become important, and will adopt the procedure of averaging the wave function around the origin in the region where the softening of the singularity begins to become important. Without a full theory of the small-distance quantum effects the validity of this procedure cannot be judged. It is encouraging to note, however, that such an averaging yields a correction to the hyperfine splitting[11] of hydrogen of the right order of magnitude.

The organization of the paper is as follows. In Sec. II we present the Klein-Gordon equation formalism employed, including the presence of both a quark confinement potential and a Coulomb-like $1/r$ potential. In Sec. III we present and discuss the results of our calculation, while Sec. IV contains conclusions and speculations.

II. THE RELATIVISTIC FORMALISM

We will use the same formalism as in our earlier paper, I. For simplicity, we consider only the Klein-Gordon equations. The quark field, $\phi_q(x)$, is assumed to satisfy a relativistic wave equation with both a quark-confinement linear potential V_1 and a Coulomb-like potential V_2, of the form

$$[(\partial_0 - V_2)^2 - \vec{\nabla}_x{}^2 + (m + V_1)^2]\,\phi_q(x) = 0. \qquad (1)$$

The short-distance Coulomb-like potential V_2 is assumed to be the time component of a 4-vector, in analogy to the usual electromagnetic Coulomb potential, appropriate, for instance, in the case of the hydrogen atom,[9] because they both originate from a gauge-type interaction and because at short distances the color gauge theory potential can be treated perturbatively and clearly remembers its gauge theory origin. The quark-confinement potential, V_1, enters in Eq. (1) as a Lorentz scalar. Treatment as the time component of a 4-vector does not lead to true quark confinement, as discussed in I. Such treatment is not necessarily in conflict with the underlying color gauge theory since confinement involves a severe infrared catastrophe or similar mechanism the *effective* result of which need not have the same Lorentz properties as the originating gauge theory. Our choice seems the only relevant one for complete confinement within the potential-theory framework.

The bound-state wave function $\psi(r)$ is defined[12] by

$$\langle 0|T(\phi_{\bar{q}}(x')\phi_q(x))|B\rangle = \psi(r)\,e^{-iP\cdot R}, \qquad (2)$$

where $r = x - x'$ is the relative coordinate, $R = \frac{1}{2}(x + x')$ is the c.m. coordinate of the bound state, and P is the 4-momentum of the bound state. In the rest frame of the bound state, where $P = (M, 0, 0, 0)$, we assume that the potentials V_1, V_2 depend only on the relative spatial coordinate $|\vec{r}|$. Then the bound-state wave function $\psi(\vec{r})$ satisfies the equation

$$\{[\tfrac{1}{2}M + V_2(r)]^2 + \nabla_r{}^2 - [m + V_1(r)]^2\}\,\psi(\vec{r}) = 0. \qquad (3)$$

The quark-confinement potential is taken to be of the form $V_1 = kr$, where k characterizes its strength. Any possible constant term in V_1 can be absorbed into the definition of the mass because it is additive to the mass term in Eq. (2). The usual electromagnetic Coulomb potential takes the form α/r, which has its zero point at $r = \infty$, corresponding to the ionization point. The applicability of this definition of the zero point of the $1/r$ potential in our case is rather unclear, because the quarks can never escape to infinity owing to the presence of the confining potential $V_1(r)$. For generality, we will allow the presence of a constant term in the Coulomb-like potential and take it to be of the form $V_2(r) = \alpha_s/r + c$, where α_s characterizes the strength. With these explicit forms for the potentials, Eq. (3) becomes

$$\left[\left(\frac{M}{2} + \frac{\alpha_s}{r} + c\right)^2 + \nabla_r{}^2 - (m + kr)^2\right]\psi(r) = 0. \qquad (4)$$

It is clear from Eq. (4) that the effect of the constant c is simply to produce an over-all shift of the energy levels M of the ground states. This means that if c is present, only the spacing between levels is relevant rather than the absolute values. The eigenvalues M of Eq. (3) are solved for numerically on a computer, with α_s, k, and m as free parameters. For $c = 0$, we constrain the parameters such that they give masses 3.1, 3.7, and $\simeq 4.1$ GeV as the first three s-wave bound states. For $c \neq 0$ we require only the correct mass splittings.

We calculate the leptonic decay width of any s-wave bound state from the quark-antiquark annihilation through one photon. We perform the calculation in the approximation in which quark three-momenta are neglected and in which only the most singular quark pole in the appropriate Feynman diagrams[13] is retained. In this approximation

$$\Gamma_l = \frac{16\pi}{9}\left(\frac{2\alpha}{M}\right)^2 |\psi(0)|^2 \qquad (5)$$

depends only on the bound-state mass, M, when ψ is normalized so that $\int \psi^*\psi\, d^3r = 1$. Thus, the effective quark mass, m, which may include a constant term from V_1, enters only through ψ.

Here $\alpha = \frac{1}{137}$ and $\psi(0)$ is the wave function at the origin. As we discussed in the Introduction, we will use an averaging procedure to smear out the singularity at the origin. We will average the wave function from the origin to the point $r_0 = \alpha_s/2m_0$, below which the potential is strong enough to produce quark pairs. The mass m_0 is the light-quark mass rather than the heavy-charmed-quark mass; virtual-light-quark-pair vacuum polarization becomes important at an earlier stage, i.e., for smaller values of the potential. Then $\psi(0)$ is given by

$$\psi(0) = \frac{1}{r_0}\int_0^{r_0} \psi(r)\,dr. \qquad (6)$$

Other possible averaging procedures yield essentially identical results. Also, the value of $\psi(0)$ obtained in this way is not sensitive to the value of r_0.

The $E1$ transition between the $2S$ and $1P$ states is calculated in the standard way[14] using the appropriate wave functions obtained from the numerical solution of Eq. (4). Since the $E1$ transition varies as the third power of the energy difference, the γ-ray width will be very sensitive to the position of the $1p$ state relative to the $2s$ state. One can check that the absence of explicit mass dependence in the transition width formula

$$\Gamma(2S\rightarrow 1P_J)=\frac{(\frac{2}{3}e)^2\omega^3}{3\pi}I\,\frac{2J+1}{9} \tag{7}$$

(where I is the standard dipole overlap integral in the nonrelativistic normalization) means that relativistic corrections to the formal expression are small.

III. DISCUSSION OF THE RESULTS

In this section we will discuss our results for both $c=0$ and $c\neq 0$.

A. $c=0$

In this case the parameters α_s, k, and m are determined by the three S-wave bound-state ψ masses (3.1, 3.7, and $\simeq 4.1$). It turns out that there are many parameter choices which yield nearly the same spectrum. An increase in α_s, which increases the level spacing, can be compensated by a decrease in k. We list two reasonable possibilities in Table I. Also given are corresponding leptonic decay widths and $E1$ γ-ray widths.

As far as the absolute size of the leptonic decay widths is concerned, smaller values of α_s are clearly preferred; only for the extreme $c=0$ case $\alpha_s=0.05$ in Table I are the widths as small as observed. Such a small α_s has two possible disadvantages. First, the leptonic widths do not decrease very rapidly for increasing resonance mass. This is because the average wave function at the origin rises with the energy level for $\alpha_s\simeq 0$. The experimental situation[15] is not entirely clear, but the $\psi(3.7)$ is believed to have less than half the leptonic width of $\psi(3.1)$. In our approach only large α_s values can achieve such an effect, but, as indicated, the absolute widths are much too large. The second disadvantage concerns the $E1$ γ-ray transitions. We first note that the dipole overlap integral between $2S$ and $1P$ wave functions is not very sensitive to relativistic effects. The $E1$ width is primarily sensitive to the $2S$-$1P$ energy difference. This difference decreases with increasing α_s because the attractive short-range $1/r$ potential is more important for S states than for P states. For larger values of α_s the γ-ray width can be reduced by a factor of 3. Experimentally the smaller width may be preferred, but,

TABLE I. ψ spectra, leptonic widths, and $E1$ transition widths for various choices of potential parameters and $c=0$.

State	Mass (GeV)	Γ_l (keV)	$E1$ transition width
(a) $m=1.14$ GeV, $\alpha_s=0.05$, $k=0.137$ GeV^2			
$1\,^3S$	3.093	4.76	
$2\,^3S$	3.695	3.862	
$3\,^3S$	4.155	3.464	
$1\,^3P$	3.455		$\frac{\Gamma(2\,^3S\rightarrow 1\,^3P_J)}{2J+1}=30.2$ keV
$2\,^3P$	3.955		
(b) $m=1.25$ GeV, $\alpha_s=0.145$, $k=0.12$ GeV^2			
$1\,^3S$	3.096	8.04	
$2\,^3S$	3.686	5.62	
$3\,^3S$	4.123	4.47	
$1\,^3P$	3.483		$\frac{\Gamma(2\,^3S\rightarrow 1\,^3P_J)}{2J+1}=19.3$ keV
$2\,^3P$	3.956		
(c) $m=1.45$ GeV, $\alpha_s=0.3$, $k=0.095$ GeV^2			
$1\,^3S$	3.114	28.9	
$2\,^3S$	3.716	15.07	
$3\,^3S$	4.125	10.7	
$1\,^3P$	3.582		$\frac{\Gamma(2\,^3S\rightarrow 1\,^3P_J)}{2J+1}=7.2$ keV
$2\,^3P$	4.005		

once again, we see that we can only achieve this by sacrificing agreement with the leptonic-width absolute magnitudes.

B. $c\neq 0$

If we allow the presence of this additional constant, our constraints are relaxed in that we must fit only the spacing between the three observed bound-state levels rather than their absolute values. Since the spacing is quite insensitive to the quark mass, this freedom essentially allows us to change the quark mass at will. First we remark that, qualitatively, increasing the quark mass while keeping α_s and k roughly the same, so as to preserve the correct spacing, increases the wave function at the origin. Since the leptonic-width formula is insensitive to the quark mass, this is not desirable. Clearly $c>0$, corresponding to reduced values of the quark mass and smaller values of the average wave function at the origin, is the experimentally preferred region.

As the examples in Table II indicate, positive c allows one to obtain reasonable leptonic widths, at least for moderate values of α_s. However, the $E1$ γ-ray widths for $c>0$ are somewhat larger

TABLE II. As for Table I with $c \neq 0$.

State	Mass (GeV)	Γ_l (keV)	$E1$ transition width
(a) $m=1$ GeV, $\alpha_s=0.145$, $k=0.118$ GeV^2, $2c=0.455$			
$1\,^3S$	3.1	6.01	
$2\,^3S$	3.698	4.22	
$3\,^3S$	4.141	4.06	
$1\,^3P$	3.489		$\frac{\Gamma(2\,^3S\rightarrow 1\,^3P_J)}{2J+1}=25.1$ keV
$2\,^3P$	3.971		
(b) $m=1$ GeV, $\alpha_s=0.3$, $k=0.092$ GeV^2, $2c=0.773$			
$1\,^3S$	3.095	14.84	
$2\,^3S$	3.691	10.8	
$3\,^3S$	4.105	6.0	
$1\,^3P$	3.546		$\frac{\Gamma(2\,^3S\rightarrow 1\,^3P_J)}{2J+1}=11.1$ keV
$2\,^3P$	3.984		

than those for $c=0$ and similar values of α_s and k. Noting that there is no explicit quark mass dependence in Eq. (7) and that the $2S$-$1P$ energy spacing is insensitive to m, we see that this reflects a slow rise with c of the overlap integral. Unless one employs more extreme values of c than those tabulated, the $E1$ transition will have a substantial width for choices of α_s and k yielding small leptonic widths.

Finally, we wish to mention briefly the situation for the ρ spectrum. We restrict ourselves to $c=0$. Intuitively one would like to use the same k value in this case as was appropriate to the charmonium spectrum. This turns out to be possible only if the first excited S state above $\rho(0.77)$ is $\rho(1.6)$; see Table III. But then the $1P$ state lies above 1.5 GeV, which is grossly inconsistent with the 1.2-GeV centrum of the $\{(f, A_2)B, A_1, \delta\}$ $1P$ resonances. Also, the wave function at the origin is very large, leading one to predict (using our approximate formula, which is admittedly inadequate for more than an order-of-magnitude estimate) leptonic widths far in excess of those seen.

The $1P$ centrum position can be improved only by using the somewhat questionable $\rho(1.25)$ as the first excited S state. However, the spacing requirements restrict one to relatively small values of k (an example appears in Table III) quite different from those appropriate to the ψ spectrum.

TABLE III. ρ spectra for $c=0$ and two choices of first excited s state; all numbers in GeV.

ρ spectra			
$L=0$		$L=1$	
State	Mass (GeV)	State	Mass (GeV)
(a) $\rho(0.77$ GeV$)-\rho(1.25$ GeV$)$			
$m=0.22$ GeV, $\alpha_s=0.39$, $k=0.048$ GeV^2, $2c=0$			
$1\,^3S$	0.785	$1\,^3P$	1.144
$2\,^3S$	1.248	$2\,^3P$	1.488
$3\,^3S$	1.57	$3\,^3P$	1.76
(b) $\rho(0.77$ GeV$)-\rho(1.6$ GeV$)$			
$m=0.2$ GeV, $\alpha_s=0.54$, $k=0.125$ GeV^2, $2c=0$			
$1\,^3S$	0.77	$1\,^3P$	1.522
$2\,^3S$	1.603	$2\,^3P$	2.094
$3\,^3S$	2.162	$3\,^3P$	2.545

IV. CONCLUSION

We have attempted here to assess the importance of relativistic effects, particularly in the presence of a Coulomb-like short-range force for the naive potential-model picture of the ψ spectra, leptonic decay widths, and $E1$ γ-ray transition width. The leptonic decay widths are sensitive to the singularity of the wave function at the origin present for a $1/r$ potential in a relativistic wave equation. We adopted the procedure of averaging the wave function over the region where shielding due to pair creation becomes significant and employing this average in the leptonic-width formula. This procedure is clearly not exact, but a detailed inclusion of quantum effects is not possible at present.

We found that any attempt to make the $1P$ level fairly near the $2S$ level, by increasing the short-range potential strength, α_s, led to leptonic widths much above those observed experimentally. This remained true even given moderate freedom in the definition of the energy zero point. Thus reasonable leptonic widths, which are obtainable only for relatively small α_s, appear to imply a large $2S$-$1P$ energy difference and a correspondingly large $E1$ transition width.

We emphasize that even within the potential approach other effects might cause a significant decrease of the $2S$-$1P$ energy difference. For example, the continuum channels may have strong virtual coupling to the below-threshold resonances we consider. If the coupling to S states were stronger than that to P states, this would clearly shift all the S states down with respect to the P states.[16] This is clearly a very model-dependent situation; in addition one must be careful that S-state leptonic widths are not increased in the process.

It will be interesting to see whether models[17] such as the MIT "bag" will also be forced into a similar situation. The physical picture in which certain Lagrangian fields are imagined to establish a boundary inside which quarks are effectively free, to a first approximation, is very different.

ACKNOWLEDGMENTS

We would like to acknowledge helpful conversations with D. Yennie and L. Wolfenstein.

---

*Work supported in part by the NSF.

†Work supported in part by the ERDA.

[1]J. J. Aubert *et al*., Phys. Rev. Lett. 33, 1404 (1974); J.-E. Augustin *et al*., *ibid*. 33, 1406 (1974); C. Bacci *et al*., *ibid*. 33, 1408 (1974); G. S. Abrams *et al*., *ibid*. 33, 1453 (1974); J. J. Aubert *et al*., *ibid*. 33, 1624 (1974).

[2]D. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973).

[3]K. Wilson, Phys. Rev. D 10, 2445 (1974); and Cornell Report No. CLNS-271, 1974 (unpublished).

[4]E. Eichten *et al*., Phys. Rev. Lett. 34, 369 (1975); B. J. Harrington, S. Y. Park, and A. Yildiz; *ibid*. 34, 168 (1975); K. Jhung, K. Chung, and R. S. Willey, Phys. Rev. D 12, 1999 (1975); J. F. Gunion and R. S. Willey, *ibid*. 12, 174 (1975).

[5]J. Kogut and L. Susskind, Phys. Rev. D 9, 3501 (1974); J. Schwinger, Phys. Rev. 128, 2425 (1962).

[6]R. Hofstadter, in proceedings of the 1975 Washington APS meeting (unpublished).

[7]J. F. Gunion and L. F. Li, Phys. Rev. D 12, 3583 (1975); referred to as I from here on.

[8]T. Appelquist and D. H. Politzer, Phys. Rev. Lett. 34, 43 (1975).

[9]See for example, J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[10]J. Schwinger, *Particles, Sources and Fields*, Vol. II (Addison-Wesley, Reading, Mass., 1973).

[11]H. Grotch and D. R. Yennie, Rev. Mod. Phys. 41, 350 (1969).

[12]See for example D. Lurié, *Particles and Fields* (Interscience, New York, 1968).

[13]Two Feynman diagrams need to be considered: (a) the charge normalization diagram of the standard Bethe-Salpeter framework and (b) the quark loop annihilation diagram. In both cases, one performs the energy integration of the loop by contour methods. In either half plane one quark propagation pole is substantially more singular than any other pole; it is this pole which yeilds the weak binding approximation and which we retain.

[14]See for example J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, Reading, Mass., 1967).

[15]F. J. Gilman, invited talk presented at the Orbis Scientiae II, Univ. of Miami, Coral Gables, Florida, 1985 (unpublished).

[16]We understand that such questions are being studied extensively at Cornell University. See E. Eichten *et al.*, Cornell Report No. CLNS-316 (unpublished).

[17]W. A. Bardeen *et al*., Phys. Rev. D 11, 1094 (1975); A. Chodos *et al*., *ibid*. 9, 3471 (1974); T. P. Cheng and P. B. James, Univ. of Missouri report, 1975 (unpublished).