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Tachyon behavior in theories with broken Lorentz invariance

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We consider questions relating to the production and detection of tachyons obeying one of the models of a non-Lorentz-invariant theory of tachyons developed in the preceding paper. We find one of these models to be promising, in that the existence of tachyons described by the model does not appear to contradict existing experiments, while it appears that the detection of such tachyons experimentally may be possible. The other models lead either to conflicts with experiment or to the implication that the existence of tachyons would be very difficult to observe.

I. INTRODUCTION

In the preceding paper[1] we discuss possible theories in which there exists a preferred reference frame in nature, so that there is not exact invariance under proper Lorentz transformations. (Hereafter we refer to Ref. 1 as I, and designate equations from I as I-1, etc.) In such a theory one can have tachyons without the problems of causal loops which arise when one attempts to construct Lorentz-invariant tachyon theories (LITT's). In the present paper we wish to discuss various purely kinematic aspects of tachyon production, and of the possible instability of tachyons against various kinds of decay processes in such a theory, and their implications for the possibility of producing and detecting tachyons experimentally. We are interested essentially in two questions. First, can a non-Lorentz-invariant tachyon theory (non-LITT) of the type suggested in I be made consistent with the fact that, so far, tachyons have not been observed experimentally? Second, if it can, does the theory suggest that the possibility of being able to observe tachyons, if they exist, in the future is good?

We review briefly the results from I which we will need. All of our equations, unless otherwise indicated, will be written in terms of quantities in the preferred reference frame, S, which we take to be practically indistinguishable from the frame of reference of the earth in which our experiments are done. We distinguished two general classes of behavior of the energy as a function of velocity in I. In the first, case 1, the energy is large and finite at $v=1$ (we take $c=1$ throughout) and increases to ∞ for $v\to\infty$. As a specific example of a case-1 theory in I, we took the possible energy and momentum relations, for $v>1$, of a particle with rest mass m to be

$$E=mv/k, \tag{1a}$$

$$p=(m/k)(\ln v+1). \tag{1b}$$

The momentum equation is determined, once a form for $E(v)$ has been assumed, by the requirement that

$$dE/dp=v, \tag{2}$$

which allows for the physical interpretability of a quantum-mechanical version of the theory by ensuring that v represents the group velocity of a wave packet. For $v<1-d$, where d is some very small positive number, the energy and momentum relations, in all cases, are assumed to become just those of special relativity (SR), as required by experiment. In I we introduced the term "kinetic mass" with the symbol m' defined by

$$m'^2=E^2-p^2. \tag{3}$$

One has to distinguish m and m' since, for $v>1-d$, where the theory departs from special relativity, $m'\neq m$ but becomes a function of v. As shown in I it follows easily from Eqs. (1) and Eq. (2) that $m'^2>0$ in case 1.

In a second class of theory, which we designate as case 2, the energy reaches a maximum at or near $v=1$ and then decreases. In I we introduced two specific examples of case-2 theories. In the first, case 2a, the energy and momentum were given by, for $v>1+d'$, with d' a positive constant,

$$E=m/(v^2-1)^{1/2}, \tag{4a}$$

$$p=mv/(v^2-1)^{1/2}. \tag{4b}$$

Equations (4) agree with the definitions in the usual LITT's[2] with the important exception that the positive square root is to be understood, so that the energy, in S, is always positive. Equations (4) give $m'^2=-m^2<0$. In a second example, case 2b, the energy and momentum relations are, for $v>1+d'$,

$$E=m/(1-1/v^2)^{1/2}, \tag{5a}$$

$$p=m[(v^2-1)^{-1/2}+\sec^{-1}v]-m\pi/2. \tag{5b}$$

In case 2b, E decreases monotonically to m and p

to 0 as $v \to \infty$. As shown in I, $E-p$ decreases monotonically as a result of (2). Hence $E^2-p^2=m'^2>m^2$. It is convenient to write the relation between E and p for case 2b which follows from Eqs. (5a) and (5b) in the form

$$E=p+a(p), \tag{5'}$$

the point being that, as we have seen, $a(p)$ is a slowly varying function of v, varying from $m\pi/2$ at $v=1$ or $p=p_{\max}$ to m at $v=\infty$ or $p=0$. Hence one can approximate, for many purposes, the energy-momentum relation in case 2b by $E=p+a$, where a is a constant of order m.

In Sec. II below we will make some general remarks about tachyon kinematics in non-LITT's without regard to the details of the energy and momentum relations; indeed, many of these remarks also apply in the Lorentz-invariant case. In Sec. III we examine the properties of cases 1, 2a, and 2b in detail, and the question of whether existing experimental searches for tachyons could have been expected to detect tachyons described by the present theory. In Sec. IV we conclude briefly, and some kinematical details are given in the Appendix.

II. GENERAL KINEMATICS

In order to prevent obvious conflicts with experiment, one prerequisite of any theory of tachyons is that it not predict unobserved spontaneous decay of stable bradyons by tachyon emission. In LITT's this problem is particularly bothersome because tachyons in such theories can have negative energy, so that a bradyon at rest can decay spontaneously by emitting a negative-energy tachyon, gaining energy in the process. Throughout this paper we will use the notation A_t, e.g., to indicate a particle of type A in a state with $v>1$, i.e., in a state where it is a tachyon. Thus we are saying that in LITT's the reaction

$$A(\text{at rest}) \to A(\text{moving}) + B_t(\text{negative energy}) \tag{6}$$

is allowed, where B_t is a tachyon, or system of tachyons, having all conserved quantum numbers, such as charge, equal to those of the vacuum. Extremely small experimental limits can be placed on the rates of such processes.[3] Hence, if they are kinematically allowed, tachyon-bradyon couplings have to be so extremely small that there would seem to be no chance of observing tachyons. Such problems do not exist in the present theory, since in the preferred reference frame all tachyons have positive energy, so that, working in S, one can see that reactions of type (6) are obviously forbidden by energy conservation. [As pointed out in I, conservation of energy and momentum holds in all frames if it does in one in this theory as in special relativity, since the energy and momentum transform as a four-vector under the generalized transformations, Eqs. (I-4). We work in the preferred frame only to simplify the argument.] There remains the possibility, however, that the reaction

$$A(\text{in motion}) \to A + B_t \tag{7}$$

might be kinematically allowed, with the energy required to create the B_t coming from the initial kinetic energy. Note that the usual simple argument which forbids such reactions in relativistically invariant theories, based on the observation that the reaction is clearly not allowed in S', the frame of reference where the initial particle is at rest, is no longer necessarily valid, precisely because of the lack of Lorentz invariance. In particular the spectrum of possible energies is not the same in S' as in S; in fact, it may be that tachyons can have negative energies in S'. Hence, in the absence of Lorentz invariance one has to exercise care in analyzing the kinematics in a nonpreferred frame.

There is a theorem which is exactly the same as a result familiar in SR which is useful in analyzing the possibility of reaction (7) and similar processes. We will use the following notation in discussing (7) and similar reactions such as (9) below: We designate by $p_1=(E_1,\vec{P}_1)$ the four-momentum of the initial particle, by $p_2=(E_2,\vec{P}_2)$ the four-momentum of the same particle after the decay, and by $q=(q_0,\vec{Q})$ the four-momentum transfer p_2-p_1. We let m' represent the kinetic mass of the emitted tachyon, B_t; by conservation of four-momentum, we have $q^2=m'^2$. Then the statement of our theorem is that $q^2<0$, so that (7) can occur only if $m'^2<0$. This result is simple to prove in SR by working in the rest frame of the initial particle. Here the theorem need hold only in S, since q^2 is no longer invariant. It is convenient to prove the theorem by means of (2) (which likewise is guaranteed to hold only in S), since this will make it easy to understand the corresponding result for tachyons. We consider initially the limit $Q\to 0$, and let $dP=P_2-P_1$. Then one has immediately that $dP^2=2P_1dP=2P_1Q\cos\theta$, where θ is the angle between $\vec{P}_1$ and $\vec{Q}$. Combining this with (2) gives

$$q_0=dE=v\,dP=Qv\cos\theta, \tag{8}$$

and hence for bradyons with $v<1$ one has $|q_0|<Q$ or $q^2<0$. If Q is finite, we can think of $\vec{Q}$ as being made up of a large number N of infinitesimal increments $d\vec{Q}_i=\vec{Q}/N$, so that $Q=NdQ_i$. Let dE_i be the change in energy produced by dQ_i, so that $q_0=\sum_i dE_i$, and $|q_0|\le\sum_i dE_i$. From (8), $dE_i<dQ_i=Q/N$, and hence $q_0{}^2<Q^2$ so that again $m'^2<0$. As an immediate corollary one has in the present theory, as in SR, that the analog of (7) with B_t replaced by a bradyon, B, cannot occur, at least if B has $v<1-d$, since bradyons, of course, have $m'^2>0$. The statement $q^2<0$ is also true *a fortiori* for the case of the reaction

$$A \to A^* + B_t, \qquad (7')$$

where A^* is another bradyon with rest mass $m^* > m_A$, the rest mass of the A, and the same quantum numbers as the A, since a given Q and P_2 in (7′) correspond to larger E_2, and hence smaller $q_0{}^2$ and more negative m'^2 than they do in the case of (7). The case $m^* < m_A$ is not of interest, since A will, in general, then be unstable and decay into A^* quite apart from any considerations of the existence of tachyons.

It is also true that if B_t does have $m'^2<0$, then reaction (7) will be kinematically allowed for sufficiently large E_1. Namely, suppose that it requires a minimum energy E_m to produce a B_t of kinetic mass m'. Then (7) becomes energetically allowed if $E_2 = E_1 - E_m$, so that (7) can occur if there is some P_2 such that $P_2 = [(E_1 - E_m)^2 + m_A{}^2]^{1/2} \equiv P'$ and $E_m{}^2 - |P_1 - P_2|^2 = E_m{}^2 - Q^2 = m'^2$. (We assume that the initial particle has $v<1-d$, so that it is described by the usual relativistic kinematics, and has kinetic mass = rest mass = m_A.) For given P_2, the minimum (maximum) values of Q and hence the least (most) negative values of q^2 occur for $\vec{P}_2$ parallel (antiparallel) to $\vec{P}_1$. In these configurations, with $P_2 = P'$ a simple calculation gives

$$q^2 = E_m{}^2 - Q^2$$

$$= -2P_1^2\{1 - E_1E_m/P_1{}^2 \pm [1 - (2E_1E_m - E_m{}^2)/P_1{}^2]^{1/2}\}.$$

For $P_1 \gg E_m$, one thus has $-2P_1{}^2 \leqslant q^2 \leqslant 0$, so that (7) becomes possible for any tachyon with $m'^2<0$ for large enough P_1, i.e., for $P_1 \gg E_m$ and $P_1{}^2 > -m'^2/2$.

Returning to Eq. (8), one sees that the kinematic restrictions on reactions of the type

$$A_t \to A_t + X, \qquad (9)$$

where X denotes any kind of a particle, bradyon, tachyon, or photon, are much less restrictive. In the limit of small Q to which (8) is applicable, when $v>1$, depending on the choice of $\cos\theta$ one can have q_0 either less than or greater than Q. Since q^2 can have either sign when Q is infinitesimal, one naturally expects that for Q finite (9) can also occur with $m_X'^2$ either positive or negative, and this is indeed the case. A simple argument shows that, for sufficiently large initial energy, (9) is allowed for arbitrarily large $m_X'^2$. Namely, consider the case when the decay is collinear, and the final four-momentum of the A_t is fixed. Because of the collinearity $Q = P_1 - P_2$, and because of (2) and the fact that $v>1$ one will have $Q = E_1 - E_2 - y = -q_0 - y$, where y is positive and nondecreasing (actually increasing) with E_1. Hence $q^2 = q_0{}^2 - Q^2 = -2q_0y - y^2$. As E_1 gets large, which implies Q gets large, q_0 becomes large and negative, and $-q_0 > y$, so that by taking E_1 large enough, q^2 can be made as positive as we wish. Thus, for tachyons, one can have the reaction

$$A_t \to A_t + B_t, \qquad (9')$$

regardless of the kinetic mass of the B_t, and we can also have the reaction

$$A_t \to A_t + B, \qquad (9'')$$

where B has $v<1$. Moreover, there is the interesting possibility

$$A_t \to A_t + \gamma. \qquad (10)$$

We can obtain the condition for this in the case of very small Q from (8) and the requirement that the energy of a photon equal the magnitude of the momentum, so that $-q_0 = Q$ or $v\cos\theta = -1$, giving

$$\cos\theta = -1/v. \qquad (11)$$

Equation (11) is just the familiar condition for the direction of Čerenkov radiation (the minus sign appears since the photon direction is that of $-\vec{Q}$), and we have in fact just the argument, in terms of photon kinematics, for the usual assumption that since tachyons travel faster than the speed of light they will emit Čerenkov radiation.[4] (We hereafter refer to the two papers in Ref. 4 as AK and DKA, respectively.) Note that our argument yields the usual Čerenkov relation (11) only in the limit that Q is small. This is not surprising since (11) can be derived from purely classical arguments[5] which neglect the quantum structure of the emitted radiation, and assume a constant value for v, thereby neglecting recoil of the radiating particle; this is, of course, generally an excellent approximation, as one is interested in Čerenkov radiation in or near the visible part of the spectrum in most conventional applications. However, here we will be concerned with the decay of possibly high-energy tachyons via reaction (10), and it is quite possible that the emitted photon might carry off an appreciable fraction of the initial energy, in which case one would expect deviations from the classical formulas. It is easy to see that (10) remains kinematically allowed for photons for which Q is not small. However, Eq. (11) no longer need hold then. In fact, in case 2a, where the tachyons have the usually assumed energy-momentum relation (at least for $v>1+d'$), (11) holds for any emitted photon of whatever energy, but this is not true for other tachyon energy-momentum relations. Also, in case 2a, the emission of a photon carrying any energy up to the full energy E_1 of the incident tachyon is kinematically allowed, but, as shown in the Appendix, this is not true in case 2b.

The existence of this Čerenkov radiation allows one to estimate a lower bound on the rate, $-dK/dx$, at which a moving tachyon will spontaneously lose kinetic energy, K, by radiation, using the classical formula[5] for the energy/unit length emitted as Čerenkov radiation by a charged particle. In case 2, the kinetic energy, K_1, for the incident tachyon is defined as $E_1(v_1)-E(\infty)$, while in case 1, where the particles slow down as they lose energy, we take $K=E_1-E(1)$, since the particle will cease to radiate when its speed becomes <1. The formula for dK/dx involves an integration over the frequency or photon energy of the emitted radiation. As we have indicated, the classical formula can be expected to be correct in detail only when the energy of the emitted photon is small compared with K. Suppose that the classical result gives a reasonable approximation for $q_0< Q_0$. Then a lower bound for $-dK/dx$, coming from the emission of "soft" photons in the classical part of the spectrum, is given by

$$-dK/dx=e'^2\int_0^{Q_0}(1-1/v^2)q_0\,dq_0, \qquad (12)$$

where e' is the tachyon charge, and we use throughout units with $\hbar=1$. For purposes of estimation here we will take $Q_0=K_1/10$. The exact solution of (12) requires specifying $K(v)$. However, if the range of energies involved is such that v never gets very close to unity, we can obtain the order-of-magnitude behavior, which is all we require, by treating the quantity in parentheses in (12) as a constant of order unity. One then obtains $dK/dx=-e'^2K^2/200$, and hence

$$x=(200/e'^2)(1/K_f-1/K_i), \qquad (13)$$

where x is the distance traveled while K changes from K_i to K_f. Equation (13) implies that, unless the coupling of particles in states with $v>1$ to the electromagnetic field is much weaker than for bradyons, they will lose almost all of their energy in a very short distance, except in cases where the initial kinetic energy is very small. If we take $e'=e$, (13) implies that K for an energetic tachyon would decrease to, e.g., 1 keV in a distance of order 10^{-4} cm, provided v is always greater than, say, 1.1 during the motion. (We consider the situation $v\approx 1$ below.) Moreover, one must remember that (13) gives an upper bound on x; it will probably be a few times smaller when hard-photon emission is taken into account. Also, as we have seen, tachyons are kinematically allowed to decay via (9′) or (9″) if the initial energy is large enough to allow the production of the second particle. If the decaying tachyon has strong couplings, these processes may dominate the electromagnetic Čerenkov radiation above threshold, giving decay lengths shorter than given by (13) by two or three orders of magnitude. In case 2b (13) must be modified for $K\lesssim 1$ keV, since it follows from the discussion in the Appendix that the maximum photon energy allowed kinematically is then less than $K/10$. Thus (13) gives too small a value for x in case 2b when $K_f\lesssim 1$ keV.

Equation (13) applies for charged tachyons. However, one expects comparable rates of energy loss for neutral tachyons of spin $\neq 0$ because of their coupling to the electromagnetic field via their anomalous magnetic moments, unless all tachyon anomalous moments vanish or are very small, which one would expect to be true only if nonelectromagnetic couplings of tachyons are very weak. Thus, except for the spin-0 case, one expects in general that neutral tachyons, like charged tachyons, with v appreciably greater than 1 either decay in effectively zero path length, via photon emission and possibly (9′) and (9″), until the energy of the residual tachyon is so small as to make it very difficult to detect, or else that tachyon couplings are quite weak, again making tachyon detection difficult.

There remains the consideration of Eq. (12) for v very close to unity. For the sake of brevity we present the discussion only for the more interesting case, case 2; the results for case 1 contain no surprises. One finds, for either case 2a or case 2b, that for $v\approx 1$, $(1-1/v^2)\approx m_A{}^2/K^2$ and (12) becomes $dK/dx\approx -e'^2m_A{}^2/200$ for $v\approx 1$, giving

$$x=(200/e'^2m_A{}^2)(K_i-K_f),\quad v\approx 1,\text{case 2} \qquad (13')$$

so that a particle with $v\approx 1$ and rest mass m_A loses essentially all its energy and acquires a v appreciably greater than 1, so that (13) is valid, in a distance of order $200K_i/(e'^2m_A{}^2)$. Thus, e.g., a 1000-GeV proton in a tachyon state would lose, taking $e'=e$, almost all its energy through "soft" photon emission in about 10^{-7} cm, while it would require an energy $\approx 10^{10}$ GeV in order to travel 1 cm.

III. SPECIFIC MODELS

We now turn to a discussion of our three specific models of tachyon behavior with broken Lorentz invariance, using the general results of the preceding section. It is clear that our conclusions apply not only to these specific models, but to wide classes of models having properties generally similar to them.

We will not consider case 1 at any length for the reason that the possibility of producing or observing tachyons in case 1 seems quite remote. From the discussion in Sec. II one would expect a proton with $v > 1$, in case 1, to lose energy in a very short distance by the radiation of photons and other hadrons until $v \lesssim 1$, and hence its energy $E \lesssim E(1)$. On the other hand, protons with energies $E \approx 10^{11}$ times the proton mass exist in cosmic rays,[6] and hence retain this energy over long time intervals. Thus, for protons $E(1) \gtrsim 10^{11}$ GeV, or $\Gamma(1) \gtrsim 10^{11}$, where $\Gamma(v)$ is the ratio of energy to rest mass for a particle of speed v. In view of this very large value of $\Gamma(1)$, it is difficult to imagine any kind of process which would ever be practicable for producing tachyons in case 1.

In case 2 there are two states for a given particle with the same momentum and spin, one with $v < 1$ and one with $v > 1$. This means that for a given set of particles, e.g., p, $\bar{p}$, and π^0, there are several new trilinear couplings possible in addition to the familiar ones which describe the coupling when all particles have $v < 1$. We can refer to the possible new couplings as BBT, BTT, and TTT, where B and T refer to $v < 1$ and $v > 1$, respectively, and the usual coupling would be designated by BBB. In fact, however, we are going to make the assumption that there are no direct BBT or BTT couplings, and that the only interaction between bradyons and tachyons is by photon exchange, i.e., via vertices of the type γBB and γTT, with coupling constants e and e', respectively. We also assume there are no couplings of the type γBT, which is a rather natural extension of the principle of minimal electromagnetic coupling, but is also justified experimentally as discussed below. These assumptions imply that the net number of charged bradyons, i.e., number of positive bradyons minus number of negative bradyons, and the net number of charged tachyons are separately conserved; this, incidentally, leaves open the possibility $e' \neq e$ without violating charge conservation. Our coupling scheme also implies that the net number of bradyon fermions (i.e., number of fermions minus number of antifermions with $v < 1$) and the net number of tachyon fermions are separately conserved. We shall allow the possible existence of TTT couplings, i.e., of interactions of tachyons among themselves. Indeed, our general philosophy that tachyons are to be viewed simply as bradyons whose speed has become > 1 would suggest that TTT couplings are quite similar to the familiar BBB couplings, although this is not crucial.

The main experimental justification for choosing this coupling scheme is that the resulting selection rules prevent reactions which could give apparent violation of charge conservation. For example, if there were direct BBT, BTT, or γBT couplings, one could have the reaction

$$p + p \rightarrow p + n + \pi_t^+ \tag{14}$$

followed by an essentially instantaneous decay of the π_t^+ via Čerenkov radiation into a photon plus a very low energy, high speed π_t^+. Since the latter would be invisible in, say, a bubble chamber, reaction (14) would involve the apparent disappearance of one unit of positive charge. Reactions of this type are forbidden in our scheme since the total charge carried by bradyons is separately a conserved quantity. A second, somewhat less direct, experimental argument in favor of having bradyons and tachyons coupled only through photons is that, as discussed in I, violations of Lorentz invariance are not observed above the 1% level with bradyons, even for values of v rather near unity. In view of the fact that virtual tachyon contributions are not Lorentz-invariant in this theory, that might present a problem if there were strong couplings between bradyons and tachyons. However, if bradyon-tachyon couplings are purely electromagnetic, and if one makes the plausible assumption that $e' = e$, then one would presumably not expect violations of Lorentz invariance for bradyons of more than the order of one per cent. Finally, although our choice of coupling scheme is based on experimental considerations, it is perhaps not entirely unappealing, in view of the special role played by c in electromagnetism, that electromagnetic interactions play a special role in coupling particles with $v < c$ to those with $v > c$.

In most of our discussion of case 2 we will adhere to our general philosophy that tachyons are to be regarded as ordinary particles with $v > 1$. In particular, we suppose that there is a one-to-one correspondence between types of tachyons and types of bradyons in the sense that for every bradyon of rest mass m there exists a tachyon whose energy is given by $m\Gamma(v)$. It is with this notion in mind, e.g., that we used the notation π_t^+ in (14). However, as discussed in I, it is perfectly possible in the present approach to consider the limit $d, d' \rightarrow 0$ in which the energy becomes infinite for $v = 1$, and one has an impenetrable light barrier as in SR. With a discontinuity at $v = 1$, it is no longer clear whether one would expect to find the same types of particles with $v > 1$ as with $v < 1$, and there might, in fact, in that case be no tachyon with $E/\Gamma(v) = m_\pi$, for example.

Case 2a

As we have seen, a distinguishing feature of case 2a is that the kinetic mass of a tachyon of rest mass m obeys $m'^2 = -m^2 < 0$. From the discussion in Sec. II we know that this means that reaction (7) becomes kinematically allowed provided the incident energy is high enough. In fact, the threshold condition here is very simple since the minimum energy required to produce any tachyon is 0, corresponding to $v=\infty$. This means that (7) is kinematically allowed provided the incident momentum satisfies $P_1 \geq m/2$; for $P_1 = m/2$, the kinematics of (7) is given by $p_1 = (E_1, \vec{P}_1)$, $p_2 = (E_1, -\vec{P}_1)$, and $q = (0, -2\vec{P}_1)$, so that $q^2 = -m^2$, as required. Thus, e.g., in case 2a the reaction

$$p \to p + \pi_t^+ + \pi_t^- \tag{15}$$

becomes possible for protons of energy equal to $(m_p{}^2 + m_\pi{}^2)^{1/2}$, which corresponds to a proton kinetic energy of about 10 MeV.

One knows, of course, that reactions such as (15) do not occur, or at least occur only exceedingly slowly, since energetic protons survive for times of the order of a second during acceleration in accelerators, and, more dramatically, for many years as cosmic rays crossing galactic distances. For this reason case 2a is rather unattractive. It is only viable if BBT and BTT couplings (which is to say, in our coupling scheme, e') are so exceedingly weak, weaker by many orders of magnitude that the usual interactions in particle physics, that the rate of (15) is suppressed sufficiently so as to be compatible with experiment. However, if the couplings are chosen weak enough to eliminate the unwanted decay reactions such as (15), then there would seem to be no possibility of ever being able to observe tachyon production reactions such as, e.g.,

$$p + p \to p + p + \pi_t^0. \tag{16}$$

Thus one might as well simply consider a theory in which tachyons are not present. Because of these difficulties we will not consider case 2a further.

Case 2b

The crucial point in case 2b is that a particle of rest mass m in a state with $v > 1$ has $m'^2 > 0$, and in fact, as we have seen above, $m'^2 > m^2$. From the fact that $m'^2 > 0$ for an individual tachyon it follows at once that $m'^2 > 0$ for any system of tachyons. Consequently, by the theorem proved in Sec. II, all spontaneous decays of bradyons of the form (7) are kinematically forbidden in case 2b. Before we can be sure that there is no problem with the prediction of unobserved spontaneous decays of isolated moving bradyons we must also consider reactions of the form

$$A \to A_t + B_t. \tag{17}$$

In our coupling scheme reaction (17) is forbidden by the selection rules for all initial particles except neutral bosons, which are unstable in any event. However, it is not necessary to rely on a specific coupling scheme. Reaction (17) is, in fact, kinematically forbidden, since if (17) is allowed with energy and momentum E_2 and P_2 for the A_t in the final state, then so is the reaction $A \to A^* + B_t$, where A^* is a bradyon of rest mass (equal to kinetic mass) m^* given by $m^{*2} = E_2{}^2 - P_2{}^2 = m'^2 > m_A{}^2$, where m_A is the rest mass of the A particle and m' its kinetic mass in its final tachyon state designated by A_t. Hence the kinematical possibility of (16) is equivalent to the kinematical possibility of a reaction of type (7′) which, as we saw in Sec. II, is forbidden when the kinetic mass of the B_t is real, as it is here in case 2b. Hence in case 2b all unobserved spontaneous decays of isolated stable bradyons are kinematically forbidden; a high-energy proton, e.g., could not undergo reaction (15). Hence in case 2b tachyon-bradyon couplings can be of such a magnitude as to allow a reasonable possibility of producing tachyons in collisions without contradicting the experimental fact that reactions such as (15) are not observed.

One must next ask how tachyons can be observed if they are produced. From our previous discussion we expect that a charged tachyon will lose almost all its energy in a very short distance. This will certainly occur through Čerenkov radiation, reaction (10). It may also occur through the emission of other tachyons if the initial energy is above threshold for one or more reactions of type (9′), and if TTT couplings for the particle in question are not negligible relative to its coupling to photons. In this event the tachyons which are the decay products will again decay immediately, and so on. At some point in the chain the tachyon energies will become small enough that reactions of type (9′) are no longer energetically allowed; this will occur when the kinetic energy of the incident particle is no longer large enough to allow (9′) to occur with the emission of the lowest-mass strongly interacting tachyon, presumably a π_t if tachyon couplings are similar to those of bradyons. After that point, each tachyon will lose almost all of its energy by Čerenkov radiation. Hence, if one begins with a high-energy tachyon, one expects that almost all of the initial energy will be converted into photons by Čerenkov radiation, and one will be left with one or several tachyons whose residual energy is too small to allow the tachyon to be easily detected, say through its producing a track of ionized atoms in a bubble chamber. Thus to detect tachyon production, one must either detect the photons produced by the Čerenkov radiation, or one must manage to detect the residual low-energy, very high-velocity tachyons.

Since in our model all tachyon production results from coupling of tachyons to photons, the most favorable incident beam for producing tachyons would be a photon beam; cross sections for tachyon production using an incident charged particle beam would be down by an additional factor of α. A possible reaction for producing tachyons described by the present theory would be

$$\gamma + Z \to e_t^+ + e_t^- + Z , \tag{18}$$

i.e., tachyon electron pair production in the field of a nucleus. Because of the small electron rest mass, this reaction allows one to produce tachyons with very large kinetic energies with relatively modest photon energies. Assuming tachyon electrons, like electrons with $v < 1$, are coupled primarily by electromagnetic interactions, the electron pair produced in (18) lose their energy entirely via Čerenkov radiation. To know the exact spectrum of emitted photons would require a detailed calculation with a nonexistent dynamical theory. As shown in the Appendix, a high-energy electron in case 2b is kinematically allowed to lose almost its entire energy by the emission of a single high-energy photon. However, it seems reasonable to expect that the most likely decay sequence would be one in which most of the energy goes into several photons, each of which carries an appreciable fraction of the energy of the emitting electron. That is, it seems unlikely that, on the one hand, the electron will emit a single photon of the maximum possible energy, thereby losing almost all its energy at once, or, on the other hand, that it will lose all its energy by the emission of many soft photons before the emission of several hard photons. One might imagine using incident energies in (18) of the order of 100 MeV, so that any tachyonic electrons produced would have enough energy to emit several photons in the 10-MeV range. On the other hand, one would not have to consider the possibility of photons from π^0 production; moreover, as we shall see below, it is likely that the cross section for (18) falls rapidly with energy, making it desirable to keep the incident energy small. Hence, assuming a reasonable probability of detecting the photons, what one will observe will be the reaction

$$\gamma + Z \to n\gamma + Z , \tag{18$'$}$$

where n is some small integer. As shown in the Appendix, the produced photons lie in a cone of vertex angle $\cos^{-1}(1/v)$ about the direction of the emitting electron. Since, from Eq. (5), v is very close to unity for an electron whose energy is large compared to its rest mass, the photons from a given electron will tend to be collinear.

The cross section for reaction (18) is proportional to $Z^2\alpha'^2\alpha$, where $\alpha' = e'^2$ and α is the usual fine-structure constant. Experimentally measured γ-ray absorption lengths in the energy range 11–20 MeV, where pair production is the dominant process, agree on the whole with the values calculated on the basis of the usual theory of pair production to within the experimental errors, which are about 1%, although the uncertainty in the theoretical values due to the use of the Born approximation is of comparable, or somewhat larger, magnitude.[7] One might think that this would require the possible, but unappealing, choice $e' < e$ in order to avoid the experimentally unacceptable prediction that the cross section for the production of tachyon pairs is equal to that for ordinary pairs. However, this is not so. The energy denominators which enter the perturbation expansion are, for the kinematics of case 2b, appreciably larger in the case of tachyon production than for ordinary particles, and appear capable of giving the required suppression of tachyon production even if $e' = e$, though a detailed theory and calculation would be necessary to verify this with complete certainty. To estimate this effect, consider an incident photon of momentum p going to an intermediate state containing a pair with momenta p_1 and $p - p_1$. (We use "old fashioned" perturbation theory, and neglect the other possible intermediate state in lowest order which contains the electron pair plus the incident photon and whose contribution will be strongly suppressed by a much larger energy denominator.) We denote by G_b and G_t the inverse of the energy difference between the intermediate and the initial state in the case of bradyon and tachyon pair production, respectively. The cross sections σ_b and σ_t for the production of bradyon and tachyon pairs are proportional to G_b^2 and G_t^2. G, and therefore σ, will be a maximum when p and p_1 are collinear. In this case one has, letting $p' = p - p_1$,

$$G_b = [(p_1^2 + m^2)^{1/2} + (p'^2 + m^2)^{1/2} - p]^{-1}$$

$$\approx [m^2/(2p_1) + m^2/(2p')]^{-1}, \tag{19}$$

where m is the electron mass and we are assuming both p_1 and $p' \gg m$. For G_t one has, using (5$''$),

$$G_t = [a(p_1) + a(p')]^{-1}. \tag{19$'$}$$

Almost all of the production cross section occurs between, say, $p_1 = 0.1p$ and $p_1 = 0.9p$; within this range the cross section is relatively flat with a slight maximum at $p_1 \approx 0.5p$.[8] At this value of p_1, taking $p = 11$ MeV $\approx 2p_1$ one gets $G_b \approx p_1/m^2 \approx 11/m$ and $G_t \approx 1/(\pi m)$, where we have taken $a(p_1) \approx a(p_{\max}) = \pi m/2$, since v is very close to unity for $\Gamma = 11$.

This gives $\sigma_t/\sigma_b = G_t^2/G_b^2 \approx \frac{1}{1100}$. For the asymmetric case $p_1 = 0.1p$ one finds $\sigma_t/\sigma_b \approx \frac{1}{120}$. The suppression at large angles is much less, but in view of the sharp peaking of σ_b within a cone of angles of order $\theta = m/p$,[8] it seems quite likely that the production of tachyon pairs by photons in the 10-MeV range is sufficiently suppressed in case 2b with $e' = e$ to be consistent with the lack of discrepancy between standard pair-production theory and experiment. The parameter controlling the suppression of σ_t relative to σ_b is essentially $(2m/p)^2$. Therefore, σ_t becomes relatively more important for $p < 10$ MeV, but at those photon energies pair production itself becomes less important relative to Compton scattering or the photoelectric effect, so that the production of tachyon pairs would still probably not disturb agreement between theory and experimental measurements on γ-ray absorption.

For higher incident photon energies σ_t becomes still smaller relative to σ_b; for $p = 100$ MeV one expects σ_b/σ_t of the order of 10^4. Hence in searching for events of the form (18′) as evidence of (18) one must be concerned about small backgrounds. One can indeed obtain (18′) from ordinary electrodynamic processes. However, for $n > 1$ the cross section for such processes is down by a factor α^n relative to ordinary pair production. Hence (18) should dominate ordinary higher-order processes as a source of (18′) for $n \geq 3$.

The other approach to observing tachyons if they are produced is to look for the low-energy residual tachyon remaining after the energy of the produced tachyon has been radiated away. AK developed a technique for doing this. Although we shall see below that the experiments in Ref. 4 would probably not have detected tachyons in case 2b, this kind of experiment may be the most promising approach for finding tachyons.

The emerging tachyons will typically have kinetic energies of the order of an electron volt. Since they have $v \gg 1$, they will be very penetrating, and will have no trouble leaving the target in which they are produced and penetrating large amounts of matter. The problem is to detect their presence. We suppose that the equations governing the behavior of a charged tachyon in electric and magnetic fields $\vec{E}$ and $\vec{B}$ are those with which we are familiar for bradyons,[9] namely

$$dK/dt = e'\vec{E}\cdot\vec{v}, \tag{20a}$$

$$d\vec{p}/dt = e'(\vec{E} + \vec{v}\times\vec{B}), \tag{20b}$$

where K is the tachyon kinetic energy. Note that Eqs. (20) are consistent with one another only because of our imposition of the condition (2), since it follows from (20b) and the fact that

$$\vec{p} = b\vec{v}, \tag{21}$$

where b is a scalar function of v, that $dp/dt = e\vec{E}\cdot\vec{v}/v$. Equation (20a) implies of course the usual assumption that tachyons gain energy in an electric field. In the AK apparatus, tachyons enter a region between two conducting plates containing an electric field. They gain energy from the field until an equilibrium is reached, at which the rate of energy gain from the field is equal to the rate of Čerenkov radiation. Taking the field to be in the x direction, the equilibrium condition is

$$e'E = -dK/dx, \tag{22}$$

where dK/dx is the rate of change of the tachyon kinetic energy due to radiation.

There are several important differences between the present theory and the tachyon theory being tested in Ref. 4 which bear on the experiment. First, unlike the Lorentz-invariant case, here a minimum energy, presumably twice the electron mass, is required to produce tachyons. In AK, an 800-keV photon beam was used to attempt to produce tachyons; however, this was raised to 1.3 MeV by DKA, above the presumed threshold in the present theory. However, one should keep in mind the possibility that there is not a one-to-one correspondence between tachyons and bradyons. In that case, it is possible that the lowest tachyon mass would be greater than the electron mass, in which case even the photons used by DKA would be below threshold for tachyon production in case 2b.

If the tachyons are moving in a straight line in the x direction, then one can write using (12) and (22),

$$E = ek^2/2, \tag{23}$$

where we let k = the maximum photon energy allowed kinematically, and in (12) we have let $e' = e$, neglected $1/v^2$ for the low-energy tachyons with which we are concerned, and assumed that $Q_0 = k$. Note that in (23) the electric field strength E is determined by k; however, the maximum kinetic energy of the tachyons depends on the kinematics, which determines the relation of k and K. DKA used (23) with E chosen so that k corresponded roughly to the upper limit of sensitivity of the photomultiplier used to detect the Čerenkov radiation, about 3.5 eV. In the usual theory, assumed in DKA, $k = K$. In the present theory one has for low-energy tachyons, $K \ll m$, as shown in the Appendix

$$k \approx 2p \approx (4\sqrt{2}/3)K^{3/2}/\sqrt{m}, \tag{24}$$

which gives $K \approx 85$ eV for $k = 3$ eV. This would pose no problem in the experimental arrangement of

DKA, since the total accelerating potential is 12 kV. However, with $k \approx 2p \ll K$, one has that the tachyon energy after photon emission is almost unchanged, but the recoil angle can be anything from nearly $0°$ to $180°$; in other words, the tachyon trajectory as it passes between the plates is almost a random walk with a systematic effect due to the electric field superimposed. Because of the random motion the total path length, which we designate by s, will be $> x$. In the Appendix we estimate the characteristic distance between large-angle deflections for a low-energy tachyon as being $2K^{-3/2}m^{1/2}/e^2$; for an 80-eV tachyonic electron this gives 0.005 cm, so the random nature of the motion is quite significant in crossing the 4-cm gap between the plates of apparatus in Ref. 4. To see the effect of this, we write (22) in the form

$$E dx/ds = -dK/ds = ek^2/2\,, \tag{25}$$

where the last step comes from noting that the right-hand side of (12) is now equal to $-dK/ds$, not $-dK/dx$. A careful evaluation of k from (25) would require detailed study of the complicated random tachyon motion, but we expect that dx/ds is appreciably less than unity so that (25) will yield a smaller value of k than (23); a very crude estimate based on (25) gives $k \approx 1.1$ eV for $E = 3$ kV/cm. Since a 1-eV photon is below the threshold of the counters used by DKA, they would apparently not have succeeded in detecting tachyonic electrons governed by case 2b. However, it appears that increasing the electric field by about a factor of 10 might allow one to do so. It should also be noted that, in the present case, because of the random nature of the tachyon motion (as well as the fact that the angle of the emitted photons may range from about $0°$ to $90°$ with respect to the tachyon momentum) the directions of the emitted photons will be almost uniformly distributed in space, so that one predicts that a lower proportion of photons will reach a detector of given solid angle than expected by DKA.

Equation (20b) is also relevant to experiments attempting to detect low-energy tachyons in case 2b, since it has an interesting implication for the trajectories they will follow. Equations (20a), (20b), and (21) are sufficient to imply the usual relation $r = p/eB$ for the radius of curvature of a charged particle in a magnetic field, independent of the particular form of b. In case 2b the momentum of the very low-energy tachyons with which we are concerned is exceedingly small, and hence the particle trajectories will be very tight spirals about the magnetic field lines, even in a very weak magnetic field. For example, tachyonic electrons with kinetic energy = 1 eV in a magnetic field of 1 G would have a radius of curvature of about 10^{-5} cm. Hence if any magnetic field is present the tachyon trajectories can be considered to be, for practical purposes, along the field lines. This is a second reason why DKA would presumably have failed, in case 2b, to detect any tachyons which were present. Unless the lines of whatever magnetic field was present in their laboratory happened to run from their source into the solid angle $\approx 10^{-3}$ subtended by their detector, any tachyons which were produced would have failed to reach the detector.

Finally we consider briefly other existing experimental searches for tachyons, besides those of Ref. 4, and note that they would not have been sensitive to tachyons described by a non-LITT. One type of experiment[10] has involved searches for systems with $m'^2 < 0$. As we know, in case 2b (and case 1) tachyons in the present theory have $m'^2 > 0$, and cannot be distinguished by missing-mass experiments. Such experiments do apply in case 2a, but as we have seen any tachyons described by case 2a must be so weakly coupled to bradyons as to make their observation impossible in practice.

Other experiments have looked for tachyons as precursors of high-energy cosmic-ray showers[11,12]; in Ref. 12, the possibility of a positive result is reported. However, such experiments do not appear to be a promising method of detecting tachyons, either here or in LITT's, since it seems highly probable that almost all of the tachyon energy goes into photons, either from Čerenkov radiation or π_t^0 decay, leaving residual tachyons whose energy is much too small to be detected by conventional detectors. The same comments apply to attempts to observe tachyon tracks in bubble chambers, so that no real conclusion about the nonexistence of tachyons can be drawn from the negative result obtained in Ref. 13. There thus appear to be no strong experimental arguments against the existence of tachyons obeying a non-LITT.

IV. CONCLUSIONS

From the foregoing discussion, it appears that case 2b is a viable theory of tachyons. Existing experiments do not appear to provide any strong evidence against the existence of tachyons described by such a theory. On the other hand, there appears to be some hope, at least, that it might be possible to observe them in future experiments. However, in case 1 and 2a, if the parameters of the theory are chosen to prevent contradictions with experiment, it becomes difficult to see how tachyons obeying those theories could be observed.

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APPENDIX

In this appendix we discuss some of the kinematic details of Čerenkov radiation by tachyons in case 2b. We consider reaction (10), using the notation of Sec. II. For (10) to be allowed one must have

$$\vec{\mathrm{P}}_1+\vec{\mathrm{Q}}=\vec{\mathrm{P}}_2 \tag{A1}$$

and

$$P_2+a_2+Q=P_1+a_1, \tag{A2}$$

where $a_i=a(p_i)=E_i-P_i$ in accord with (5″). The condition for (A1) and (A2) to be simultaneously valid is easily seen to be

$$-\cos\theta=1-y/Q+y/P_1-y^2/(2P_1Q), \tag{A3}$$

where $y=a_1-a_2$, and θ is the angle between $\vec{\mathrm{Q}}$ and $\vec{\mathrm{P}}_1$; $\theta=\pi-\phi$, where ϕ is the angle of the emitted photon relative to $\vec{\mathrm{P}}_1$. For very small photon energies one will have $y \ll P_1$ and the last two terms in (A3) may be neglected. From its definition $y=E_1-E_2-(P_1-P_2)$; for small Q this becomes $y=dE(1-dP/dE)=Q(1-1/v)$ using conservation of energy and Eq. (2), so that (A3) gives back, as we have seen it must, the classical Čerenkov condition $\cos\theta=-1/v$ in the limit of small Q. As Q increases, $\cos\theta$ becomes more negative. To find Q_m, the maximum photon energy allowed, we put $\cos\theta=-1$ in (A3), giving

$$Q_m-P_1-y/2=0. \tag{A4}$$

With $\theta=\pi$, as Q_m is varied from P_1 to $2P_1$, $\vec{\mathrm{P}}_2$ varies from 0 to $-\vec{\mathrm{P}}_1$, y varies from a_1-m to 0, and thus the left-hand side of (A4) varies from $m-a_1$ to P_1. Since a_1 decreases monotonically to m as $P_1\to 0$, $m-a_1<0$ for $P_1>0$, and hence (A4) certainly has a solution for $P_1<Q_m<2P_1$. Thus the maximum photon energy corresponds to the situation where the photon is emitted at $\phi=0$, i.e., in the forward direction. One can find the approximate value of Q_m in the limit $P_1 \gg m$. Since $a<\pi m/2$, for $P_1\gg m$, $y/2$ is negligible in (A4) and one has $Q_m\approx P_1$. Thus a high-energy tachyon can give up almost its entire energy to one photon. Except for Q very close to P_1, the angular deflection which the tachyon undergoes in emitting the photon will be small, since $\phi \lesssim \cos^{-1}(1/v)$ which is very close to 0 for a high-energy tachyon with $v\approx 1$; thus the transverse momentum of the photon will be very small.

To consider the limit $P_1 \ll m$, which is relevant to the discussion of the experiment in Ref. 4, we must examine the low-energy limit of the energy-momentum relations (5a) and (5b). Expanding these equations for $v\gg 1$ and using $K=E-m=p+a-m$, one finds

$$K\approx m/(2v^2), \tag{A5}$$

$$p\approx m/(3v^3)\approx (2\sqrt{2}/3)K^{3/2}/\sqrt{m}. \tag{A6}$$

From (A6) one sees $p \ll K$ for $K\ll m$; i.e., in this limit $K\approx a-m$. Thus for low-energy tachyons, the maximum photon energy, $2p$, allowed by conservation of momentum leaves K, and hence p, almost unchanged, so that the low-energy solution to (A4) is $Q_m\approx 2P_1$, $\vec{\mathrm{P}}_2\approx-\vec{\mathrm{P}}_1$. Thus, following the emission of a photon, the momentum of a low-energy tachyon may have essentially any direction, while its magnitude is nearly unchanged, so that the tachyon trajectory is, as we asserted in Sec. III, nearly a random walk at constant speed. We estimate the distance traveled by a tachyon before undergoing a major change in direction. To undergo such a change, the tachyon must emit a photon with energy of order p. If we let x_0 represent the distance traveled by the tachyon before this occurs, then we expect $x_0\approx p/(-dK/dx)$, with $-dK/dx$ given by (12) with $Q_0=2p$. This gives

$$x_0\approx(3/\sqrt{2})K^{-3/2}\sqrt{m}/e^2. \tag{A7}$$

Note that $x_0\to\infty$ for $K\to 0$, so that as the speed v of a tachyon increases, it does on the average travel larger distances between points where it undergoes large directional changes.

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[9]Equations (20) must obviously fail for particles with $v \approx 1$ and $p \approx p_{\max}$. In a quantum picture this failure will reflect the fact that conservation of momentum together with the fact that momenta with $p > p_{\max}$ are not allowed means that there are some quanta which cannot be absorbed by a given particle. However, for a low-energy particle, this will affect only the absorption of quanta with momenta near $p_{\max}$ Since $p_{\max}$ is so large, such quanta will have little to do with particle motion in ordinary macroscopic fields. These difficulties are entirely absent in the version of the theory where the energy barrier at $v = 1$ is infinite; in that case, Eqs. (20) may be exact.

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