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Possible interpretation of the $\rho'(1600)$ as a threshold enhancement*

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The possibility that the $\rho'(1600)$ is not a true resonance is examined. In the model considered the reactions $\gamma p \to \rho\pi^+\pi^- p$ and $e^+e^- \to \rho\pi^+\pi^-$ proceed through the production of a virtual ρ meson. The threshold factors associated with the opening of the $\rho\pi\pi$ channel are shown to combine with the virtual-ρ propagators to produce peaks in the $\rho\pi^+\pi^-$ mass spectra, similar in mass and width to the experimental spectra. However, a comparison of the relative magnitudes of the two reactions, when combined with experimental limits on the reaction $\pi^+p \to \Delta^{++}\rho\pi^+\pi^-$, rules out this mechanism. The roles of final-state interactions and of the virtual-mass dependence of the $\gamma p \to \rho p$ production amplitude are considered. It is then assumed that the ρ' is a real resonance. The ρ'-γ coupling and the $\rho' p$ total cross section are extracted. We find $f_{\rho'}{}^2/4\pi = 21 \pm 10$ and $\sigma_{\rho' p} = 23 \pm 4$ mb, as compared with the ρ-meson values of 2.1 ± 0.5 and 22.2 ± 2.6 mb. The background due to a ρ tail in the production of the recently reported $\rho'(1250)$ is briefly discussed.

I. INTRODUCTION

It seems to be fairly well established that a resonance, known as the ρ', exists[1] at a mass of about 1600 MeV with a width ~400 MeV. It has been seen in both electron-positron colliding beams,[2-5] which establishes its quantum numbers as $J^{PC} = 1^{--}$, and in photoproduction,[6-8] where the momentum-transfer dependence and helicity structure again suggest that it behaves much like the familiar ρ meson.

It nevertheless has some very peculiar properties. First among these is that it apparently does not couple strongly to two pions.[6] Eisenberg *et al.*[9] have set a limit[10] on the coupling constant of $g_{\rho'\pi\pi}{}^2/g_{\rho\pi\pi}{}^2 \lesssim 0.02$. Second, the ρ' appears as an isolated resonance rather than as a member of a nonet, as we might expect. An $\omega'(1675)$ has been reported[1] with normal decay properties, but it probably should be associated with the[1] $g(1680)$ as the 3^- recurrences of the ω and ρ particles along their Regge trajectories, rather than with the ρ'. [It would, of course, be very valuable to establish experimentally that this is the case and to find the strange mesons belonging to the nonet as well: A candidate for the latter could be the[1] $L(1770)$, though $J^P = 3^-$ does not seem to be indicated; perhaps the tentative[1] $K_N(1850)$ is a more likely candidate.]

The dominant decay mode of the ρ' is into four pions, and more specifically into a ρ meson plus a pair of pions in a relative S state with isospin zero. There is no evidence, e.g., for a $\rho\rho$ decay mode. By contrast, the $g(1680)$ decays[1] about 30% of the time into two pions and 70% into four pions; the four-pion mode includes such states as the $\pi\pi\rho$, $\rho\rho$, $A_2\pi$, and $\omega\pi$, however.

There are several possible interpretations of the ρ'; for instance, in the quark model it might be a radial excitation of the $L=1$ $\bar{q}q$ state.[11,12] In a Regge model it could be a granddaughter of the ρ trajectory. A dual-resonance model was constructed by Shapiro[13] which predicted $J^P = 1^-$ daughters of the ρ at 1300 and 1670 MeV. The coupling of the $\rho'(1670)$ to two pions was predicted to be weak $[g_{\rho'\pi\pi}{}^2(1670)/g_{\rho\pi\pi}{}^2 \approx 0.05]$ in the Shapiro model, but not so weak as the experimental limit of 0.02. Furthermore, the $\rho'(1300)$ was predicted to couple strongly to two pions $[g_{\rho'\pi\pi}{}^2(1300)/g_{\rho\pi\pi}{}^2 \approx 0.54]$. Although two recent experiments[5,14] have reported the detection of a resonance around 1250 MeV, only the $\rho'(1250) \to \pi^+\pi^-\pi^0\pi^0$ (probably $\omega\pi^0$) decay mode has been observed.

Because of all these uncertainties and because of increased interest in the $\rho'(1600)$ decay mode generated by a possible similarity to the $\psi' \to \psi\pi\pi$ decay mode[15] of the $\psi'(3684)$, we decided to reopen the question of the existence of the ρ'. In particular, we attempted to fit present data without invoking the ρ', but only using the tail of the ρ meson and adjusting an over-all free parameter corresponding to the strength of the $\rho\rho\pi\pi$ coupling. A combination of threshold enhancement and propagator dampening allows us to have mass distributions that effectively recreate the ρ' shape. The major theoretical uncertainty, other than the size and form of the $\rho\rho\pi\pi$ amplitude, is the dependence of the virtual-ρ photoproduction amplitude on the virtual-ρ mass. There are three sets of experiments that need to be explained: They are[2-5] $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, photoproduction,[6-8] i.e., $\gamma p \to \pi^+\pi^-\pi^+\pi^- p$, and[9] $\pi^+p \to \Delta^{++}\pi^+\pi^-\pi^+\pi^-$. We shall examine the three reactions in Secs. II, III, and IV, respectively, employing a variety of parametrizations. The result is that the ρ' tail is not sufficient to explain simultaneously all three sets of data.

In Sec. V we will reexamine the data assuming that the $\rho'(1600)$ is a real resonance. The ρ'-γ coupling and the $\rho' p$ total cross section are extracted under conventional assumptions. We find that $\sigma_{\rho' p} \approx \sigma_{\rho p}$, while the ρ'-γ coupling is much weaker than the ρ-γ coupling. The effects of the final-state interaction are discussed. Section VI is devoted to a brief discussion of the $\rho'(1250)$. The background due to the tail of the ρ is calculated for both $e^+e^- \to \omega\pi^0$ and $\gamma p \to \omega\pi^0 p$. For the latter reaction, it is crucial that the amplitude to photoproduce a virtual ρ be given the correct mass dependence. Section VII is a summary of our conclusions.

II. ELECTRON-POSITRON RESULTS

We examine in this section the evidence[2–5] from e^+-e^- collisions for the supposed existence of a ρ' to see if we can interpret it using only the ρ meson.

Consider the diagram of Fig. 1, for $e^+ + e^- \to \rho + \pi + \pi$, where the two pions are in a relative S state with zero isospin. It is clear that the cross section first increases as more phase space becomes available, then decreases as the dampening due to the ρ and photon propagators sets in. If we call the two-pion invariant mass squared $\mu^2 = (q_1 + q_2)^2$ and put $M^2 = (p_1 + p_2)^2$, a straightforward evaluation of the diagram in Fig. 1 gives

$$\frac{d\sigma}{d\mu} = \frac{32\,\alpha^2}{f_\rho^{\,2}/4\pi}\,\frac{m_\rho^{\,4}}{M^5}\,\frac{q_{\rho\pi\pi}(1 + q_{\rho\pi\pi}{}^2/3m_\rho^{\,2})}{(M^2 - m_\rho^{\,2})^2 + m_\rho^{\,2}\Gamma_\rho^{\,2}}\,K^2 G(\mu)\,, \tag{1}$$

where m_ρ and Γ_ρ are the ρ-meson mass and width, $f_\rho^{\,2}/4\pi = 2.1$ is the inverse squared of the γ-ρ coupling, $q_{\rho\pi\pi}$ is a kinematical factor given by

$$q_{\rho\pi\pi}(M, \mu) = \frac{1}{2M}\,(M^4 + \mu^4 + m_\rho^{\,4} - 2M^2\mu^2 - 2M^2 m_\rho^{\,2} - 2\mu^2 m_\rho^{\,2})^{1/2}\,, \tag{2}$$

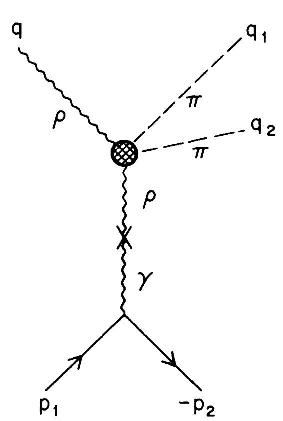

FIG. 1. Diagram for $e^+e^- \to \rho^0\pi^+\pi^-$.

and $K^2 G(\mu)$ is a parametrization of the $\rho\rho\pi\pi$ interaction. If we call R the invariant amplitude for $\rho^0\rho^0 \to \pi^+\pi^-$, we obtain

$$G = \frac{q_{\pi\pi}(\mu)\,|R|^2}{(16\,\pi K)^2}\,, \tag{3}$$

where the kinematical factor $q_{\pi\pi}(\mu)$ is

$$q_{\pi\pi}(\mu) = \left(\frac{\mu^2}{4} - m_\pi^{\,2}\right)^{1/2}, \tag{4}$$

with, of course, $\mu = 2m_\pi$ being the lower limit on the range of μ. The upper limit on μ is $M - m_\rho$, and we are principally interested in 500 MeV $\leq \mu \leq 1000$ MeV, a region in which the π-π final-state interaction may well be of considerable importance. In (3), K is a constant enhancement factor, which we will vary to normalize our formula to experiment.

We have considered three different forms for R: The first, which we call R_1, is

$$|R_1(\mu)|^2 = \left(\frac{16\,\pi K\mu\,\sin\delta_0^0(\mu)}{2\,q_{\pi\pi}(\mu)}\right)^2\,, \tag{5}$$

where δ_0^0 is the S-wave $I = 0$ pion-pion scattering phase shift as determined by[16] Protopopescu *et al.* This corresponds to an effective Fermi-Watson-type model in which the phase of the $\rho^0\rho^0 \to \pi^+\pi^-$ interaction is given by the $\pi^+\pi^- \to \pi^+\pi^-$ phase, and the magnitude is left as a free parameter. Our second model is one in which $\rho^0\rho^0 \to \pi^+\pi^+$ proceeds by way of a spin-zero isospin-zero resonance, which we call ϵ.

$$|R_2(\mu)|^2 = \frac{g_{\rho_0\rho_0\epsilon}{}^2\, g_{\epsilon\pi^+\pi^-}{}^2}{(\mu^2 - m_\epsilon^{\,2})^2 + \Gamma_\epsilon^{\,2}\, m_\epsilon^{\,2}}\,. \tag{6}$$

If, for this case, we define K as

$$K \equiv \frac{g_{\rho_0\rho_0\epsilon}}{g_{\epsilon\pi^+\pi^-}}\,, \tag{7}$$

we have, using the relation between the coupling constant $g_{\epsilon\pi^+\pi^-}$ and the width, Γ_ϵ, of the ϵ meson (which we assume decays entirely into two pions), that

$$G = G_2 = \frac{\frac{4}{9}[1/(16\,\pi)^2]\,[8\pi m_\epsilon^{\,2}\Gamma_\epsilon/(m_\epsilon^{\,2}/4 - m_\pi^{\,2})^{1/2}]^2}{(\mu^2 - m_\epsilon^{\,2})^2 + \Gamma_\epsilon^{\,2} m_\epsilon^{\,2}}\,, \tag{8}$$

the $\frac{4}{9}$ factor being due to the fact that $\Gamma_{\epsilon\to\pi^+\pi^-} = \frac{2}{3}\Gamma_\epsilon$. As parameters we fix m_ϵ at 700 MeV and Γ_ϵ at 500 MeV.

Our final choice is a simple point coupling, for which the $\rho_0\rho_0 \to \pi^+\pi^-$ interaction Lagrangian is

$$\mathcal{L}_{\rm int} = 16\,\pi K\,\rho_0^\alpha\rho_{0\alpha}\,\frac{\pi^+\pi^-}{2}\,, \qquad G = G_3 = q_{\pi\pi}(\mu)\,. \tag{9}$$

This is the simplest point coupling: We might also have considered derivative couplings for the pions. This would have emphasized higher values of μ^2 in our calculation, but not changed our results qualitatively. We compare our results with the accumulated data of Conversi *et al.*[5] in Fig. 2. Our procedure is to take our expression in Eq. (1) and integrate it numerically over the range $2m_\pi \leqslant \mu \leqslant M - m_\rho$ and then adjust the constant K so that the resulting $\sigma(M)$ has a peak of 17 nb, the peak of the Breit-Wigner fit of Conversi *et al.*[5] As we see in Fig. 2, the agreement of the shapes of the curves is very good. The necessary values of K to fit the height are, for the phase-shift ($\sin^2\delta_0^0$) fit, ϵ fit, and point-coupling fit, respectively,

$$K = 13.3\ ,\quad K = 16.5\ ,\quad K = 9.3\ , \tag{10}$$

which are quite large numbers. We have no theoretical insight into why they should be this size and so turn to photoproduction now.

III. ANALYSIS OF PHOTOPRODUCTION

The second reaction in which the ρ' is supposedly seen[6-8] is $\gamma p \to \rho^0\pi^+\pi^- p$. The contributing Feynman diagram is shown schematically in Fig. 3, and the corresponding invariant amplitude is

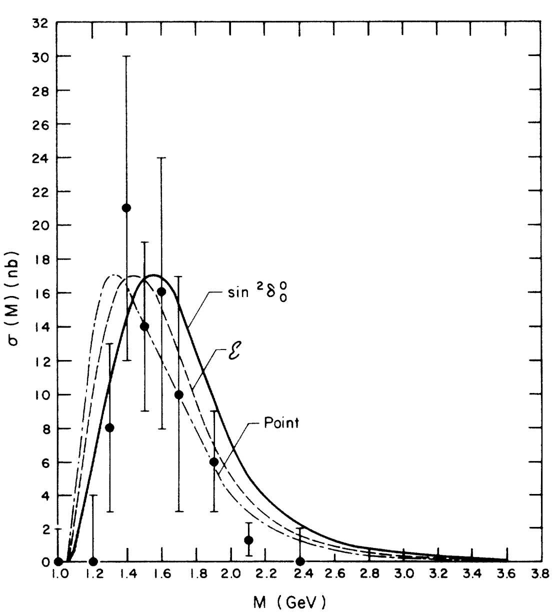

FIG. 2. Comparison of theory and experiment for $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$.

$$\mathfrak{M} = \epsilon_\mu(k, \lambda_k) A^{\mu\nu}(s, t, M^2)\, \Delta_{\nu\sigma}^{(\rho)}(M) \times R^{\sigma\tau}(\mu)\, \alpha_\tau(q, \lambda_q)\ , \tag{11}$$

$$s = (k + p_1)^2,\quad t = (p_2 - p_1)^2,$$

$$\mu^2 = (q_1 + q_2)^2\ ,$$

where ϵ_μ and α_τ are the photon and ρ polarization vectors, and λ_k and λ_q are their s-channel helicities, $\Delta_{\nu\sigma}^{(\rho)}$ is the ρ propagator, $R_{\sigma\tau}(\mu)$ is the $\rho\rho\pi\pi$ amplitude, and $A^{\mu\nu}(s, t, M^2)$, when dotted into polarization vectors, is the scattering amplitude for $\gamma p \to p$ plus off-mass-shell ρ.

In what follows we make assumptions about $A^{\mu\nu}$, $\Delta_{\nu\sigma}^{(\rho)}$, and $R_{\sigma\tau}$. The first is that

$$\epsilon_\mu(k, \lambda_k) A^{\mu\nu}(s, t, M^2) = A(s, t, M^2)\, \alpha^\nu(M, \lambda_k) = \alpha^\nu(M, \lambda_k) A(s, 0, M^2)\, e^{at/2}\ , \tag{12}$$

namely that the intermediate ρ has the initial photon helicity and that the scattering is diffractive.[17,18] For the parameter a, we will take a value of ~6 GeV^{-2}, the same as the ρ on-mass-shell values [17,18] (we will discuss this in more detail later). Similarly, we make the ansatz that helicity is conserved at the $\rho\rho\pi\pi$ vertex, so that

$$\alpha_\sigma(M, \lambda_k) R^{\sigma\tau}(\mu)\, \alpha_\tau(q, \lambda_q) = \delta_{\lambda_k, \lambda_q} R(\mu)\ . \tag{12'}$$

[In the present context the alternative assumption $R_{\sigma\tau} = -g_{\sigma\tau}R(\mu)$ leads to very similar results.] Our assumption about $\Delta_{\nu\sigma}^{(\rho)}(M)$ is that the $M_\nu M_\sigma$ terms in the numerator do not contribute when we dot $\Delta_{\nu\sigma}$ into the polarization vectors, i.e.,

$$\alpha_\nu(M, \lambda_k)\, \Delta^{\nu\sigma} R_{\sigma\tau}\, \alpha^\tau(q, \lambda_q) = \frac{\delta_{\lambda_k, \lambda_q} R(\mu)}{M^2 - m_\rho{}^2 + i\Gamma_\rho m_\rho}\ . \tag{13}$$

The $M_\nu M_\sigma$ terms did not contribute for electron-positron annihilation because of current conservation. Here they do in principle, and it can make

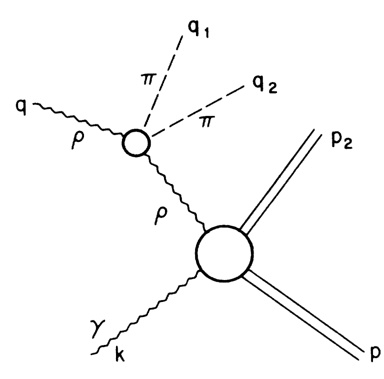

FIG. 3. Diagram for $\gamma p \to \rho^0\pi^+\pi^- p$.

a significant difference how they are evaluated; e.g., one could in a dispersion-theoretical sense set $M^2 = m_\rho^2$ in the numerator (or residue) terms or keep M^2 at its physical value. We have adopted the position of neglecting such terms altogether, i.e., Eq. (13).

Taking the absolute value squared of $\mathfrak{M}$ in Eq. (11) and making use of Eqs. (12) and (13) we find

$$\frac{d\sigma}{dM\,d\mu} = \sigma_{\gamma p\to\rho p}(s)\exp\{a[t(M^2) - t(m_\rho^2)]\} \times \frac{16K^2}{\pi^2}\, q_{\rho\pi\pi}\, G(\mu)\ , \tag{14}$$

where $t(m_\rho^2)$ and $t(M^2)$ are the minimum kinematically allowed values for t for the production of a real ρ and for a virtual ρ of mass M, respectively. The additional assumption that

$$A(s,0,M^2) = A(s,0,m_\rho^2) \tag{15}$$

has been made in relating $|A|^2$ to $\sigma_{\gamma p\to\rho p}(s)$. The exponential factor in (14) arises from the fact that in our t integration the lower limit is $t(M^2)$, not $t(m_\rho^2)$. For the slope parameter and the total cross section in (14), we use the values[17,18]

$$\sigma_{\gamma p\to\rho p}(s) = 13.5\ \mu\text{b},\qquad a = 6.5\ \text{GeV}^{-2}. \tag{16}$$

Our procedure is to integrate Eq. (14) over μ for $2m_\pi \le \mu \le M - m_\rho$ and evaluate the resulting $d\sigma/dM$ for incoming photon energies of 9.3 and 15 GeV, leaving K as a free parameter, using the three different forms of $G(\mu)$ discussed in Sec. II. To compare with experiment, we use the broad-beam data of Davier *et al.*[7]; we convert their histograms to cross sections by integrating under their histograms and demanding that the result agree with their stated cross sections

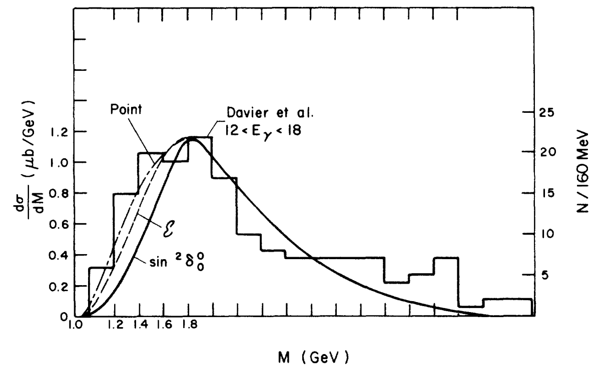

FIG. 5. $d\sigma/dM$ from theory and experiment for $\gamma p \to \rho\pi\pi p$ with $E_\gamma = 15.0$ GeV.

$$\sigma_{\gamma p\to\rho\pi\pi p} = \begin{cases} 1.4\ \mu\text{b}, & E_\gamma = 15.0\ \text{GeV} \\ 1.8\ \mu\text{b}, & E_\gamma = 9.3\ \text{GeV}. \end{cases} \tag{17}$$

We normalize the height of our curves to the data of Ref. 7 by adjusting K. Comparison of theory and experiment is given in Figs. 4 and 5. In Fig. 6 we compare Eq. (14) integrated over M rather than μ with the π-π mass plot of Bingham *et al.*[6] The shapes of the curves in all three figures agree fairly well with experiment. The values of K we obtain are given in Table I. In principle, we should obtain the same values of K for $E\gamma = 9.3$ and 15 GeV. The discrepancy is at least in part due to the fact that we have used the same value of $\sigma_{\gamma p\to\rho p}$

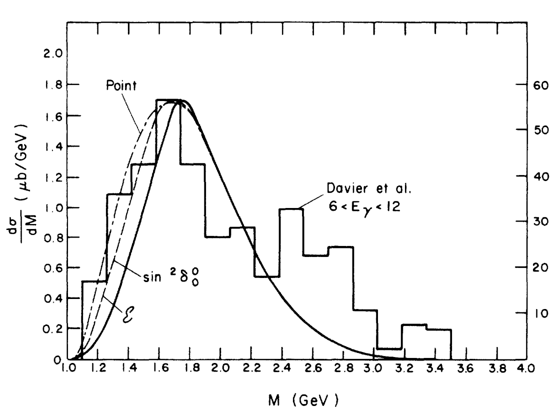

FIG. 4. Comparison of $d\sigma/dM$ from theory and experiment for $\gamma p \to \rho\pi\pi p$ with $E_\gamma = 9.3$ GeV.

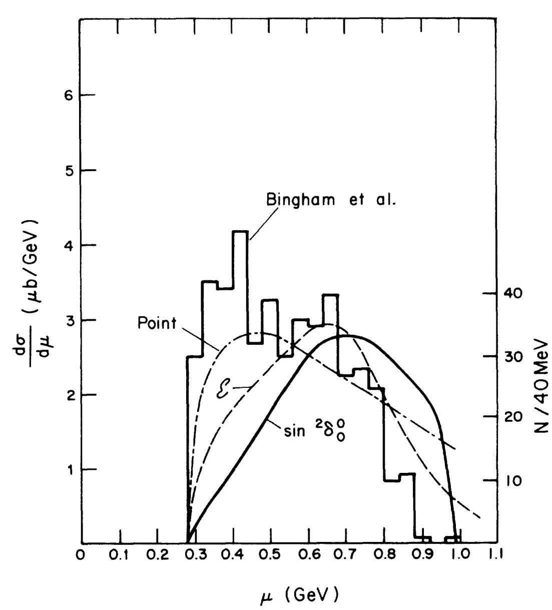

FIG. 6. $d\sigma/d\mu$ from theory and experiment for $\gamma p \to \rho\pi\pi p$ with $E_\gamma = 9.3$ GeV.

TABLE I. The values of the $\rho\rho\pi\pi$ parameter K for three parametrizations of $G(\mu)$.

	$E\gamma = 9.3$ GeV	$E\gamma = 15.0$ GeV
Phase shift	3.0	2.3
ϵ resonance	4.2	3.3
Point coupling	2.7	2.1

whereas, as seen in (17), $\sigma_{\gamma p \to \rho\pi\pi p}$ is lower at 15 GeV than at 9.3 GeV. This has the effect of reducing the effective K.

The major problem is that these values are inconsistent with the K's determined from electron-positron annihilation, as given in Eq. (10). There is still a loophole left to us, however, and that is the off-mass-shell behavior of $A(s,0,M^2)$. Instead of assuming (15) to be true, we could say that

$$A(s,0,M^2) = \frac{m_\rho^{\,2}}{M^2} A(s,0,m_\rho^{\,2}). \qquad (18)$$

This possibility arises because the diffractive scattering amplitude has asymptotic behavior in s of s/m^2, where m is some mass scale, and we can at least conceive that the scale should be M^2, the large mass in the problem. If we adopt (18) we obtain a new form for $d\sigma/dMd\mu$ which we call $d\hat{\sigma}/dMd\mu$ related to the $d\sigma/dMd\mu$ of (14) by simply

$$\frac{d\hat{\sigma}}{dM\,d\mu} = \left(\frac{m_\rho^{\,2}}{M^2}\right)^2 \frac{d\sigma}{dM\,d\mu}\,. \qquad (19)$$

We now integrate this new expression over M or μ and compare with experiment. The results are shown in Figs. 7, 8, and 9. On the whole, agreement is reasonably satisfactory, though the theoretical curves do peak at lower values of M than the experimental ones. It is hard to say how much the discrepancy is due to background events. The values of K are, of course, dramatically different, since $(m_\rho^{\,2}/M^2)$ is much smaller than one for M in the 1–2-GeV mass region. The new values, given in Table II, are in quite good agreement with the values of electron-positron annihilation as given in Eq. (10).

We thus seem to be able to fit simultaneously the data for photoproduction and for electron-positron annihilation in the "ρ'" region, albeit by stretching theoretical considerations a bit. To do this, however, we require a large value for K, i.e., an effective large $\rho\rho\pi\pi$ coupling. The question then arises as to whether the ρ-tail effect

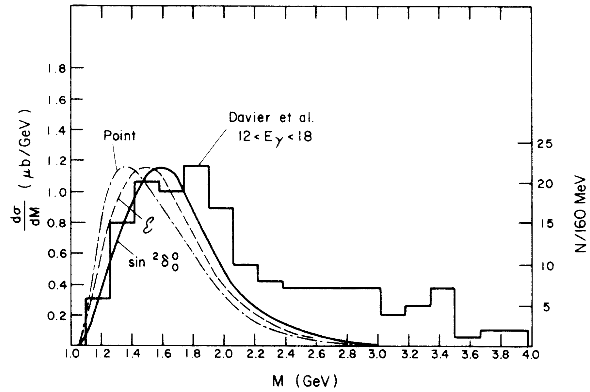

FIG. 8. Comparison of experimental $d\sigma/dM$ with $d\hat{\sigma}/dM$ for $E_\gamma = 15.0$ GeV.

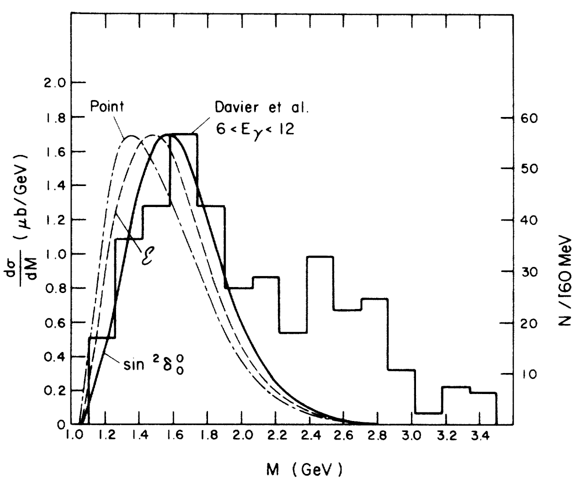

FIG. 7. Comparison of experimental $d\sigma/dM$ with $d\hat{\sigma}/dM$ for $E_\gamma = 9.3$ GeV.

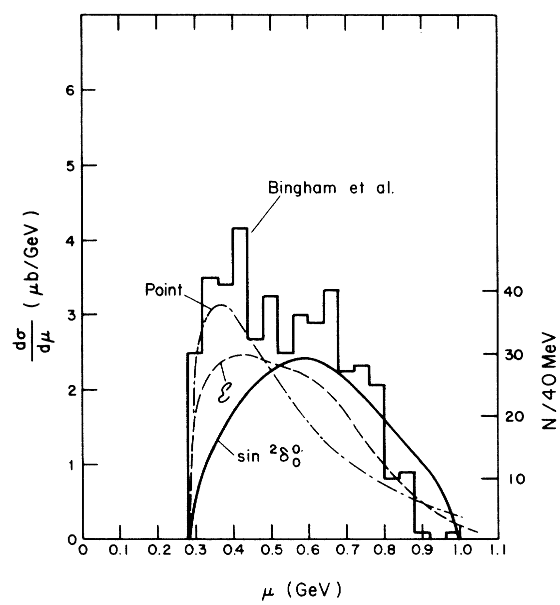

FIG. 9. Comparison of experimental $d\sigma/d\mu$ with $d\hat{\sigma}/d\mu$ for $E_\gamma = 9.3$ GeV.

would have been seen in other experiments if K is in fact of the order of ten in magnitude. To examine this question, we turn to the data for $\pi^+ p \to \Delta^{++}\rho^0\pi^+\pi^-$.

IV. ANALYSIS OF $\pi^+ p \to \Delta^{++}\rho^0\pi^+\pi^-$

Eisenberg *et al.*[9] have measured the cross sections for $\pi^+ p \to \Delta^{++}\pi^+\pi^-$ and $\pi^+ p \to \Delta^{++}\pi^+\pi^-\pi^+\pi^-$ using an incident 5 GeV/c π^+ beam. By confining themselves to a small-t region, they could safely assume the process to be dominated by one-pion exchange; they looked for ρ' production and ρ production in, respectively, the four-pion and two-pion final states. The latter was large and unmistakable, whereas the four-pion region was fitted quite well by phase space weighted by an $e^{2.3t}$ factor. Their conclusion was that the ρ' did not couple appreciably to two pions, $g_{\rho'\pi\pi}{}^2/g_{\rho\pi\pi}{}^2 < 0.02$, since otherwise it would have been produced in greater numbers. They also conclude, however, that the observed ρ' enhancement in photoproduction cannot be due to the ρ^0 tail. We believe that this conclusion does not follow from their analysis alone.

In our model, diagrammatically displayed in Fig. 10, we would expect

$$\frac{d\sigma}{dM\,d\mu} = \sigma_{\pi^+p\to\rho^0\Delta^{++}}(s)\exp\{B[t(M^2)-t(m_\rho{}^2)]\} \times \frac{16}{\pi^2}\frac{K^2 q_{\rho\pi\pi} G(\mu)}{(M^2-m_\rho{}^2)^2+\Gamma_\rho{}^2 m_\rho{}^2}\,. \tag{20}$$

We set $B = 2.3\ \mathrm{GeV}^{-2}$ and use the observed values for $\sigma_{\pi^+p\to\Delta^{++}\rho^0}(s)$ given in Ref. 9. There presumably is no ambiguity of the sort displayed in Eq. (18) caused by mass factors because the scattering amplitude $A_{\pi^+p\to\rho^0\Delta^{++}}(s)$ is dominated by one-pion exchange for small t and hence behaves approximately like a constant (in a Regge-pole model we would say $s^{\alpha_\pi(0)} \sim s^0 \sim 1$), whereas the $\gamma p \to \rho^0 p$ scattering is diffractive.

We compare with experiment in Fig. 11 for only one form of $G(\mu)$, namely the final-state interaction with the π-π phase shift. The lower solid curve corresponds to the value of K given in Table I and the upper solid curve to the value of K from Table II. The first is clearly compatible with experiment, while the second is not.

TABLE II. The values obtained for K using Eq. (18) for $A(s,0,M^2)$.

	$E\gamma$=9.3 GeV	$E\gamma$=15.0 GeV
Phase shift	14.1	11.2
ϵ resonance	17.7	14.3
Point coupling	10.1	8.2

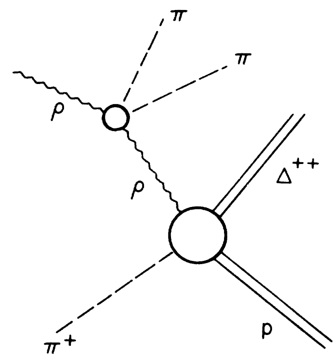

FIG. 10. Diagram for $\pi^+ p \to \rho^0\pi^+\pi^-\Delta^{++}$.

We therefore conclude that the ρ-tail mechanism cannot account for the $\rho'(1600)$ enhancement. If the ansatz of Eq. (15) is made for the virtual ρ photoproduction amplitude, the magnitudes of $\gamma p \to \rho\pi^+\pi^- p$ and $e^+e^- \to \rho\pi^+\pi^-$ are incompatible. With the ansatz of Eq. (18) they are compatible, but then a large effect would be seen in $\pi^+ p \to \Delta^{++}\rho\pi^+\pi^-$.

V. REAL ρ' RESONANCE

In the previous three sections we have tried to analyze data under the assumption that the various

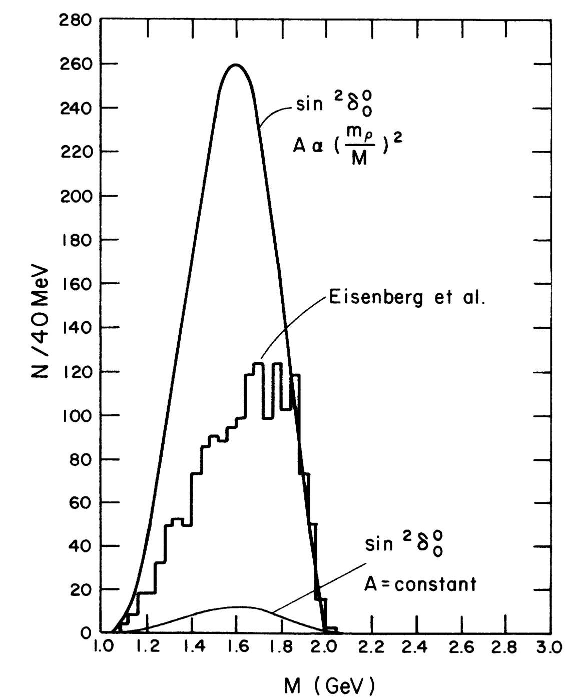

FIG. 11. Comparison of theory and experiment for $\pi^+ p \to \Delta^{++}\pi^+\pi^-\pi^+\pi^-$.

differential cross sections could be accounted for without invoking the presence of a $\rho'(1600)$; the ρ tail would simulate it.

We would now like to briefly discuss fitting data if a ρ' does in fact exist, as seems likely. The absence[9] of $\pi^+p\to\Delta^{++}\rho'$ is, of course, explained by saying that the process is dominated by one-pion exchange and that the coupling constant $g_{\rho'\pi\pi}$ is small compared to $g_{\rho\pi\pi}$. (It might be interesting to estimate the three-pion exchange contribution in this context.)

The photoproduction data of Bingham *et al.*[6] gives for the ρ' a mass and width of 1.43 ± 0.05 GeV and 0.65 ± 0.1 GeV, respectively, though they point out that these determinations are sensitive to how one parametrizes the ρ' meson. The photoproduction data of Davier *et al.*[7] give a ρ' with mass and width of 1.62 ± 0.03 GeV and 0.31 ± 0.07 GeV, respectively. We have analyzed the photoproduction data using the three final-state interaction models of Sec. II, but have little to add to the discussions of Ref. 6 and 7.

As far as $e^+e^-\to\pi^+\pi^-\pi^+\pi^-$ is concerned, the data are displayed in Fig. 2. Conversi *et al.*[5] have fitted it by a Breit-Wigner form with a ρ' resonance, whose mass and width are found to be 1.55 ± 0.06 GeV and 0.36 ± 0.10 GeV. Their fit is shown in Fig. 12. Using the same formalism as in Sec. II, we find a differential cross section formula

$$\frac{d\sigma}{d\mu}=\frac{32\alpha^2}{f_{\rho'}^2/4\pi}\frac{m_{\rho'}^4}{M^5}\frac{q_{\rho\pi\pi}(1+q_{\rho\pi\pi}^2/3m_\rho^2)G(\mu)K'^2}{(M^2-m_{\rho'}^2)^2+m_{\rho'}^2\Gamma_{\rho'}^2}, \tag{21}$$

to be contrasted to formula (1). Here $K'^2G(\mu)$ is a parametrization of the $\rho'\rho\pi\pi$ interaction and $f_{\rho'}^2/4\pi$ is the inverse squared of the γ-ρ' coupling. The smooth Breit-Wigner fit of Conversi *et al.*[5] corresponds most closely to our taking a point coupling for the $\rho'\rho\pi\pi$ interaction. If we instead employ a π-π phase-shift model, i.e., Eq. (5), we find an appreciable deviation; in particular, the shape is no longer the Breit-Wigner type. In Fig. 12 we contrast the Breit-Wigner curve drawn by Conversi *et al.*[5] through their data with the result of integrating Eq. (21) with a $G(\mu)$ given by Eq. (5), adjusting the curve so that it also peaks at $\sigma(M)=17$ nb. As input we use $m_{\rho'}=1.55$ GeV and $\Gamma_{\rho'}=0.36\pm0.10$ GeV, the Conversi *et al.*[5] values.

If we allow for the fact that the shape of the fitting curve may be a distorted Breit-Wigner shape, we can try to fit the data differently. As an example, we show a curve in Fig. 12, in which we still have $\Gamma_{\rho'}=0.36\pm0.10$ GeV, but have demanded that it peak at the lower value of M of 1.40 GeV. Because of the distortion, it appears

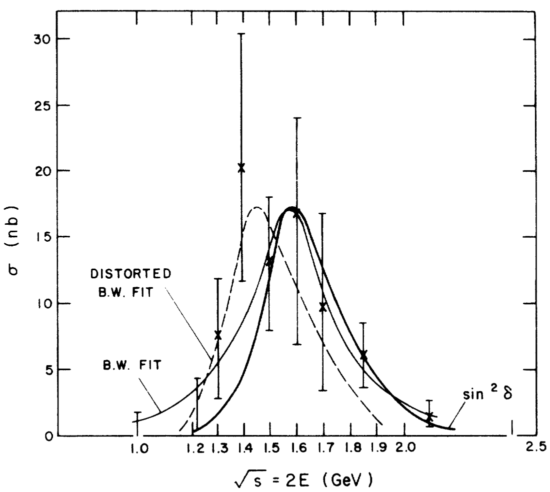

FIG. 12. Fit to $e^+e^-\to\pi^+\pi^-\pi^+\pi^-$ with Breit-Wigner and distorted Breit-Wigner shapes.

to provide an equally satisfactory fit to the data. In this sense, one should probably allow for a larger uncertainty in the mass of the ρ' than is quoted. Better experimental information would, of course, not only reduce the uncertainty, but also provide information about the $\rho\pi\pi$ interaction.

We now extract the ρ' parameters $f_{\rho'}$, K' (which measures the strength of the $\rho'\rho\pi\pi$ coupling), and $\sigma_{\rho'p}$ from the data. We assume $m_{\rho'}=1.55\pm0.06$ GeV and $\Gamma_{\rho'}=0.36\pm0.10$ GeV, as given by Conversi *et al.*[5] From Eq. (21) we can determine $K^2/(f_{\rho'}^2/4\pi)$ for each of the parametrizations for the $\rho'\rho\pi\pi$ coupling by integrating over μ and requiring that the maximum value of the cross section agree with the experimental value[5] of 17 ± 5 nb. The results are listed in Table III.

As in Sec. III we write the cross section for $\gamma p\to\rho\,\pi^+\pi^-p$ with the ρ' as

$$\frac{d\sigma}{dMd\mu}=\sigma_{\gamma p\to\rho'p}(s)\exp\{a[t(M^2)-t(m_{\rho'}^2)]\}\times\frac{16K'^2}{\pi^2}\frac{q_{\rho\pi\pi}G(\mu)}{(M^2-m_{\rho'}^2)^2+m_{\rho'}^2\Gamma_{\rho'}^2}. \tag{22}$$

The vector-meson-dominance model (VMD) for $\sigma_{\gamma p\to\rho'p}$ assumes that the reaction proceeds via the photon converting into a ρ' (with no form factor) which then scatters diffractively. Hence,

$$\sigma_{\gamma p\to\rho'p}(s)=\frac{e^2}{16\pi f_{\rho'}^2}\frac{e^{at(m_{\rho'}^2)}}{a}[\sigma_{\rho'p}(s)]^2, \tag{23}$$

where $\sigma_{\rho'p}$ is the $\rho'p$ total cross section.

Experimentally,[6,7,19] the integral of $d\sigma/d\mu dM$ over M and μ is 0.85 ± 0.30 μb at $E\gamma=9.3$ GeV/c, and the slope parameter is[7] $a=5.8\pm0.4$ GeV^{-2}.

TABLE III. The ρ' parameters determined for three parametrizations of $G(\mu)$.

	$4\pi K'^2/f_{\rho'}^2$	$\sigma_{\gamma p\to\rho' p}$ (9.3 GeV/c)	$\sigma_{\rho' p}$ (9.3 GeV/c)	$f_{\rho'}^2/4\pi$	K'
Phase shift	0.45 ± 0.22	1.40 ± 0.39 μb	22.7 ± 3.4 mb	21.2 ± 8.4	3.1 ± 0.4
ϵ resonance	0.76 ± 0.37	1.62 ± 0.45 μb	23.9 ± 3.6 mb	20.2 ± 8.0	3.9 ± 0.5
Point coupling	0.31 ± 0.15	1.60 ± 0.45 μb	23.6 ± 3.5 mb	20.1 ± 8.0	2.5 ± 0.4

Fitting (22) and (23) to the integrated cross section, and using the results already determined for $K'^2/(f_{\rho'}^2/4\pi)$ we find the values given in Table III for $\sigma_{\gamma p\to\rho' p}$ and $\sigma_{\rho' p}$ at 9.3 GeV/c. The result, $\sigma_{\rho' p}\sim 23\pm4$ mb is relatively stable under changes in the $\rho'\rho\pi\pi$ parametrization and in $\Gamma_{\rho'}$. It is also in suprisingly good agreement with the ρp cross section $\sigma_{\rho p}=22.2\pm2.6$ mb determined from the $\gamma p\to\rho p$ cross section[17,18] under similar assumptions. This provides indirect evidence for the validity of VMD and the quark-model assumption that meson-baryon diffractive cross sections are the same for all mesons.

Finally, if one assumes that the ρ' decays only into $\rho\pi\pi$, with the two pions in an $I=0$ state, then one has

$$\Gamma_{\rho'}=\tfrac{3}{2}\Gamma_{\rho'\to\rho\pi^+\pi^-}$$
$$=\frac{12K'^2}{\pi m_{\rho'}^2}\int_{2m_\pi}^{m_{\rho'}-m_\rho} d\mu\, q_{\rho\pi\pi}G(\mu)\,, \qquad (24)$$

where $q_{\rho\pi\pi}$ is given by Eq. (2), with M replaced by $m_{\rho'}$. By comparing (24) to the experimental width, one can determine K' and $f_{\rho'}^2/4\pi$. The results are given in Table III. The large errors are due to the uncertainty in $\Gamma_{\rho'}$; $f_{\rho'}^2/4\pi$ is relatively independent of the parametrization.

The value of $f_{\rho'}^2/4\pi\approx21\pm10$ is to be compared with the ρ-γ coupling of $f_\rho^2/4\pi\approx2.1\pm0.5$ and the $\rho'(1250)$-γ coupling[5] of $f_{\rho'}^2(1250)/4\pi\approx7\pm2$. Within the present model the smallness of the $\gamma p\to\rho' p$ cross section compared with the ρ photoproduction cross section[17,18] of 13.5 ± 0.5 μb is mainly due to $f_{\rho'}^2$. The magnitude K' of the $\rho'\rho\pi\pi$ coupling is still rather large.

VI. THE ρ' (1250)

Peaks in the $\pi^+\pi^-\pi^0\pi^0$ mass spectra at a mass of 1250 MeV and width 150 MeV have recently been observed in the reactions[5,14] $\gamma p\to\pi^+\pi^-\pi^0\pi^0 p$ and $e^+e^-\to\pi^+\pi^-\pi^0\pi^0$. One might speculate that these peaks are threshold enhancements due to the opening of the $\omega\pi^0$ channel. That is, the enhancements would be caused by diagrams similar to Figs. 1 and 3, except that the virtual ρ decays into an $\omega\pi^0$ pair.

The $\rho\omega\pi^0$ coupling is assumed to be

$$g_{\omega\rho\pi}\epsilon^{\mu\nu\tau\sigma}\epsilon_\mu^{(\rho)}\epsilon_\nu^{(\omega)}p_\tau^{(\rho)}q_\sigma^{(\omega)}\,, \qquad (25)$$

where $\epsilon_\mu^{(\rho)}$ $(\epsilon_\nu^{(\omega)})$ and $p_\tau^{(\rho)}$ $(q_\sigma^{(\omega)})$ are the polarization vector and momentum of the ρ (ω). The coupling $g_{\omega\rho\pi}$ is taken to be 22 GeV^{-1} from the Gell-Mann–Sharp–Wagner model[20] of ω decay.

One can then calculate the virtual-ρ contribution to $e^+e^-\to\omega\pi^0$ as

$$\sigma(M)=\frac{\alpha^2 m_\rho^4}{3f_\rho^2/4\pi}g_{\rho\omega\pi}^2\left(\frac{P_\omega}{M}\right)^3\frac{1}{(M^2-m_\rho^2)^2+m_\rho^2\Gamma_\rho^2}\,, \qquad (26)$$

where P_ω is the ω momentum in the virtual-ρ center-of-mass system, and M is the mass of the $\omega\pi^0$ pair. Equation (26) has previously been given by Renard.[21] Although it peaks at around 1150 MeV, it is much smaller than the experimental data [Eq. (26) is compared with the data in Ref. 5].

Therefore, the ρ tail cannot be responsible for the 1250 enhancement. Nevertheless, it could be an important background to the $\rho'(1250)$ in the reaction $\gamma p\to\pi^+\pi^-\pi^0\pi^0 p$. We have calculated this background; as in Sec. III we assume that the virtual ρ carries the s-channel helicity of the photon. The only real ambiguity concerns the M dependence of the virtual $\gamma p\to\rho(M)p$ amplitude. If one assumes an amplitude independent of M [Eq. (15)], then

$$\frac{d\sigma}{dM}=\sigma_{\gamma p\to\rho p}(s)\exp\{a[t(M^2)-t(M_\rho^2)]\}\times\frac{g_{\omega\rho\pi}^2M^2P_\omega^3}{6\pi^2[(M^2-m_\rho^2)^2+m_\rho^2\Gamma_\rho^2]}\,. \qquad (27)$$

If one assumes (18) for the amplitude, then

$$\frac{d\hat\sigma}{dM}=\left(\frac{m_\rho^2}{M^2}\right)^2\frac{d\sigma}{dM}\,. \qquad (28)$$

The two possibilities are plotted in Fig. 13 for an incident energy of 9.3 GeV/c. The parameters are in Sec. III. By way of comparison, Ballam *et al.*[14] find a total cross section for $\gamma p\to p\rho'(1250)\to\pi^+\pi^-\pi^0\pi^0 p$ of 1–3 μb with a width of 150 to 310 MeV. Clearly, if Eq. (15) were correct for $A(s,0,M^2)$, then the ρ tail would produce an effect much larger than is seen. On the other hand, Eq. (18) leads to a relatively small background.

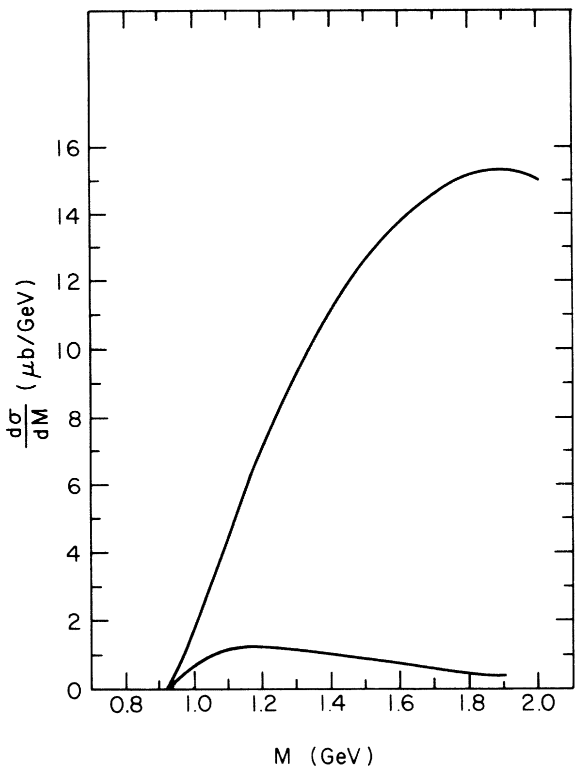

FIG. 13. Background contribution to $\gamma p \to \omega\pi^0 p$ due to virtual ρ. Upper curve is $d\sigma/dM$. Lower curve is $d\hat{\sigma}/dM$.

The parameters of the $\rho'(1250)$ are not sufficiently well determined to extract the $\rho'(1250)p$ total cross section in the VMD model. However, Conversi *et al.*[5] have determined $f_{\rho'}{}^2(1250)/4\pi = 7 \pm 2$.

VII. CONCLUSIONS

There appears to be good evidence for a broad resonance with quantum numbers 1^{--} and a mass at about 1.60 GeV which decays primarily into $\rho\pi\pi$. The evidence for the existence of the so-called ρ' comes from $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, $\gamma p \to p\pi^+\pi^-\pi^+\pi^-$, and $\pi^+p \to \Delta^{++}\rho^0\pi^+\pi^-$. We have tried to see whether the data could be explained without invoking the presence of a new resonance, attributing the effect entirely to the tail of the ρ meson. The analysis of each experiment claims to rule this out as a possible interpretation, but we have found the matter to be somewhat more complicated, though our final conclusion is that in fact the ρ tail cannot simultaneously explain all three sets of experiments. It can, however, account for both the electron-positron and photoproduction data by allowing for a very large $\rho\rho\pi\pi$ coupling and an appropriate mass factor in the photoproduction amplitude of off-mass-shell ρ mesons. Alternatively, without these factors, and with a smaller $\rho\rho\pi\pi$ coupling, one can simultaneously fit the photoproduction data and the $\pi^+p \to \Delta^{++}\rho^0\pi\pi$ data. One cannot, however, fit all three sets of experiments in a consistent manner without invoking a real ρ' resonance. It is, of course, possible that some entirely different type of background could be responsible for the ρ' enhancement. A Deck-type mechanism[22] could not do the job, however, because the enhancement is seen in e^+e^- annihilation as well as in photoproduction.[23]

Admitting the existence of this particle, we show how there may nevertheless be some uncertainty in what its mass is, owing to final-state interaction effects. Assuming vector-meson dominance, we have determined the $\rho' p$ total cross section to be 23 ± 4 mb, in remarkable agreement with $\sigma_{\rho p} = 22.2 \pm 2.6$ mb. Further, assuming that the ρ' always decays into $\rho\pi\pi$, we find $f_{\rho'}{}^2/4\pi \approx 21 \pm 10$, compared with $f_\rho^2/4\pi \approx 2.1 \pm 0.5$. The value of the $\rho'\rho\pi\pi$ coupling, which depends on the parametrization, is found to be large.

We have also examined the background due to the ρ tail for the $\rho'(1250)$ experiments. In particular, the background in the $\gamma p \to \rho'(1250)p \to \pi^+\pi^-\pi^0\pi^0 p$ would be enormous if the virtual ρ could be photoproduced without any suppression for large virtual-ρ mass. With a suppression as given by Eq. (19), however, the background is small.

The question of why neither the $\rho'(1600)$ nor the $\rho'(1250)$ couple strongly to two pions remains a mystery.

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