Phenomenological analysis of the properties of narrow resonances and possible evidence for a new quantum number

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A phenomenological analysis is carried out to show that the narrow resonances at 3095 and 3685 MeV probably carry a new quantum number which forbids them to decay into ordinary hadrons by strong and medium-strong interactions. It is also shown that the radiative decay modes of these particles are considerably suppressed relative to that of ordinary hadrons. Production in hadronic collisions and photoproduction of these particles are analyzed in this framework. It is suggested that no resonance carrying the same new quantum number can exist with a mass below 2.5 GeV and that the 4.1-GeV resonance can only decay into 3095-MeV and 3685-MeV particles with a width of less than 1 MeV. It is shown, moreover, that the assignment of the ordinary SU(3) quantum numbers of $\omega(783)$ to these states, besides the new quantum number, is favored by experiments.

I. INTRODUCTION

The recent exciting discovery[1] of two very narrow resonances $\psi(3095)$ and $\psi'(3685)$ has given rise to a number of speculations concerning the nature of these particles. Subsequently, the decay modes of these particles[2] and the photoproduction of the 3095-MeV particle[3] have been observed. Since the 3095-MeV resonance is photoproduced with a large cross section, one can conclude it is a hadron. Whether the existence of these particles could be explained by various models so far advanced, such as the charm scheme,[4] color Han-Nambu model,[5] or something else, is difficult to tell at present because of the lack of more detailed experimental data concerning the properties of these resonances; in particular for the Han-Nambu model, one would expect that these resonances decay mainly into ordinary hadrons accompanied by the emission of a photon with a considerable rate. While this question will be eventually settled in the near future, one can at present carry out a phenomenological analysis to help to clarify the situation.

The purpose of this paper is to show that all the characteristics and the decay modes of these two narrow resonances point to the existence of a new quantum number X of unknown origin which forbids them to decay into ordinary hadrons. The strength of the X-conserving reactions is, however, suppressed relative to ordinary strong interaction. Using this fact we then show that the radiative modes $\psi\to\pi\pi\gamma$ and $\psi\to\eta\gamma$ are suppressed.

The paper is organized as follows: In Sec. II we begin with the analysis of the production of these particles at SLAC. The results show that these particles couple to the virtual photon almost like other well-known vector mesons. We then give an analysis of the decay of the second particle ψ'. Its large width compared to the first one and its substantial cascade decay mode into the first one with the emission of two pions allow us to infer that there must exist a new quantum number X which distinguishes these two particles from ordinary hadrons and that the smallness of the decay width of these two particles into ordinary hadrons can only be attributed to the violation of the new quantum number by interactions other than usual strong interactions. This hypothesis will be used in Sec. III to carry out calculations for other possible decay modes of ψ and ψ' as well as their production in hadron and photon collisions with matter. For the radiative decay modes of ψ, using vector-meson dominance, it is shown that the processes $\psi\to\pi\pi\gamma$ and $\psi\to\eta\gamma$ are much suppressed relative to the radiative rates of ordinary vector mesons. Because of the small widths of these particles ψ and ψ', it is shown that new particles, be they pseudoscalar or vector mesons, which have the new quantum number, cannot exist below 2.5 GeV. The last section (Sec. IV) is devoted to a discussion of assigning ordinary SU(3) quantum numbers to $\psi(3.1)$ and $\psi'(3.7)$ besides the new quantum number X. It is shown that the assignment of the SU(3) quantum numbers of $\omega(783)$ to $\psi(3.1)$ and $\psi'(3.7)$ seems to be favored by experiments.

II. EVIDENCE FOR A NEW QUANTUM NUMBER

As a basis for the following analysis, we shall summarize here the main characteristics of the two narrow resonances[1,2] ψ and ψ'. The numerical results obtained below eventually will have to be corrected in accordance with the final experimental data.

For the ψ

$J^P = 1^-$,

$m_\psi = 3095 \pm 10$ MeV,

$\Gamma(\psi \to e^+e^-) = 5.2 \pm 1.3$ keV,

$\Gamma_{tot}(\psi) = 77 \pm 19$ keV.

For the ψ'

$J^P = 1^-$

$m_{\psi'} = 3685$ MeV,

$\Gamma(\psi' \to e^+e^-) = 2.2 \pm 0.5$ keV,

$200 \leq \Gamma_{tot}(\psi') \leq 800$ keV,

$\Gamma(\psi' \to \psi\pi^+\pi^-) = (0.31 \pm 0.04)\Gamma_{tot}(\psi')$,

$\Gamma(\psi' \to \psi + \text{anything}) = (0.54 \pm 0.08)\Gamma_{tot}(\psi')$.

A. Production of ψ and ψ' in e^+e^- colliding-beam experiments at SLAC

We assume that the ψ and ψ' couple to the leptons via a photon according to the usual one-photon-exchange diagram. Let the $\psi\gamma$ coupling constant be $f_{\gamma\psi}$; we have a coupling $f_{\gamma\psi}\psi^\mu A_\mu$. A straightforward calculation gives

$$\Gamma(\psi \to e^+e^-) = \Gamma(\psi \to \mu^+\mu^-) = \frac{\alpha}{3}\left(\frac{f_{\gamma\psi}}{m_\gamma^2}\right)^2 m_\psi \tag{1}$$

and similar expressions for ψ'. From experimental data, one gets $f_{\gamma\psi}/m_\psi^2 = 2.7 \times 10^{-2} = 3.7\alpha$ and $f_{\gamma\psi'}/m^2\psi' = 1.7 \times 10^{-2} = 2.3\alpha$. These are to be compared with $f_{\gamma\omega}/m_\omega^2 = 2.6\alpha$, $f_{\gamma\phi}/m_\phi^2 = 3.2\alpha$, and $f_{\gamma\rho}/m_\rho^2 = 7.7\alpha$, respectively, for the ω, ϕ, and ρ resonances. It appears then that as far as their couplings with leptons are concerned, the new particles ψ and ψ' just behave as ordinary vector mesons. The extraordinary behavior of these particles with respect to other interactions becomes clear in their decay modes into hadrons as will be discussed below.

B. Decay modes of ψ and ψ'

For our purpose, we shall first consider the cascade decay of ψ'.

1. $\psi' \to \psi\pi^+\pi^-$

The general matrix element for this process is complicated and involves many amplitudes. Let us for simplicity assume the $\pi^+\pi^-$ are in the S state, which is a good approximation here because higher angular momentum states ($D, G, \ldots$) are suppressed by the centrifugal barrier. The general matrix element is

$$\mathfrak{M} = \epsilon_\mu(\psi')\mathfrak{M}^\mu, \tag{2}$$

where $\epsilon_\mu(\psi')$ is the polarization vector of ψ'. On general invariance grounds $\mathfrak{M}^\mu$ can be written as

$$\mathfrak{M}^\mu = a(s,t,u)\epsilon^\mu(\psi) + b(s,t,u)(\epsilon(\psi)\cdot p')p^\mu + c(s,t,u)(\epsilon(\psi)\cdot p')p'^\mu, \tag{3}$$

where p' and p are, respectively, the 4-momenta of ψ' and ψ. a, b, and c are, in general, functions of the following variables:

$$s = (p'-p)^2 = (q_1+q_2)^2, \quad t = (p'-q_1)^2, \quad u = (p'-q_2)^2, \tag{4}$$

with

$$s + t + u = m_{\psi'}^2 + 2\mu^2,$$

where q_1 and q_2 are the 4-momenta of the two pions of mass μ. In general, there is no relation between a, b, and c unless we assume that ψ' is itself a gauge particle. The experimental situation dealt with here does require such a condition. This is because in the production by e^+e^- collisions, ψ' is coupled to a virtual photon; since the hadronic electromagnetic current is conserved, we must impose the condition

$$p'_\mu \mathfrak{M}^\mu = 0. \tag{5}$$

This condition is automatically satisfied for an effective $\psi'\psi\pi\pi$ coupling of the following form:

$$\mathcal{L}_{int} = g\vec{\phi}\cdot\vec{\phi}\, F^\psi_{\mu\nu}F^{\mu\nu}_{\psi'} \tag{6}$$

($F_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu$, ϕ_μ being the vector-meson fields), which is equivalent to assuming $c = 0$ and imposing condition (5). This form is useful in computing radiative decay rates of ψ as discussed in the following sections. For this reason we shall use this type of coupling to calculate $\mathfrak{M}$. One then obtains

$$\mathfrak{M}^\mu = b(s,t,u)[-p'\cdot p\epsilon^\mu(\psi) + \epsilon(\psi)\cdot p'p^\mu]. \tag{7}$$

Because the interaction between ordinary hadrons and ψ, as will be seen below, is suppressed, we neglect final-state interactions between ψ and the two pions. Under this approximation b is a function of a single variable s. With the $\pi\pi$ final-state interaction taken into account by the usual Omnès function,[6] $b(s)$ is given by

$$b(s) = b\exp\left[\frac{s-4\mu^2}{\pi}\int_{4\mu^2}^\infty \frac{\delta^0(s')\,ds'}{(s'-4\mu^2)(s'-s-i\epsilon)}\right], \tag{8}$$

where we have defined $b(4\mu^2) = b$ and δ^0 is the S-wave isospin-0 $\pi\pi$ scattering phase shift. There are experimental evidences for an S-wave $\pi\pi$ resonance at a mass m_ϵ of 700 MeV with a large width $\Gamma \simeq 500$ MeV; therefore we shall approximate the right-hand side of Eq. (8) by

$$b(s)=b\,\frac{m_\epsilon^2-4\mu^2}{(m_\epsilon^2-s)-i\,m_\epsilon\Gamma(1-4\mu^2/s)^{1/2}\theta(s-4\mu^2)}\,. \tag{9}$$

The decay width is given by

$$\Gamma(\psi'\to\psi\pi^+\pi^-) = \frac{1}{3}\,\frac{1}{2m_{\psi'}}\sum_{\text{pol}}\int|\mathfrak{M}|^2\prod_n\frac{d^3p_n}{2\omega_n(2\pi)^3}\,\delta^4\Big(p_i-\sum_n p_n\Big), \tag{10}$$

where the factor $\frac{1}{3}$ comes from averaging of polarization in the initial state. Using the expression for $\mathfrak{M}^\mu$ in Eq. (7), one gets

$$|\mathfrak{M}|^2=2|b(s)|^2(p'\cdot p)^2\left(1+\frac{m_\psi^2}{2E^2}\right) = \frac{|b(s)|^2}{2}(m_{\psi'}^2+m_\psi^2-s)^2\left(1+\frac{2m_\psi^2m_{\psi'}^2}{(m_{\psi'}^2+m_\psi^2-s)^2}\right). \tag{11}$$

From this expression it is clear that the pion angular distribution as measured relative to the ψ momentum in the c.m. system of the two pions is isotropic. The angular integration is then trivial and yields

$$\Gamma(\psi'\to\psi\pi^+\pi^-)=\frac{1}{12(64\pi^3)m_{\psi'}^3}\times\int ds\left(\frac{s-4\mu^2}{s}\right)^{1/2}\times[(m_{\psi'}^2-m_\psi^2-s)^2-4m_\psi^2 s]^{1/2}|\mathfrak{M}|^2.$$

A numerical integration gives

$$\Gamma(\psi'\to\psi\pi^+\pi^-)=0.15\times10^{-3}|b|^2. \tag{12}$$

At present no value for the total width of ψ' is given but probably it is much larger than that of ψ, of the order 600–800 keV. Therefore, we shall assume $\Gamma(\psi'\to\psi\pi^+\pi^-)\simeq 200$ keV. Defining $|b| = g_{\psi'\psi\pi\pi}/m^2\psi'$, we have

$$g_{\psi'\psi\pi\pi}^2/4\pi=20. \tag{13}$$

It matters little whether we use the coupling given by Eq. (6) or an effective interaction which does not satisfy explicitly the current-conservation condition, for example, a coupling of the form $g\vec{\phi}\cdot\vec{\phi}\psi_\mu\psi'^\mu$, since terms which involve $b(s)$ in Eq. (3), being proportional to the small ψ recoil momentum, contribute little compared with the a terms. [In fact, the matrix element without the b term gives a factor $(1+E^2/2m_\psi^2)$ instead of $(1+m_\psi^2/2E^2)$; as $E\simeq m_\psi$ these two expressions are almost the same.] To find how much the rate given above depends on $\pi\pi$ interaction, let us put $\delta^0=0$ in Eq. (8); one then deduces the same value for $g_{\psi'\psi\pi\pi}^2$. As one can see, the rate is not affected by the $\pi\pi$ final-state interaction since the two pions do not have enough energy to be in the ϵ resonance region. It is difficult to compare the coupling constant $g_{\psi'\psi\pi\pi}$ obtained above with the usual strong-interaction coupling constant encountered, for example, in $\rho'\to\rho\pi\pi$. In the latter case one gets $g_{\rho'\rho\pi\pi}^2/4\pi \simeq 800$ for a partial width of 200 MeV if only the S wave is involved and no final-state interaction is taken into account. The $\pi\pi$ final-state interaction as dominated by the ϵ resonance will enhance the matrix element and hence reduce the effective coupling constant. Using the enhancement factor given by Eq. (9), one gets $g_{\rho'\rho\pi\pi}^2/4\pi \simeq 500$. Thus it appears that the coupling constant $g_{\psi'\psi\pi\pi}^2/4\pi$ is suppressed relative to $g_{\rho'\rho\pi\pi}^2/4\pi$ by a factor of 30–50. More or less the same conclusion holds if the matrix element contains a zero.[7] For a detailed analysis of the shape of the two-pion spectrum we refer to a separate paper.[7]

2. $\psi\to\omega\pi\pi$

This decay process looks similar to $\psi'\to\psi\pi\pi$ except that only ordinary hadrons are observed in the final state. It is because of the total decay rate of ψ into hadrons and of this decay rate that the extraordinary behavior of ψ becomes evident. Since ψ, as ψ' in the case of $\psi'\to\psi\pi\pi$, is produced in e^+e^- collisions through its interaction with a virtual photon, we must consider ψ as a gauge particle which subsequently decays into hadrons. Assuming an explicitly gauge-invariant effective Lagrangian similar to that given in Eq. (6),

$$\mathcal{L}_{\text{int}}=g\vec{\phi}\cdot\vec{\phi}F^\psi_{\mu\nu}F^{\mu\nu}_\omega, \tag{6$'$}$$

we have for the matrix element

$$\mathfrak{M}=b(s,t,u)[(\epsilon^\psi\cdot\epsilon^\omega)(p^\omega\cdot p^\psi)-(\epsilon^\psi\cdot p^\omega)(\epsilon^\omega\cdot p^\psi)].$$

It is assumed here for simplicity that the two pions are in a relative S state. The partial rate, with $\pi\pi$ final-state interaction taken into account in the same way as in the case $\psi'\to\psi\pi\pi$, is given by

$$\Gamma=\frac{|b|^2}{12(64\pi^3)M^3}\int_{4\mu^2}^{(M-m)^2}ds\,(1-4\mu^2/s)^{1/2}\times[(M^2-m^2-s)^2-4m^2s]^{1/2} \times\frac{(m_\epsilon^2-4\mu^2)^2}{(m_\epsilon^2-s)^2+(1-4\mu^2/s)m_\epsilon^2\Gamma_\epsilon^2}\,\frac{(s-M^2-m^2)^2}{2}\left[1+\frac{2m^2M^2}{(s-M^2-m^2)^2}\right], \tag{14}$$

where M, m, and μ are the masses of ψ, ω, and π, respectively. A numerical calculation gives

$$\Gamma = 0.53\times10^{-3}|b|^2. \tag{15}$$

Using the preliminary data for $\Gamma(\psi\to\omega\pi^+\pi^-)\simeq 1$ keV (see Ref. 8) with the dimensionless effective coupling constant $g_{\psi\omega\pi\pi}$ defined as $|b|m_\psi^2$, one deduces

$$g_{\psi\omega\pi\pi}^2/4\pi = 1.3\times10^{-2}, \tag{16}$$

It is clear from comparing Eq. (13) and Eq. (16) that the decay modes $\psi'\to\psi\pi\pi$ and $\psi\to\omega\pi\pi$ are completely different in nature. It is natural then to assign a new quantum number to ψ and ψ' and to assume that it is conserved by strong and semistrong interactions. A typical order of magnitude of the strength of the violation of this selection rule is given by

$$g_{\psi\omega\pi\pi}^2/g_{\psi'\psi\pi\pi}^2 \sim 10^{-3}.$$

Whether the conservation of this new quantum number is violated by electromagnetic interaction or by some new interaction of unknown origin[9] remains to be seen. In the following we shall examine the consequences of this hypothesis by carrying out a calculation for other decay modes and the production process of ψ and ψ', using conventional dispersion relation techniques. We shall denote the new quantum number by X.

III. CONSEQUENCES OF THE EXISTENCE OF A NEW QUANTUM NUMBER

A. Possible decay modes of ψ and ψ'

1. $\psi'\to\psi\eta$

Among the X-conserving decay modes of ψ', there is a possibility that ψ' might decay into ψ and η, if ψ' and ψ have isospin $I=0$. Assume that ψ' is coupled to ψ and η in a way similar to the coupling of π^0 to $\omega\rho$, i.e., a coupling of the form

$$i\,\frac{g_{\psi'\psi\eta}}{m_{\psi'}}\,\epsilon_{\mu\nu\rho\sigma}p^\mu_{\psi'}\,\epsilon^\nu_{\psi'}\,p^\rho_\psi\,\epsilon^\sigma_\psi. \tag{17}$$

The partial width of this mode is then given by

$$\Gamma(\psi'\to\psi\eta) = \left(\frac{g_{\psi'\psi\eta}^2}{4\pi}\right)\frac{Q^3}{m_{\psi'}^2} \tag{18}$$

with

$$Q = \frac{[(m_{\psi'}^2 - m_\psi^2 - m_\eta^2)^2 - 4m_\psi^2 m_\eta^2]^{1/2}}{2m_{\psi'}}.$$

Numerically one gets

$$\Gamma(\psi'\to\psi\eta) = 190\times\left(\frac{g_{\psi'\psi\eta}^2}{4\pi}\right)\text{ keV}.$$

To have an idea of how much this contributes to the ψ' width we shall take $g_{\psi'\psi\eta}^2/4\pi = 0.2-0.3$, which is smaller than $g_{\omega\rho\pi}^2/4\pi$ by a factor of 30–50; one gets

$$\Gamma(\psi'\to\psi\eta) = 30-65\text{ keV}. \tag{19}$$

This represents a sizable contribution to the total rate of ψ'. However, the detection of this mode is rather difficult since $\frac{2}{3}$ of the decays of η consist of only neutrals. We may note that in the simplest color scheme one usually considers this decay forbidden. The study of the radiative decay modes of ψ will show that this mode could be suppressed relative to that of an ordinary vector meson.

2. *Radiative decays*

Since the ψ and ψ' are coupled to the photon in a similar way as other vector mesons are, it is natural to ask the following question: Is the partial decay rate of ψ and ψ' into a photon and hadrons a substantial fraction of the total rate? It should be stressed that this question rises naturally from our phenomenological analysis independently of any model. To answer this question we shall use a vector-meson-dominance model based on the dispersion-relation technique to estimate[10] the partial rates for $\psi\to\eta\gamma$, $\psi\to\eta'\gamma$, and $\psi\to\pi\pi\gamma$.

(i) $\psi\to\eta\gamma$. Consider the matrix element for the decay $\psi\to\eta\gamma$, where the outgoing photon is on the mass shell, i.e.,

$$t = k_\gamma^2 = (k_\psi - k_\eta)^2 = 0.$$

We have

$$\mathfrak{M} = \epsilon^\mu_\gamma\,\mathfrak{M}_\mu \tag{20}$$

with

$$\begin{aligned}\mathfrak{M}_\mu &= \langle\eta|J_\mu(0)|\psi\rangle\\ &= i\,\epsilon_{\mu\nu\rho\sigma}\,\epsilon^\nu_\psi\,k^\rho_\gamma\,k^\sigma_\eta\,F(t).\end{aligned} \tag{21}$$

In the spirit of dispersion relations, $F(t)$ is an analytic continuation in the photon mass variable s of the quantity $F(s)$ defined by

$$\begin{aligned}\mathfrak{M}_\mu(s) &= \langle 0|J_\mu(0)|\eta\psi\rangle\\ &= i\,\epsilon_{\mu\nu\rho\sigma}\epsilon^\nu_\psi\,k^\rho_\gamma\,k^\sigma_\eta\,F(s)\end{aligned} \tag{22}$$

with

$$s = (k_\psi + k_\eta)^2.$$

Assuming an unsubtracted dispersion relation for $F(s)$, one can express $F(t)$ in terms of the absorptive part of $F(s)$,

$$F(t) = \frac{1}{\pi}\int_{s_0}^{\infty}\frac{\text{Abs}\,F(s)}{s-t}\,ds, \tag{23}$$

where Abs$F(s)$ is given by

$$\mathrm{Abs}\mathfrak{M}_\mu(s)=\sum_n (2\pi)^4\langle 0|J_\mu(0)|n\rangle\langle n|T|\psi\eta\rangle \times\delta^4(k_\psi-k_\eta-k_n); \quad (24)$$

here T is the transition matrix for the scattering process $\psi+\eta\to n$. Since the system $\eta\psi$ is characterized by a quantum number X, it is clear that only intermediate states which contain at least one ψ or ψ' contribute appreciably to the absorptive part; all other states involving ordinary hadrons give a contribution suppressed by a factor of 30 (in amplitude). Thus we need to estimate only the pole contributions of ψ and ψ'. In particular, the matrix element due to the ψ' pole, using Eq. (17), is given by

$$\mathfrak{M}=\frac{g_{\psi'\psi\eta}}{m_{\psi'}}\left(\frac{f_{\gamma\psi'}}{m_{\psi'}^2}\right)\epsilon_{\mu\nu\rho\sigma}\epsilon_\psi^\mu\epsilon_\gamma^\nu k_\psi^\rho k_\gamma^\sigma. \quad (25)$$

This matrix element contributes to the partial rate an amount

$$\Gamma(\psi\to\eta\gamma)=\frac{1}{24}\left(\frac{g_{\psi'\psi\eta}^2}{4\pi}\right)\left(\frac{f_{\gamma\psi'}}{m_{\psi'}^2}\right)^2\frac{m_\psi^3}{m_\psi^2}\left(1-\frac{m_\eta^2}{m_\psi^2}\right)^3. \quad (26)$$

Taking $g_{\psi'\psi\eta}^2=0.3$ as discussed above and with $f_{\gamma\psi'}$ obtained from Eq. (1) we obtain 8 keV for the contribution of the ψ' to the rate of $\psi\to\eta\gamma$. Equation (26) may be used also to estimate the ψ' pole contribution to the decay rate $\Gamma(\psi\to\eta'\gamma)$; one obtains a similar result when $g_{\psi'\psi\eta}$ and $g_{\psi'\psi\eta'}$ are the same order of magnitude. The contribution of the ψ pole to the rate is more difficult to estimate as one does not have a model-independent way to relate the coupling constant $g_{\psi\psi\eta}$ to measurable quantities. If one assumes a value for the coupling constant $g_{\psi\psi\eta}$ of the same order as $g_{\psi'\psi\eta}$ then the rate is somewhat larger by a factor of 3 than the value given above unless the interference effects are important in reducing the rate.

The above calculation shows that the radiative decay rate of ψ into η or η' and γ is suppressed relative to that of $\Gamma(\omega\to\pi^0\gamma)$ because of a suppression in the coupling constant $g_{\psi'\psi\eta}$ (or $g_{\psi'\psi\eta'}$) relative to the usual strong coupling constant $g_{\omega\rho\pi}$.

Preliminary data[11] from DESY and Frascati indicate that the width for $\psi\to\eta\gamma$ is less than 1.7 keV, implying that the squared effective coupling constant for $\psi'\to\psi\eta$ is quite small, of the order 0.1, or that there might be a destructive interference between the contributions to the rate from the ψ and ψ' poles.

(ii) $\psi\to\pi\pi\gamma$. The gauge-invariant amplitude for this decay mode can be written as

$$\mathfrak{M}=(k_\psi\cdot k_\gamma)b_\gamma(s,t,u)\left[\epsilon_\psi\cdot\epsilon_\gamma-\frac{(k_\psi\epsilon_\gamma)(k_\gamma\epsilon_\psi)}{k_\psi k_\gamma}\right], \quad (27)$$

which follows from the vector-meson dominance hypothesis and an effective Lagrangian given by Eq. (6) for the $\psi'\psi\pi\pi$ coupling. It is to be noted here that for a general coupling between the $\psi'\psi\pi\pi$ the calculation of the radiative rate for $\psi\to\pi\pi\gamma$ is made difficult and model-dependent since only a part of the $\psi'\psi\pi\pi$ coupling which satisfies the gauge condition effectively contributes to the radiative rate. [This can easily be seen by adding to the right-hand side of (7) a term $c(s,t,u)p_\gamma^\mu$ which does not contribute to the amplitude $\psi\to\pi\pi\gamma$ for the photon on the mass shell.] $b_\gamma(s,t,u)$ is the form factor for the $\psi\pi\pi\gamma$ vertex and s,t,u are the usual kinematical variables defined in the same way as in Eq. (4). When the $\pi\pi$ S-wave interaction is taken into account $b_\gamma(s,t,u)$ has the s dependence given by Eq. (8). With the matrix element given by Eq. (27) one finds

$$\Gamma(\psi\to\pi\pi\gamma)=\frac{1}{12(64\pi^3)}\frac{1}{2m_\psi^3}\int_{4\mu^2}^{m_\psi^2}ds\left(\frac{s-4\mu^2}{s}\right)^{1/2}(m_\psi^2-s)^3|b_\gamma(s)^2|^2. \quad (28)$$

In terms of the ψ' pole dominance b_γ is given by

$$b_\gamma(s)=b\left(\frac{f_{\gamma\psi'}}{m_{\psi'}^2}\right)\frac{(m_\epsilon^2-4\mu^2)}{(s-m_\epsilon^2)+im_\epsilon\Gamma_\epsilon(1-4\mu^2/s)^{1/2}\theta(s-4\mu^2)}, \quad (29)$$

with $|b|$ obtained from the decay rate $\Gamma(\psi'\to\psi\pi\pi)$. A numerical calculation gives

$$\Gamma(\psi\to\pi^+\pi^-\gamma)\simeq 0.1\text{ keV}. \quad (30)$$

If there is no $\pi\pi$ final-state interaction this rate is larger by a factor of 4 because of the fact that the emitted pions can carry large momenta.

(iii) Contributions of the ψ' *and* ψ *poles to* $\Gamma(\eta\to2\gamma)$. The role of the ψ and ψ' poles on the decay $\eta\to2\gamma$ has been discussed by Krammer *et al.*[12] Using the vector-meson-dominance model one can estimate various contributions to this rate. In particular, for the (ϕ,ϕ) pole diagram, we have

$$\Gamma_{\phi\phi}(\eta\to 2\gamma)=\frac{1}{4\pi}\frac{g_{\phi\phi\eta}{}^2}{m_\phi^2}\frac{m_\eta{}^3}{8}\left(\frac{f_{\gamma\phi}}{m_\phi^2}\right)^4. \tag{31}$$

Similarly, for the $(\psi'\psi)$ pole which involves only an X-conserving vertex,

$$\Gamma_{\psi\psi'}(\eta\to 2\gamma)=\frac{1}{4\pi}\frac{g_{\psi'\psi\eta}{}^2}{m_{\psi'}{}^2}\frac{m_\eta{}^3}{8}\left(\frac{f_{\gamma\psi}}{m_\psi{}^2}\right)^2\left(\frac{f_{\gamma\psi'}}{m_{\psi'}{}^2}\right)^2. \tag{32}$$

Similar expressions are obtained for other pole diagrams. From the experimental value of $\Gamma(\phi\to\eta\gamma)$ one estimates $g_{\phi\phi\eta}{}^2/4\pi\simeq 21$. Using this value in Eq. (31) we obtain

$$\Gamma_{\phi\phi}(\eta\to 2\gamma)\simeq 130 \text{ eV},$$

which contributes already a substantial fraction to $\Gamma(\eta\to 2\gamma)$, not to mention other important contributions from the ρ and ω poles. The contribution to the rate from the (ψ',ψ) pole occurs mainly through its interference with ordinary vector-meson poles. One finds that this interference term contributes to the rate a small fraction of the order 12 eV. In some models, for example, in the Han-Nambu model discussed by Krammer *et al.*, the $\psi'\psi\eta$ coupling usually vanishes unless one takes into account a small mixing of the SU(3) singlet state into the η. If one takes a mixing angle of 10° between η and η', then the contribution from the interference term is much more suppressed, being of the order 2 eV. Thus, in general it is not possible to obtain a good upper limit for $\Gamma(\psi\to\eta\gamma)$ from $\Gamma(\eta\to 2\gamma)$ without going into details of specific arguments based on symmetry.

3. *Rare decay modes*

(i) $\psi\to p\bar p$. Preliminary data indicate that the production cross section of $p\bar p$ pairs at the ψ resonance corresponds to a partial rate for this mode $\Gamma(\psi\to p\bar p)\simeq 0.06$ keV.[8] Assuming that the ψ decays into $p\bar p$ via one-photon exchange, one has

$$r=\frac{\sigma(e^+e^-\to\psi\to p\bar p)}{\sigma(e^+e^-\to\psi\to e^+e^-)}=\left(1-\frac{4m_p{}^2}{m_\psi{}^2}\right)^{1/2}\left(|G_M|^2+\frac{2m_p{}^2}{m_\psi{}^2}|G_E|^2\right), \tag{33}$$

where the electron mass has been neglected and G_E and G_M are respectively the proton electric and magnetic Sachs form factors. Assuming a scaling relation for these form factors in the timelike region [i.e., $G_M(s)=2.79G_E(s)$], we obtain

$$r=6.4|G_E(s)|^2,\quad s=m_\psi{}^2.$$

At present it is difficult to calculate r as one does not have the form factor measured for timelike transfer far above the $p\bar p$ threshold. One experimental point near the threshold seems to indicate that for $s=4.5$ GeV^2 the form factors do not decrease as fast as the dipole form but seems to behave like a single-pole fit. Using the usual single-pole fit (ρ meson, etc.) for G_E, one gets

$$r=\frac{6.4}{(1-m_\psi{}^2/m_\rho^2)^2}=0.027,$$

which is not far from the experimental value[13] of 0.01. A dipole form for G_E would give too small a value for r. In any case, because of the lack of experimental data for the form factors in the timelike region, it is not possible to rule out a large contribution to $\Gamma(\psi\to p\bar p)$ from the one-photon-exchange process. Another possibility is a direct X-violating interaction between ψ and $p\bar p$. In this case the rate is given by

$$\Gamma(\psi\to p\bar p)=\frac{1}{3}\frac{g_{\psi p\bar p}{}^2}{4\pi}m_\psi\left(1-\frac{4m_p{}^2}{m_\psi{}^2}\right)^{1/2}. \tag{34}$$

For $\Gamma(\psi\to pp)=0.06$ keV, one gets $g_{\psi pp}{}^2/4\pi=8.10^{-8}$.

(ii) $\psi\to\pi^+\pi^-$. This decay mode has not been seen. If the decay proceeds via one-photon exchange, a similar calculation gives an extremely small rate corresponding to $\sigma(e^+e^-\to\psi\to\pi^+\pi^-)=0.1$ nb, assuming a timelike pion electromagnetic form factor given by the ρ-dominance model. An X-violating direct coupling of the form $g_{\psi\pi\pi}\,\epsilon(\psi)\circ(p_{\pi^+}-p_{\pi^-})$ gives a partial width

$$\Gamma(\psi\to\pi^+\pi^-)=\frac{1}{12}\left(\frac{g_{\psi\pi\pi}{}^2}{4\pi}\right)m_\psi. \tag{35}$$

For example, a rate $\Gamma(\psi\to\pi^+\pi^-)<1$ keV implies a suppression factor 2×10^{-6} for $g_{\psi\pi\pi}{}^2$ relative to $g_{\rho\pi\pi}{}^2$ which would be explained by the fact that it is an X-violating process and by the variation of the $\psi\pi\pi$ form factor.

It is clear that all the partial rates of ψ discussed above amount to only a small fraction of the total rate. The rest of the decay modes can be due to other radiative decays and/or other process involving a new interaction discussed in a previous paper.[9]

4. *Other possible decay modes of* ψ'

From preliminary data, it is known that the cascade decay of ψ' amounts to only 54% of its total width. The remaining consists of the decay into ordinary hadrons and the radiative modes, i.e., the decay into hadrons accompanied by the emission of a photon. From experimental data we have

$$100 \text{ keV}\le\Gamma(\psi'\to\text{had})+\Gamma(\psi'\to\text{had}+\gamma)\le 400 \text{ keV}.$$

Allowing a factor of 2 or 3 for the difference between the matrix elements for $\psi\to$had and $\psi'\to$had (including the radiative modes with the emission of a photon) the width given above can be consistent with only X-violating processes. It is possible, however, that a good fraction of the remaining width given above is due to X-conserving decay involving new particles with mass between 3.1 GeV and 3.7 GeV.

Another problem related to the cascade decay mode of ψ' into ψ plus anything is the observed ratio of the partial rate for $\psi' \to \psi +$ neutrals to that of $\psi' \to \psi +$ all. If the two pions emitted in the cascade mode of ψ' are in isospin $I = 0$ state, one would have

$$\Gamma(\psi' \to \psi + \pi^0\pi^0) = \tfrac{1}{2}\Gamma(\psi' \to \psi + \pi^+\pi^-). \tag{36}$$

Experimentally,[14] one has

$$\frac{\Gamma(\psi' \to \psi + \text{anything})}{\Gamma(\psi' \to \psi + \pi^+\pi^-)} = 1.80 \pm 0.10. \tag{37}$$

From Eq. (36) and Eq. (37) one deduces

$$\Gamma(\psi' \to \psi + \text{neutrals other than } \pi^0\pi^0) \simeq \tfrac{1}{10}\Gamma_{\text{tot}}(\psi').$$

Among the cascade modes with the emission of neutrals other than $\pi^0\pi^0$ a possible decay mode is $\psi' \to \psi + \eta$. However, as discussed previously, the preliminary data[11] from DESY and Frascati indicate that it occurs with a quite small rate, of the order 15 keV as inferred from the upper limit on the rate of $\psi \to \eta\gamma$. Allowing a reliability of within a factor of 2 or 3 it is quite possible that the decay $\psi' \to \psi + \eta$ can account for all the remaining partial width of $\psi' \to \psi +$ neutrals other than $\pi^0\pi^0$.

B. Production of new particles

1. Production in hadronic collisions

The BNL experiment clearly indicates that the ψ is produced via an X-violating interaction of the strength somewhat larger than that of second-order electromagnetic effects. This is not surprising, since the ψ cannot be produced in pairs with low incident proton energy. However, according to the hypothesis of a new quantum number, if the energy of the beam is sufficiently high (>35 GeV), the associated production of ψ may be possible and one would expect a large production cross section. This could be the explanation for the large cross section observed at Fermilab.[3] It is, however, difficult to calculate the associated production cross section with reliability.

2. Photoproduction

We have seen that the X-conserving interaction (involved in the decay $\psi' \to \psi\pi\pi$, for example) is somewhat weaker than the small strong interaction, the factor of suppression being about 30–50 in the squared coupling constant. Since the ψ's are coupled to the photon in the same way as ordinary vector mesons are, one can estimate the photoproduction cross section of ψ and ψ' by dispersing in the photon mass, allowing only intermediate states carrying the new quantum number. The estimated cross section is found to be[15]

$$\begin{aligned}\sigma(\gamma N \to \psi\,\text{had}) &\simeq \left(\frac{f_{\gamma\psi}}{m_\psi^2}\right)^2 \sigma(\psi N \to \psi\,\text{had}) \\ &\simeq \frac{1}{40}\left(\frac{f_{\gamma\psi}}{m_\psi^2}\right)^2 \sigma(VN \to V\text{had}),\end{aligned} \tag{38}$$

where V is an ordinary vector meson. Assuming $\sigma(VN \to V\text{had}) \simeq 1$–$5$ mb and using values of $f_{\gamma\psi}$ and $f_{\gamma\psi'}$ determined above, we obtain

$$\begin{aligned}&\sigma(\gamma N \to \psi\,\text{had}) = 10\text{–}50 \text{ nb}, \\ &\sigma(\gamma N \to \psi'\,\text{had}) = 5\text{–}25 \text{ nb},\end{aligned} \tag{39}$$

which seems to be in qualitative agreement with the data from Fermilab.[3]

Using the experimental value for $\sigma(\gamma p \to \psi p)$ and assuming that ψ photoproduction is a diffractive process, one deduces[3]

$$\sigma_{\text{tot}}(\psi p) \simeq 1 \text{ mb}$$

and

$$\frac{\sigma_{\text{el}}(\psi p)}{\sigma_{\text{tot}}(\psi p)} \simeq \frac{1}{50}$$

or equivalently

$$\frac{\sigma(\gamma + N \to X + \text{had})}{\sigma(\gamma + N \to \psi + \text{had})} = 50,$$

where X refers to particles with new quantum number X. Physically this means a large production cross section of new particles, of the order of 1μb as compared with 120 μb for purely hadronic production. This result is not excluded by the Fermilab experiment since it is designed to detect only the lepton pairs decays of vector mesons.

The above conclusion should be taken with some reservation because of the following reasons:

(i) The usual vector-meson-dominance model must be modified because of the existence of other vector mesons with the same quantum number, e.g., one must include the ψ' pole contribution to $\sigma(\gamma p \to \psi\,\text{had})$. Then a large destructive interference between the various contributions may give an effective small photoproduction cross section.

(ii) It is well known that the vector-meson-dominance model works remarkably well for small q^2 (as in the case of ρ, ω, and ϕ photoproduction), while it fails badly in electroproduction at large spacelike q^2. Thus it is not surprising to see that in the ψ photoproduction the vector-meson-dominance model does not work since it involves an extrapolation from a very large $q^2 = m_\psi^2$ to $q^2 = 0$.

C. Absence of other new narrow resonances below 2.5 GeV

It is evident that because of the narrow widths of ψ and ψ', no particles c carrying the same new quantum number X as ψ and ψ' can be found below 2.5 GeV. This is so because otherwise ψ would decay to this resonance with the emission of two pions or η by strong and medium strong interactions (the lower the mass is, the better the argument for the absence of this resonance). Using the same coupling constant as that of $\psi'\psi\eta$ $(g_{\psi'\psi\eta}{}^2/4\pi = 0.3)$, a 2.5-GeV ψ_X particle would lead to $\Gamma(\psi' \to \psi_X + \eta) \simeq 18$ MeV instead of 140 keV and $\Gamma(\psi' \to \psi_X \pi^+\pi^-) \simeq 3.5$ MeV instead of 200 keV. Similarly, $\Gamma(\psi \to \psi_X \pi^+\pi^-) \simeq 500$ keV for a coupling constant $g_{\psi_X \psi\pi\pi}{}^2/4\pi \simeq 20$.

The above argument can be used to exclude the existence of a family of new pseudoscalar mesons carrying the X quantum number lying below 2.5 GeV. For example, a new particle $\eta_X(2.5\text{ GeV})$ having all the ordinary quantum numbers of η would make the decay $\psi' \to \eta_X + \omega$ possible with a large width $\Gamma(\psi' \to \eta_X + \omega) \simeq 2$ MeV, assuming a coupling $g_{\psi'\eta_X\omega}{}^2/4\pi \simeq 0.3$.

D. Arguments against the assignment of a new quantum number to ψ'' (4.1 GeV)

After the discovery of the second narrow resonance ψ' at 3.7 GeV, a broad enhancement of the total cross section for $e^+e^- \to$ hadrons at 4.1 GeV has also been found. The hadronic and leptonic widths[1] for this vector meson ψ'' (4.1) are reported to be about 250 MeV and 4 keV, respectively. The reasons for not assigning the new quantum number X to this 4.1-GeV resonance are the following:

(i) If ψ'' (4.1) carried X as ψ and ψ', the dominant decay modes of ψ'' would be the cascade modes into ψ or ψ' with the emission of ordinary hadrons. In this case $\Gamma(\psi'' \to \psi + \text{had})$ and $\Gamma(\psi'' \to \psi' + \text{had})$ would be of the order of a hundred MeV, typical of the usual strong-interaction decay widths. This not only seems unreasonable since the X-conserving interaction should be suppressed relative to the usual strong interaction, but also implies that the radiative widths of ψ and ψ' (decaying into ordinary hadrons with emission of a photon) as calculated from the ψ'' pole dominance should be large, of the order $\Gamma(\omega \to \pi^0\gamma) \simeq 1$ MeV or larger, because of the large phase space, which is excluded by experiments. More precisely, if $\psi'' \to (\psi, \psi') + \pi^0$ or η (depending on the isospin assignment for ψ'') is a dominant decay mode, then $\Gamma(\psi \to \pi^0\gamma)$ or $\Gamma(\psi \to \eta + \gamma)$ would be of the order of 500 keV–1 MeV.

(ii) If the process $\psi'' \to \psi + 2\pi$ were a dominant decay mode, say the partial width of the order of 100–200 MeV, then the decay $\psi \to \pi\pi\gamma$ could occur through the $\gamma\psi''$ coupling and its rate could be calculated as above. Using the $\gamma\psi''$ coupling determined from the ψ'' leptonic width, a simple calculation similar to that given in Sec. III A would give $\Gamma(\psi \to \pi\pi\gamma) \simeq 10$–$40$ keV, which seems to be excluded by experiments.

From the above arguments, one can conclude that ψ'' is an ordinary vector meson and hence only a very small fraction of ψ'' of the order of less than 1% will decay into ψ (or ψ') and ordinary hadrons because the decay is then a X-violating process.

IV. POSSIBLE RELATIONS WITH SYMMETRY SCHEME

From the analysis made in previous sections, it is clear that there probably is a new quantum number associated with ψ and ψ'. We want now to make this result more precise. Three basic facts emerge from our analysis:

(i) ψ and ψ' have the same X quantum number.

(ii) ψ and ψ' couple to ordinary hadrons by X-violating amplitude with a strength slightly stronger than second-order electromagnetic effects (new interaction?).

(iii) ψ and ψ' can couple bilinearly to ordinary hadrons with a coupling constant smaller than the usual strong interaction but much larger than the X-violating amplitude.

These facts point out the possibility of classifying the new particles and ordinary hadrons by two quantum numbers (e.g., two quantum numbers of the type given by the color scheme of Han and Nambu), one with respect to the usual symmetry group of ordinary hadrons, the other associated with the new quantum number. In the following, for simplicity we shall assume X to be a new quantum number of multiplicative character. In this case ordinary hadrons will be classified according to the usual SU(3) scheme and all have $X = 1$, while ψ and ψ' have $X = -1$ and the appropriate SU(3) quantum numbers. We shall denote such a particle by (a, b) where a refers to the transformation property with respect to the ordinary SU(3) family while b refers to the X family.

It is too premature to speculate on the nature of the symmetry of the X family. We discuss only consequences deduced from the assignment of quantum numbers for the ψ and ψ' with respect to ordinary SU(3) symmetry. We tentatively assign $(\omega, X=-1)$ to the ψ and ψ'

$$\psi(3.1)=(\omega, X=-1),$$

$$\psi'(3.7)=(\omega, X=-1),$$

where ω means that the particle behaves like the physical $\omega(783)$ with respect to SU(3).

The reason for classifying ψ as ω in SU(3) space is that a large fraction of ψ is found to decay dominantly to an odd number of pions. If it turns out experimentally that ψ' prefers to decay to an odd number of pions as does ψ, then our assignment for ψ' is correct. But if it turns out that the G-parity selection rules does not hold with good accuracy for ψ' (3.7) then the above assignment may not be valid. We could try another assignment $(\phi, X=-1)$ for ψ' (3.7) since experimentally the ϕ meson does not decay strongly into 3π (this suppression is believed to hold in general for the decay of ϕ into an odd number of pions). However, this assignment seems difficult to accept because it would imply that a large number of $K\bar{K}$ events should be observed in the X-violating decay of ψ' which may be in contradiction with experiment.

As a consequence of this assignment, the decay $\psi' \to \psi\pi^0$ is forbidden because of G-parity selection rules while $\psi' \to \psi\pi\pi$ and $\psi' \to \psi\eta$ are allowed. In order to avoid the problem associated with the radiative decay widths of ψ and ψ' as discussed above, the diagonal interaction $\psi\psi\eta$, $\psi\psi\pi\pi$, and generally $\psi\psi$ hadrons (and similar couplings for ψ') must be of comparable strengths with the off-diagonal transition matrix elements $\psi'\psi$ hadrons.

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