Comments and Addenda

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Ludlam and Slansky's cluster-size estimate and correlation functions

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A relation connecting the fluctuation measure proposed by Ludlam and Slansky and the semi-inclusive correlation function is derived; it should contribute to the understanding of the discrepancy between different estimates concerning the cluster size in hadron collisions, i.e., $\langle n_{\text{decay}}\rangle = 3\text{–}4$ or 5–6.

It is important to analyze the final-state structure of inelastic collisions in a way that is as model-independent as possible. For this purpose Ludlam and Slansky[1] propose and apply a new estimator for fluctuations in the final state. They define the following measure for a sample of events with fixed associated charged multiplicity n:

$$\begin{aligned}\kappa_n &= \frac{3}{n}\int dy\,\hat{\rho}_1^{(n)}(y)\{\langle[n_L(y)-\langle n_L(y)\rangle]^2\rangle_n \\ &\qquad + \langle[n_R(y)-\langle n_R(y)\rangle]^2\rangle_n\} \\ &= \frac{6}{n}\int dy\,\hat{\rho}_1^{(n)}(y)\langle[n_L(y)-\langle n_L(y)\rangle]^2\rangle_n\,. \end{aligned} \tag{1}$$

Here

$$\hat{\rho}_1^{(n)}(y)=\frac{1}{n}\rho_1^{(n)}(y)=\frac{1}{n}\,\frac{1}{\sigma_n}\,\frac{d\sigma_n}{dy}$$

is the semi-inclusive rapidity distribution normalized to 1 and $n_L(y)$ $[n_R(y)]$ is the charged multiplicity observed to the left [right] of y in a specific event. The angular brackets indicate the averaging over the sample considered.

The experimental value of κ_n for proton-proton collisions at Fermilab energies is found to be[1] $\kappa_n \simeq 0.8\text{–}1.0$ (more or less independent of the associated multiplicity n). Mere statistical fluctuations result in $\kappa_n = 1$; phase-space restrictions due to energy-momentum conservation lower, and clustering raises, the value of κ_n. Using a Monte Carlo calculation, Ludlam and Slansky[1] find this quantity to be quite sensitive to both effects; within a specific cluster model they evaluate the needed cluster size and obtain a value of $\langle n_{\text{decay}}\rangle = 5\text{–}6$. The concern here is this high cluster decay multiplicity as compared to the other estimates of $\langle n_{\text{decay}}\rangle \simeq 3\text{–}4$, which are obtained from the measured (semi-) inclusive correlation functions in the central region.[2,3]

For the understanding of this discrepancy it is useful to consider the relation between the measure κ_n and the semi-inclusive distributions. With the help of the identities

$$\begin{aligned}\langle n_L(y)\rangle_n &= \int_{-\infty}^{y} dy'\,\rho_1^{(n)}(y')\,, \\ \langle n_L(y)[n_L(y)-1]\rangle_n &= \iint_{-\infty}^{y} dy_1 dy_2\,\rho_2^{(n)}(y_1,y_2)\,,\end{aligned} \tag{2}$$

and because of

$$\int_{-\infty}^{+\infty} dy\,\hat{\rho}_1^{(n)}(y)\int_{-\infty}^{y} dy'\,\rho_1^{(n)}(y') = \tfrac{1}{2}n\,,$$

one can evaluate the quantity in angular brackets in the last line of Eq. (1). One obtains the relation

$$\kappa_n - 3 = \frac{6}{n}\int dy\,\hat{\rho}_1^{(n)}(y)\iint_{-\infty}^{y} dy_1 dy_2\,C_2^{(n)}(y_1,y_2)\,, \tag{3}$$

where

$$C_2^{(n)}(y_1,y_2)=\rho_2^{(n)}(y_1,y_2)-\rho_1^{(n)}(y_1)\rho_1^{(n)}(y_2) \tag{4}$$

is the semi-inclusive correlation function. In the absence of correlations one would have

$$C_2^{(n)\text{uncorrelated}} = -n\hat{\rho}_1^{(n)}(y_1)\hat{\rho}_1^{(n)}(y_2)\,,$$

i.e., $\kappa_n = 1$. Alternatively, one can therefore define

$$C_2^{(n)*} = C_2^{(n)} - C_2^{(n)\text{uncorrelated}}\,, \tag{5}$$

which leads to the following expression:

$$\kappa_n - 1 = \frac{6}{n}\int dy\,\hat{\rho}_1^{(n)}(y)\iint_{-\infty}^{y} dy_1 dy_2 C_2^{(n)*}(y_1, y_2). \tag{6}$$

We find that the measure κ_n depends only on the semi-inclusive spectrum and correlation, i.e., just the same quantities which are analyzed in the standard methods to determine the cluster size. Therefore, the observed discrepancy for the estimates of $\langle n_{\text{decay}}\rangle$ is due to different model-dependent assumptions.

It may be interesting to note that in Eq. (6) the correlation function $C_2^{(n)*}$ is essentially weighted by the rapidity difference $|y_1 - y_2|$. In order to see this one has to interchange the integrations in Eq. (6) (treating both regions $y_1 > y_2$ and $y_2 > y_1$ separately), employ the forward-backward symmetry in the definition of κ_n [Eq. (1)], and rewrite Eq. (6) as

$$\kappa_n - 1 = \frac{6}{n}\iint dy_1 dy_2\, W(y_1, y_2) C_2^{(n)*}(y_1, y_2)\,, \tag{7}$$

where

$$W(y_1, y_2) = \tfrac{1}{2}\theta(y_1 - y_2)\left(\int_{y_1}^{\infty} + \int_{-\infty}^{y_2}\right)\hat{\rho}_1^{(n)}(y)dy + (y_1 \longleftrightarrow y_2)\,. \tag{8}$$

One can now approximate $\hat{\rho}_1^{(n)}(y)$ at high energies by a constant $\hat{\rho}_1^{(n)}(y) \simeq Y_{\text{eff}}^{-1}$, and one obtains the desired expression

$$\kappa_n - 1 \simeq -\frac{3}{n}\frac{1}{Y_{\text{eff}}}\iint dy_1 dy_2 |y_1 - y_2| C_2^{(n)*}(y_1, y_2)\,. \tag{9}$$

The relation (7) is model-independent and it shows that the estimator κ_n does not concentrate only on that region in phase space which is assumed to be predominantly determined by the short-range (clustering) effects. In order to illustrate this point we consider a model with independent production of neutral isotropically decaying clusters. The central semi-inclusive correlation function can be approximated asymptotically by

$$C_2^{(n)*}(y_1, y_2) \simeq n\,\gamma(n)[\Gamma(y_1, y_2) - \hat{\rho}_1^{(n)}(y_1)\hat{\rho}_1^{(n)}(y_2)]\,, \tag{10}$$

where

$$\Gamma(y_1, y_2) \simeq (2\delta\sqrt{\pi}\,Y_{\text{eff}})^{-1}\exp[-(y_1 - y_2)^2/(4\delta^2)]\,,\quad \delta \simeq 0.6 - 0.9\,, \tag{11}$$

is the normalized two-particle density resulting from the decay within one cluster, and where

$$\gamma(n) = \frac{\langle n_{\text{decay}}(n_{\text{decay}} - 1)\rangle_n}{\langle n_{\text{decay}}\rangle_n} \tag{12}$$

reflects the cluster multiplicity $[\gamma(n) \geqslant \langle n_{\text{decay}}\rangle_n - 1]$. If one puts this "central cluster" (C.C.) formula in Eq. (9) one obtains

$$\kappa_n^{\text{C.C.}} - 1 \simeq \gamma(n)\left[1 - \frac{6\delta}{\sqrt{\pi}\,Y_{\text{eff}}} + O\left(\frac{1}{Y_{\text{eff}}^2}\right)\right]\,, \tag{13}$$

which indicates a strong dependence of the measure $\kappa_n^{\text{C.C.}}$ on $\gamma(n)$. From recent CERN ISR data[3] for $C_2^{(n)}$ in the central region near $n \cong \langle n\rangle$ the value of $\gamma(n)$ is estimated to be of the order 1.0 ± 0.1 (1.2 ± 0.2) and according to Eq. (13) one can infer a value of $\kappa_n^{\text{C.C.}} \simeq 1.4 - 1.6$ at Fermilab energies. The "long-range part" in the correlation function, which is not taken into account by Eq. (10), has therefore to reduce the value of κ_n from $\kappa_n \simeq \kappa_n^{\text{C.C.}}$ to the experimental value, $\kappa_n \simeq 0.8 - 1.0$. Since this part is especially strongly affected by kinematical constraints and by diffraction, a detailed knowledge and description of $C_2^{(n)}$ over the whole region in phase space is necessary for quantitative fits.

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[1]T. Ludlam and R. Slansky, Phys. Rev. D 12, 59 (1975); T. Ludlam *et al.*, Phys. Lett. 48B, 449 (1974), and references quoted therein.

[2]See, e.g., E. L. Berger, Nucl. Phys. B85, 61 (1975); A. Morel and G. Plaut, *ibid.* B78, 541 (1974); F. Hayot and M. Le Bellac, *ibid.* B86, 333 (1975); G. Ranft and J. Ranft, *ibid.* B83, 285 (1975); R. Baier and F. Widder, Nuovo Cimento 30A, 169 (1975).

[3]K. Eggert *et al.*, Nucl. Phys. B86, 201 (1975); S. R. Amendolia *et al.*, Nuovo Cimento 31A, 1 (1975).