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Extreme example of rearrangement of spontaneously broken symmetry*

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A model of spontaneously broken SU(2) symmetry is presented in which the generators of in-field (phenomenological) transformations do not form a closed algebra. The consequences of such incomplete observational symmetries are described. Of particular interest is the emergence of current-algebra-type relations, corresponding to a partially conserved isospin current, although the isospin current is strictly conserved in this model. This suggests a possible reinterpretation of current-algebra results.

I. INTRODUCTION

One of the characteristics of spontaneous breakdown of symmetry is the replacement of the dynamical symmetry of the Heisenberg equations of motion by a different symmetry at the observational level. The crystal, where the translational and rotational invariance of the basic dynamics is replaced by the high symmetry of the lattice, provides a striking example of this phenomenon. Indeed, the occurrence of such a change of symmetry is, along with the appearance of Goldstone bosons, one of the basic features of spontaneous breakdown. We call this phenomenon the *dynamical rearrangement of symmetry*.[1,2]

Since observations on a system are expressed by matrix elements in the Hilbert space of asymptotic particles, the observational symmetry of the system is disclosed through the study of the symmetries at the level of the asymptotic fields. Therefore, stated precisely, the study of dynamical rearrangement is the investigation of the asymptotic field transformations which correspond to a given set of invariant transformations of the Heisenberg field operators. This study is of obvious significance if one wishes to deduce the phenomenological manifestations of a dynamical model with spontaneous breakdown.

The phenomenon of symmetry rearrangement has been studied extensively for various situations. Of obvious interest is the question "For a given invariance group at the dynamical (Heisenberg) level, what possible group structures can appear in observations?" Study of various models has disclosed the following possibilities up to now:

(a) The algebraic structure of the dynamical and phenomenological symmetries is the same, but the Heisenberg and asymptotic fields provide different realizations of this algebra. This is necessarily the case for Abelian groups; examples include spontaneous breakdown of phase, chiral phase, and scale invariance.[2]

(b) The algebraic structure changes in the process of dynamical rearrangement. An example of this type was presented in Ref. 3, where it was shown that a dynamical SU(2) symmetry can appear as E(2) symmetry at the observational level. The model used in that example was that of a self-interacting scalar isotriplet field. In Ref. 4 the same argument was applied to ferromagnetism.

In this paper we wish to present a model in which a third possibility emerges:

(c) The generators of the phenomenological symmetry transformations do not close into an algebra.

Therefore all the possible alternatives can be realized in the framework of rearrangement of symmetry. The three possibilities (a), (b), and (c) above were called R_1, R_2, and R_3 cases in Ref. 5.

The dynamical origin of this symmetry rearrangement was first pointed out in Ref. 6, and further elucidated in Refs. 3, 4. Observations of symmetry properties of a system refer to matrix elements of the generators between wave-packet states in the asymptotic Hilbert space. Such observations therefore miss naturally any contributions of infrared (low-energy) Goldstone bosons to the generators; such contributions extend with infinitesimally small density over all space, and are not disclosed in localized observations. In fact it has been explicitly shown that inclusion of such infrared effects restores the original group structure.[3,4,6] The same mechanism occurs in the model to be presented here.

The model that we are going to examine has an SU(2) symmetry at the dynamical level; in this respect it is similar to the one used in Ref. 3, which led to an E(2) observational symmetry. It is remarkable that the same basic symmetry can yield vastly different symmetry structures at the asymptotic level. The boundary condition through which spontaneous breakdown is imposed is of crucial importance in this respect. The boundary condition in turn has to be consistent with the dynamical model; therefore ultimately the observable symmetry depends on the specific dynamics, and not only on the symmetry properties of the original Lagrangian. It depends on interactions to which the boundary condition is adapted.

Despite the fact that the algebra of the asymptotic symmetry generators does not close, the model predicts remarkable relations among matrix elements of the S matrix. We will show that the relations emerging from such an "incomplete" symmetry bear a close resemblance to current-algebra relations and soft-limit theorems which would be normally derived by assuming a partially conserved isospin current. The fact that they appear in our model, in which the isospin current is strictly conserved, provides an alternative explanation for such relations (this phenomenon has also been discussed in, e.g., Ref. 7) In fact our results show that the task of extracting the dynamical symmetry from the phenomenological one is extremely difficult. The possibility of incomplete symmetries adds, however, new flexibility to the application of symmetry principles in high-energy phenomena.

The paper is arranged in the following way: The model is presented in Sec. II, in which the Ward-Takahashi identities needed in the following are also derived. Section III establishes the symmetry rearrangement; the algebra of the in-field generators is established and is shown not to close. The final Sec. IV is devoted to a discussion of the phenomenological consequences of incomplete symmetries and concluding remarks.

II. MODEL

The model consists of two isospin multiplets, $\phi(x)$ and $V(x)$, both scalar under Lorentz transformations. The field $\phi(x)$ is complex and transforms as a doublet under SU(2) transformations:

$$\phi(x)=\begin{bmatrix}\phi_1(x)\\ \phi_2(x)\end{bmatrix}, \tag{2.1}$$

$$\phi(x)\to\phi'(x)=e^{i\vec{\alpha}\cdot\vec{\tau}/2}\phi(x), \tag{2.2}$$

where τ_i ($i=1,2,3$) are the Pauli matrices and $\vec{\alpha}=(\alpha_1,\alpha_2,\alpha_3)$ represent the transformation parameters. $V(x)$ is a real field belonging to an n-dimensional SU(2) representation $\mathfrak{D}$, which need not be specified further:

$$V(x)=\begin{bmatrix}V_1(x)\\ \cdot\\ \cdot\\ \cdot\\ V_n(x)\end{bmatrix}, \tag{2.3}$$

$$V(x)\to V'(x)=e^{i\vec{\alpha}\cdot\vec{T}}V(x), \tag{2.4}$$

with T_i the SU(2) generators in the representation $\mathfrak{D}$.

The Lagrangian is assumed to be invariant under (2.2) and (2.4),

$$\mathfrak{L}[\phi,\phi^*,V]=\mathfrak{L}[e^{i\vec{\alpha}\cdot\vec{\tau}/2}\phi,\ \phi^*e^{-i\vec{\alpha}\cdot\vec{\tau}/2},\ e^{i\vec{\alpha}\cdot\vec{T}}V], \tag{2.5}$$

and the generating functional has the form

$$W[J,h]=\frac{1}{N}\int[d\phi][d\phi^*][dV]\exp\Big(i\int d^4x\{\mathfrak{L}(x)+i\epsilon[\phi^*(x)-v^*]\cdot[\phi(x)-v] +\phi^*(x)\cdot J(x)+J^*(x)\cdot\phi(x)+\boldsymbol{h}(x)\cdot V(x)\}\Big), \tag{2.6}$$

where

$$J(x)=\begin{bmatrix}J_1(x)\\ J_2(x)\end{bmatrix},\quad h(x)=\begin{bmatrix}h_1(x)\\ \cdot\\ \cdot\\ \cdot\\ h_n(x)\end{bmatrix},$$

and

$$v=|v|\begin{pmatrix}1\\0\end{pmatrix}. \tag{2.7}$$

As explained in Refs. 8 and 9, the "ϵ term" $\epsilon|\phi(x)-v|^2$ is used to induce spontaneous breakdown in the direction of v; the limit $\epsilon\to 0$ is taken at the end of computations. With the choice (2.7) for v, none of the SU(2) generators leaves the vacuum invariant; isospin symmetry is completely broken.

At this point we should mention the reason for introducing the field $V(x)$, which does not participate in the spontaneous breaking of the symmetry. Its inclusion is due to the fact that any SU(2)-invariant Lagrangian which consists solely of a self-interacting isodoublet field $\phi(x)$ is automatically invariant under the larger SU(2)×SU(2) group, the real and imaginary components of $\phi(x)$ combining to form a quartet representation of this group. This unwanted higher symmetry is avoided if $\phi(x)$ is coupled to other fields belonging to appropriately chosen SU(2) representations. As an added bonus, the present model will allow us to examine the dynamical rearrangement for fields which do not participate directly in the spontaneous symmetry breakdown.

We deduce the Ward-Takahashi identities by performing the change of variables (2.2) and (2.4) in the generating functional and requiring that it remains unchanged; using (2.5), we obtain the basic relation

$$i\int d^4x\langle J^*(x)\tau_i\phi(x)-\phi^*(x)\tau_i J(x)+2h(x)\cdot T_i\cdot V(x)\rangle_{J,h,\epsilon}$$
$$=-\epsilon\int d^4x\langle v^*\tau_i\phi(x)-\phi^*(x)\tau_i v\rangle_{J,h,\epsilon}\,. \quad (2.8)$$

In (2.8) we used the following abbreviations, which will be employed throughout the paper:

$$\langle F[\phi,\phi^*,V]\rangle_{J,h,\epsilon}\equiv F\left[\frac{\delta}{i\delta J^*},\frac{\delta}{i\delta J},\frac{\delta}{i\delta h}\right]W[J,h],$$
$$\langle F[\phi,\phi^*,V]\rangle_{\epsilon}\equiv\langle F[\phi,\phi^*,V]\rangle_{J=h=0,\epsilon}\,, \quad (2.9)$$
$$\langle F[\phi,\phi^*,V]\rangle\equiv\lim_{\epsilon\to 0}\langle F[\phi,\phi^*,V]\rangle_{\epsilon}\,.$$

If we differentiate (2.8) with respect to $J^*(x)$ and $J(x)$ and then set all sources equal to zero we obtain

$$\langle\tau_i\phi(x)\rangle_\epsilon=-\epsilon\int d^4z\,\langle\phi(x),v^*\tau_i\phi(z)-\phi^*(z)\tau_i v\rangle_\epsilon\,, \quad (2.10a)$$

$$-\langle\phi^*(x)\tau_i\rangle_\epsilon=-\epsilon\int d^4z\,\langle\phi^*(x),v^*\tau_i\phi(z)-\phi^*(z)\tau_i v\rangle_\epsilon\,. \quad (2.10b)$$

Multiplying the first of these relations by $v^*\tau_i$, the second by $\tau_i v$, and subtracting, we deduce the identity

$$\langle v^*\phi(x)+\phi^*(x)v\rangle_\epsilon$$
$$=-\epsilon\int d^4z\,\langle v^*\tau_i\phi(x)-\phi^*(x)\tau_i v,v^*\tau_i\phi(z)-\phi^*(z)\tau_i v\rangle_\epsilon\,. \quad (2.11)$$

The significance of this identity becomes clear if we make a field redefinition:

$$\psi(x)\equiv\frac{1}{\sqrt{2}\,|v|}\left[v^*\phi(x)+\phi^*(x)v\right], \quad (2.12a)$$

$$\vec{\chi}(x)\equiv\frac{i}{\sqrt{2}\,|v|}\left[v^*\vec{\tau}\phi(x)-\phi^*(x)\vec{\tau}v\right]. \quad (2.12b)$$

Explicitly, with the choice (2.7) for v,

$$\psi(x)=\frac{1}{\sqrt{2}}\left[\phi_1(x)+\phi_1^*(x)\right] \quad (2.13)$$

$$\chi_i(x)=\begin{cases}\frac{i}{\sqrt{2}}\left[\phi_2(x)-\phi_2^*(x)\right] & (i=1), \quad (2.14a)\\ \frac{1}{\sqrt{2}}\left[\phi_2(x)+\phi_2^*(x)\right] & (i=2), \quad (2.14b)\\ \frac{i}{\sqrt{2}}\left[\phi_1(x)-\phi_1^*(x)\right] & (i=3). \quad (2.14c)\end{cases}$$

If we define also

$$\langle\psi(x)\rangle_\epsilon\equiv\tilde{v}(\epsilon),\quad \tilde{v}\equiv\lim_{\epsilon\to 0}\tilde{v}(\epsilon)\neq 0 \quad (2.15)$$

we can rewrite (2.11) as

$$\tilde{v}(\epsilon)=\epsilon\sqrt{2}\,|v|\int d^4z\,\langle\chi_i(x),\chi_i(z)\rangle_\epsilon\quad (i=1,2,3). \quad (2.16)$$

This is a statement of the Goldstone theorem.[9] Indeed, if we set

$$\langle\chi_i(x),\chi_i(y)\rangle=i(2\pi)^{-4}\int d^4p\,e^{-ip\cdot(x-y)}\Delta_\chi^{(i)}(p^2), \quad (2.17)$$

$$\Delta_\chi^{(i)}(p^2)=\lim_{\epsilon\to 0}\frac{Z_i}{p^2-m_i^{\,2}+i\epsilon a_i}$$
$$+(\text{continuum contributions}), \quad (2.18)$$

we obtain from (2.16)

$$m_i=0\ \text{ if }\ \tilde{v}\neq 0, \quad (2.19)$$

$$\tilde{v}=\sqrt{2}\,|v|\,\frac{Z_i}{a_i}\,. \quad (2.20)$$

There are therefore three Goldstone bosons χ_i in this model.

It is convenient at this point to perform the change of variables (2.13) and (2.14) in W, and to define new sources:

$$l(x)=\frac{1}{\sqrt{2}}\left[J_1(x)+J_1^*(x)\right], \quad (2.21)$$

$$j_i(x)=\begin{cases}\frac{i}{\sqrt{2}}\left[J_2(x)-J_2^*(x)\right] & (i=1),\\ \frac{1}{\sqrt{2}}\left[J_2(x)+J_2^*(x)\right] & (i=2),\\ \frac{i}{\sqrt{2}}\left[J_1(x)-J_1^*(x)\right] & (i=3).\end{cases} \quad (2.22)$$

The generating functional then takes the form

$$W[\vec{j},l,h]=\frac{1}{N}\int[d\vec{\chi}][d\psi][dV]\exp\left\{i\int d^4x\left[\mathcal{L}(x)+\vec{j}(x)\cdot\vec{\chi}(x)+l(x)\psi(x)+h(x)\cdot V(x)+i\epsilon\left(\frac{\psi^2(x)+\vec{\chi}^2(x)}{2}-\sqrt{2}\,|v|\psi(x)\right)\right]\right\}. \tag{2.23}$$

To obtain the Ward-Takahashi identities in terms of the new fields, we perform a change of variables equivalent to (2.2) and (2.4). When $\phi(x)$ undergoes an infinitesimal SU(2) transformation, $\vec{\chi}$ and ψ transform as follows:

$$\begin{aligned}\psi(x)\to\psi'(x)&=\psi(x)+\tfrac{1}{2}\vec{\alpha}\cdot\vec{\chi}(x),\\ \vec{\chi}(x)\to\vec{\chi}'(x)&=\vec{\chi}(x)-\tfrac{1}{2}\vec{\alpha}\psi(x)-\tfrac{1}{2}\vec{\alpha}\times\vec{\chi}(x).\end{aligned} \tag{2.24}$$

This is exactly the way a quartet $(\vec{\chi},\psi)$ would transform under special SU(2)×SU(2) transformations in which the transformation parameters of the two SU(2) subgroups take the same values. Indeed, if we define

$$F_i=\int d^3x\,\psi(x)\overleftrightarrow{\frac{\partial}{\partial t}}\,\chi_i(x), \tag{2.25a}$$

$$A_i=-\int d^3x\,\epsilon_{ijk}\dot{\chi}_j(x)\chi_k(x), \tag{2.25b}$$

these operators satisfy an SU(2)×SU(2) algebra at equal times,

$$[F_i,F_j]_{\rm ET}=i\epsilon_{ijk}A_k,$$

$$[A_i,A_j]_{\rm ET}=i\epsilon_{ijk}A_k,$$

$$[F_i,A_j]_{\rm ET}=i\epsilon_{ijk}F_k,$$

and the transformation (2.24) is generated by

$$G_i=\tfrac{1}{2}(A_i+F_i). \tag{2.26}$$

Though A_i and F_i are not time-independent, G_i is a constant of motion.

Let us now turn to the Ward-Takahashi identities. If we make the change of variables (2.4) and (2.24) in Eq. (2.23) and require that

$$\left.\frac{\partial W}{\partial\alpha_i}\right|_{\vec{\alpha}=0}=0,$$

we obtain

$$i\int d^4z\,\langle l(z)\chi_i(z)-j_i(z)\psi(z)+i\vec{j}(z)\cdot t_i\vec{\chi}(z)+2ih(z)\cdot T_i\cdot V(z)\rangle\!\rangle_{l,\vec{j},h,\epsilon}=-\epsilon\sqrt{2}\,|v|\int d^4z\,\langle\chi_i(z)\rangle\!\rangle_{l,\vec{j},h,\epsilon}, \tag{2.27}$$

where the t_i's are the SU(2) generators in the triplet representation:

$$(t_i)_{jk}=-i\epsilon_{ijk}\quad(i,j,k=1,2,3). \tag{2.28}$$

The identities (2.27) have, of course, exactly the same content as (2.8), and are expressed in a more convenient form. Another equivalent form is

$$i\int d^4z\,\langle l(z)\chi_i(z)-j_i(z)\psi(z)+i\vec{j}(z)\cdot t_i\vec{\chi}(z)+2ih(z)\cdot T_i\cdot V(z)\rangle\!\rangle_{l,\vec{j},h}=i\,\frac{\bar{v}}{Z_i}\int d^4z\,K(z)\langle\chi_i(z)\rangle\!\rangle_{l,\vec{j},h}, \tag{2.29}$$

with

$$K(z)=-\partial_\mu\partial^\mu.$$

Equation (2.29) follows from (2.27) since the limit $\epsilon\to0$ serves to pick the zero-mass χ pole; indeed, for any function Q we have the identity

$$\lim_{\epsilon\to0}\epsilon\sqrt{2}\,|v|\int d^4z\,\langle\chi_i(z),Q\rangle_\epsilon=-i\,\frac{\bar{v}}{Z_i}\int d^4z\,K(z)\langle\chi_i(z)Q\rangle \tag{2.30}$$

[Eqs. (2.18) and (2.20) are used in obtaining this relation]. Differentiating (2.29) with respect to $j_i(x)$ and

$l(y)$ and setting the sources equal to zero we obtain

$$\langle\chi_i(x)\chi_i(y)\rangle\!\rangle - \langle\rho(x)\rho(y)\rangle\!\rangle = i\,\frac{\tilde{v}}{Z_i}\int d^4z\,K(z)\langle\chi_i(z)\chi_i(x)\rho(y)\rangle\!\rangle\,, \qquad (2.31)$$

where we defined

$$\rho(x) = \psi(x) - \tilde{v}\,. \qquad (2.32)$$

In terms of the propagators of χ and ρ and the proper $\chi\chi\rho$ vertex,

$$\langle\rho(x)\rho(y)\rangle\!\rangle = i(2\pi)^{-4}\int d^4p\, e^{-ip\cdot(x-y)}\Delta_\rho(p)\,, \qquad (2.33)$$

$$\langle\chi_i(x)\chi_i(y)\rho(z)\rangle\!\rangle = -(2\pi)^{-8}\int d^4p\,d^4q\,d^4r\,e^{-i(p\cdot x+q\cdot y+r\cdot z)}\,\delta(p+q+r)\Delta_\chi^{(i)}(p)\Delta_\chi^{(i)}(q)\Delta_\rho(r)\Gamma_{ii\rho}(p,q;r)\,, \qquad (2.34)$$

the identity (2.31) reads

$$\Delta_\chi^{(i)}(p) - \Delta_\rho(p) = -\tilde{v}\Delta_\chi^{(i)}(p)\Delta_\rho(-p)\Gamma_{ii\rho}(0,p;-p)\,,$$

or, using the fact that Δ_ρ is a function of p^2,

$$\Delta_\chi^{(i)}(p)^{-1} - \Delta_\rho(p)^{-1} = \tilde{v}\Gamma_{ii\rho}(0,p;-p)\,. \qquad (2.35)$$

This relation makes it very plausible that, if ρ is massive, it can decay into two χ's and therefore is unstable. We will assume this to be the case in the following.

III. DYNAMICAL REARRANGEMENT OF SYMMETRY

From the discussion of the previous section, the asymptotic fields in this model are $\chi_i^{\text{in}}(x)$ $(i=1,2,3)$ and $V_\alpha^{\text{in}}(x)$ $(\alpha=1,\ldots,n)$. For simplicity, we assumed that all the V's are stable. The Ward-Takahashi identities give no information about the masses of the V particles; such information can be obtained by detailed dynamical calculations with a specific Lagrangian. In the following we will examine separately two cases: (i) All the V's have the same mass, and (ii) the masses of the V particles are all different.

Let us first assume that all components of $V^{\text{in}}(x)$ have the same mass:

$$m_1 = m_2 = \cdots = m_n \equiv m\,. \qquad (3.1)$$

Then the in fields satisfy the free-field equations

$$K(x)\chi_i^{\text{in}}(x) = 0\,, \qquad (3.2\text{a})$$

$$\tilde{K}(x)V_\alpha^{\text{in}}(x) = 0\,, \qquad (3.2\text{b})$$

where

$$K(x) = -\partial_\mu\partial^\mu\,,\quad \tilde{K}(x) = \Box + m^2\,.$$

The in-field expansions of the S matrix and the various Heisenberg fields are given concisely in a functional form[2]:

$$S = :\langle e^{-\mathfrak{A}}\rangle:\,, \qquad (3.3\text{a})$$

$$S\phi_H(x) = :\langle\phi(x)e^{-\mathfrak{A}}\rangle:\,, \qquad (3.3\text{b})$$

$$SV_H(x) = :\langle V(x)e^{-\mathfrak{A}}\rangle:\,, \qquad (3.3\text{c})$$

where we defined

$$\mathfrak{A} = i\int d^4x\Big[\sum_{i=1}^{3} Z_i^{-1/2}\chi_i^{\text{in}}(x)K(x)\chi_i(x) + \sum_{\alpha=1}^{n}\tilde{Z}_\alpha^{-1/2}V_\alpha^{\text{in}}(x)\tilde{K}(x)V_\alpha(x)\Big]\,, \qquad (3.4)$$

with $\tilde{Z}_\alpha$ the renormalization constant of the field $V_\alpha(x)$. Equations (3.3) are to be understood as weak relations, expressing the equality of matrix elements of the two sides between states of the in-particle Hilbert space.

We seek the in-field transformation

$$\begin{aligned} &\chi_i^{\text{in}}(x) \to \chi_i'^{\text{in}}(x,\lambda\hat{n})\,,\\ &V_\alpha^{\text{in}}(x) \to V_\alpha'^{\text{in}}(x,\lambda\hat{n})\,, \end{aligned} \qquad (3.5)$$

which induces the SU(2) transformation of the Heisenberg fields:

$$\phi_H(x) \to \phi_H'(x) = \exp\left(i\lambda\frac{\hat{n}\cdot\vec{\tau}}{2}\right)\phi_H(x)\,, \qquad (3.6\text{a})$$

$$V_H(x) \to V_H'(x) = \exp(i\lambda\hat{n}\cdot\vec{\text{T}})V_H(x)\,. \qquad (3.6\text{b})$$

In these formulas, $\hat{n}$ is a unit vector which specifies the direction of the SU(2) transformation. If we introduce

$$\mathfrak{A}' = \mathfrak{A}[\vec{\chi}'^{\text{in}}(x,\lambda\hat{n}), V'^{\text{in}}(x,\lambda\hat{n})]\,, \qquad (3.7)$$

we obtain from (3.3)–(3.6) the conditions

$$-:\left\langle\phi(x)\frac{\partial\mathfrak{A}'}{\partial\lambda}e^{-\mathfrak{A}'}\right\rangle: = i\,\frac{\hat{n}\cdot\vec{\tau}}{2}:\langle\phi(x)e^{-\mathfrak{A}'}\rangle:\,, \qquad (3.8)$$

$$-:\left\langle V(x)\frac{\partial\mathfrak{A}'}{\partial\lambda}e^{-\mathfrak{A}'}\right\rangle: = i\hat{n}\cdot\vec{\text{T}}:\langle V(x)e^{-\mathfrak{A}'}\rangle:\,. \qquad (3.9)$$

At this stage, it is convenient to introduce the special sources

$$j_i'(x,\lambda\hat{n}) = -Z_i^{-1/2}\chi_i'^{\text{in}}(x,\lambda\hat{n})K(x)\,, \tag{3.10a}$$

$$l'(x,\lambda\hat{n}) = 0\,, \tag{3.10b}$$

$$h_\alpha'(x,\lambda\hat{n}) = -\tilde{Z}_\alpha^{-1/2}V_\alpha'^{\text{in}}(x,\lambda\hat{n})\tilde{K}(x)\,. \tag{3.10c}$$

Then, using (3.4) and (3.7) we can rewrite Eqs. (3.8) and (3.9) as follows:

$$:\int d^4z \left\langle \phi(x),\, i\,\frac{\partial \vec{j}'(z,\lambda\hat{n})}{\partial\lambda}\cdot\vec{\chi}(z) + i\,\frac{\partial h'(z,\lambda\hat{n})}{\partial\lambda}\cdot V(z)\right\rangle_{\vec{j}',h'} := i\,\frac{\hat{n}\cdot\vec{\tau}}{2}\,:\langle\phi(x)\rangle_{\vec{j}',h'}:\,, \tag{3.11}$$

$$:\int d^4z \left\langle V(x),\, i\,\frac{\partial \vec{j}'(z,\lambda\hat{n})}{\partial\lambda}\cdot\vec{\chi}(z) + i\,\frac{\partial h'(z,\lambda\hat{n})}{\partial\lambda}\cdot V(z)\right\rangle_{\vec{j}',h'} := i\hat{n}\cdot\vec{\mathrm{T}}:\langle V(x)\rangle:_{\vec{j}',h'}\,. \tag{3.12}$$

To evaluate these expressions notice that

$$i\tau_i\phi = \frac{1}{\sqrt{2}}\begin{bmatrix} \chi_i + i\epsilon_{ij3}\chi_j + i\delta_{i3}\psi \\ \\ i\epsilon_{ij1}\chi_j - \epsilon_{ij2}\chi_j + (i\delta_{\alpha 1} - \delta_{\alpha 2})\psi \end{bmatrix}. \tag{3.13}$$

On the other hand, the basic Ward-Takahashi identity (2.29) yields the relations

$$\left\langle \phi_1(x),\, i\int d^4z[l(z)\chi_i(z) - j_i(z)\psi(z) + i\vec{j}(z)\cdot t_i\vec{\chi}(z) + 2ih(z)\cdot T_i\cdot V(z)]\right\rangle_{l,\vec{j},h} - \left\langle \phi_1(x),\, i\,\frac{\bar{v}}{Z_i}\int d^4z\, K(z)\chi_i(z)\right\rangle_{l,\vec{j},h}$$
$$= -\frac{1}{\sqrt{2}}\langle \chi_i(x) + i\epsilon_{ij3}\chi_j(x) + i\delta_{i3}\psi(x)\rangle_{l,\vec{j},h}$$

and

$$\left\langle \phi_2(x),\, i\int d^4z[l(z)\chi_i(z) - j_i(z)\psi(z) + i\vec{j}(z)\cdot t_i\vec{\chi}(z) + 2ih(z)\cdot T_i\cdot V(z)]\right\rangle_{l,\vec{j},h} - \left\langle \phi_2(x),\, i\,\frac{\bar{v}}{Z_i}\int d^4z\, K(z)\chi_i(z)\right\rangle_{l,\vec{j},h}$$
$$= -\frac{1}{\sqrt{2}}\langle i\epsilon_{ij1}\chi_j(x) - \epsilon_{ij2}\chi_j(x) + (i\delta_{\alpha 1} - \delta_{\alpha 2})\psi(x)\rangle_{l,\vec{j},h}\,.$$

If we compare these relations with (3.13), use the special sources (3.10), and set $\int d^4z\, j_1(z)\psi(z) = 0$, we get the identity

$$i\hat{n}\cdot\vec{\tau}:\langle\phi(x)\rangle_{\vec{j}',h'} := (-i)\int d^4z:\langle\phi(x),[i\vec{j}'(z,\lambda\hat{n})\cdot(\hat{n}\cdot\vec{t})\vec{\chi}(z) + 2ih'(z,\lambda\hat{n})\cdot(\hat{n}\cdot\vec{\mathrm{T}})\cdot V(z) - \bar{v}K(z)\hat{n}\cdot Z^{-1}\vec{\chi}(z)]\rangle_{\vec{j}',h'}:, \tag{3.14}$$

where we defined the diagonal matrix $Z = \mathrm{diag}(Z_1, Z_2, Z_3)$.

In a similar fashion, we can obtain from (2.29) the result

$$i\hat{n}\cdot\vec{\mathrm{T}}:\langle V(x)\rangle_{\vec{j}',h'} := (-i)\int d^4z:\langle V(x),[\tfrac{1}{2}i\vec{j}'(z,\lambda\hat{n})\cdot(\hat{n}\cdot\vec{t})\vec{\chi}(z) + ih'(z,\lambda\hat{n})\cdot(\hat{n}\cdot\vec{\mathrm{T}})\cdot V(z) - \tfrac{1}{2}\bar{v}\hat{n}\cdot K(z)Z^{-1}\vec{\chi}(z)]\rangle_{\vec{j}',h'}: \tag{3.15}$$

(the last term in this relation is actually zero for massive V). Comparing Eqs. (3.11) and (3.12) with the identities (3.14) and (3.15) we see that the requirements for symmetry rearrangement are met with the choice

$$2i\,\frac{\partial\vec{j}'(x,\lambda\hat{n})}{\partial\lambda} = \vec{j}'(x,\lambda\hat{n})\cdot(\hat{n}\cdot\vec{t}) + i\bar{v}\hat{n}\cdot Z^{-1}K(x)\,, \tag{3.16a}$$

$$i\,\frac{\partial h'(x,\lambda\hat{n})}{\partial\lambda} = h'(x,\lambda\hat{n})\cdot(\hat{n}\cdot\vec{\mathrm{T}})\,. \tag{3.16b}$$

Since the sources are given by Eqs. (3.10), we can rewrite these equations in terms of the in fields:

$$2\,\frac{\partial\vec{\chi}'^{\text{in}}(x,\lambda\hat{n})}{\partial\lambda} = iZ^{1/2}(\hat{n}\cdot\vec{t})Z^{-1/2}\vec{\chi}'^{\text{in}}(x,\lambda\hat{n}) - \bar{v}Z^{-1/2}\cdot\hat{n}\,, \tag{3.17a}$$

$$\frac{\partial V'^{\text{in}}(x,\lambda\hat{n})}{\partial\lambda} = i\tilde{Z}^{1/2}(\hat{n}\cdot\vec{\mathrm{T}})\tilde{Z}^{-1/2}V'^{\text{in}}(x,\lambda\hat{n})\,, \tag{3.17b}$$

where we introduced the $n\times n$ matrix $\tilde{Z} = \mathrm{diag}(\tilde{Z}_1, \ldots, \tilde{Z}_n)$. The antisymmetry of the t_i and T_i matrices was also used. Therefore we conclude that the in-field transformation obeying (3.17a) and (3.17b) induces the SU(2) transformation of the Heisenberg fields (3.6a) and (3.6b).

Equation (3.17b) can be integrated immediately to yield

$$V'^{\text{in}}(x,\lambda\hat{n})=\exp[i\lambda\tilde{Z}^{1/2}(\hat{n}\cdot\vec{T})\tilde{Z}^{-1/2}]V^{\text{in}}(x). \qquad (3.18)$$

Therefore, apart from the renormalization factors, $V^{\text{in}}(x)$ transforms according to the n-dimensional representation of SU(2).

We could also integrate (3.17a) by considering independent rotations around the three axes; since, however, we are interested here in the algebra of the in-field generators, we proceed directly to the determination of these quantities. By definition,

$$\vec{\chi}'^{\text{in}}(x,\lambda\hat{n})=\exp[-i\lambda(\hat{n}\cdot\vec{J})]\vec{\chi}^{\text{in}}(x)\exp[i\lambda(\hat{n}\cdot\vec{J})], \qquad (3.19)$$

where J_i $(i=1,2,3)$ are the sought-after generators. Differentiating this relation with respect to λ we get

$$\frac{\partial\vec{\chi}'^{\text{in}}(x,\lambda\hat{n})}{\partial\lambda}=-i[\hat{n}\cdot\vec{J},\vec{\chi}'^{\text{in}}(x,\lambda\hat{n})]\,. \qquad (3.20)$$

If we now compare this relation with (3.17a) and set $\lambda=0$ we obtain

$$[\hat{n}\cdot\vec{J},\vec{\chi}^{\text{in}}(x)]=-\tfrac{1}{2}Z^{1/2}(\hat{n}\cdot\vec{t})Z^{-1/2}\vec{\chi}^{\text{in}}(x)-\tfrac{1}{2}i\,\tilde{v}\,Z^{-1/2}\hat{n} \qquad (3.21)$$

or

$$[J_i,\chi_j^{\text{in}}(x)]=\tfrac{1}{2}i\epsilon_{ijk}\left(\frac{Z_j}{Z_k}\right)^{1/2}\chi_k^{\text{in}}(x)-\tfrac{1}{2}i\tilde{v}Z_j^{-1/2}\delta_{ij}. \qquad (3.22)$$

Equations (3.18) and (3.22) determine the in-field generators; if we call $\vec{J}^{(\chi)}(\vec{J}^{(V)})$ the part of the generators which depends on $\vec{\chi}^{\text{in}}$ (V^{in}), i.e.,

$$\vec{J}=\vec{J}^{(\chi)}+\vec{J}^{(V)}, \qquad (3.23)$$

we obtain

$$J_i^{(\chi)}=\frac{1}{2}\int d^3x\left[-\epsilon_{ijk}\left(\frac{Z_j}{Z_k}\right)^{1/2}\dot{\chi}_j^{\text{in}}(x)\chi_k^{\text{in}}(x)+\tilde{v}Z_i^{-1/2}\dot{\chi}_i^{\text{in}}(x)\right], \qquad (3.24)$$

$$J_i^{(V)}=-i\int d^3x\left(\frac{\tilde{Z}_\alpha}{\tilde{Z}_\beta}\right)^{1/2}\dot{V}_\alpha^{\text{in}}(x)(T_i)_{\alpha\beta}V_\beta^{\text{in}}(x). \qquad (3.25)$$

These generators obey the following commutation relations:

$$[J_i^{(\chi)},J_j^{(\chi)}]=\tfrac{1}{2}i\epsilon_{ijk}J_k^{(\chi)}+\tfrac{1}{4}i\epsilon_{ijk}\tilde{v}Z_k^{1/2}\times\int d^3x(Z_i^{-1}+Z_j^{-1}-Z_k^{-1})\dot{\chi}_k^{\text{in}}(x), \qquad (3.26)$$

$$[J_i^{(V)},J_j^{(V)}]=i\epsilon_{ijk}J_k^{(V)}, \qquad (3.27)$$

$$[J_i^{(\chi)},J_j^{(V)}]=0. \qquad (3.28)$$

Therefore

$$[J_i,J_j]=i\epsilon_{ijk}J_k-\tfrac{1}{2}i\epsilon_{ijk}J_k^{(\chi)}+\tfrac{1}{4}i\epsilon_{ijk}\tilde{v}Z_k^{1/2}\times\int d^3x(Z_i^{-1}+Z_j^{-1}-Z_k^{-1})\dot{\chi}_k^{\text{in}}(x). \qquad (3.29)$$

Only summation over k is implied in Eqs. (3.26) and (3.29).

Equation (3.29) shows that *the in-field generators do not form an algebra;* the present model illustrates therefore an extreme case of symmetry rearrangement. We can of course complete the algebra by enlarging the set of generators. The resulting algebra is best expressed in terms of the operators:

$$R_i=J_i^{(V)}-\epsilon_{ijk}\left(\frac{Z_j}{Z_k}\right)^{1/2}\int d^3x\,\dot{\chi}_j^{\text{in}}(x)\chi_k^{\text{in}}(x)=J_i^{(V)}+2J_i^{(\chi)}-Z_i^{-1}P_i, \qquad (3.30a)$$

$$P_i=\tilde{v}Z_i^{1/2}\int d^3x\,\dot{\chi}_i^{\text{in}}(x). \qquad (3.30b)$$

These operators provide a realization of the algebra of E(3), the Euclidean group in three dimensions:

$$[P_i,P_j]=0, \qquad (3.31a)$$

$$[P_i,R_j]=i\epsilon_{ijk}P_k, \qquad (3.31b)$$

$$[R_i,R_j]=i\epsilon_{ijk}R_k. \qquad (3.31c)$$

This E(3) structure is reminiscent of the SU(2) $\times$SU(2) structure expressed by Eqs. (2.25); in fact, under suitable boundary conditions, the in-field symmetry which corresponds to an SU(2) $\times$SU(2) invariance of the Heisenberg equations of motion is E(3).

It remains to verify the invariance of the S matrix under the in-field transformations (3.17a) and (3.17b). Equation (3.3a) shows that, if the S matrix is to be invariant, the following condition must be satisfied:

$$:\left\langle\frac{\partial\mathfrak{a}'}{\partial\lambda}e^{-\mathfrak{a}'}\right\rangle:=0,$$

which can be rewritten as follows:

$$\int d^4x :\left\langle Z^{-1/2}\,\frac{\partial\vec{\chi}'^{\,\mathrm{in}}(x,\lambda\hat{n})}{\partial\lambda}\cdot K(x)\vec{\chi}(x)+\tilde{Z}^{-1/2}\,\frac{\partial V'(x,\lambda\hat{n})}{\partial\lambda}\,\tilde{K}(x)V(x)\right\rangle_{\vec{j}',h'}:=0.$$

Using Eqs. (3.17a) and (3.17b) and the antisymmetry of the t_i's, T_i's, it is seen that this relation is equivalent to

$$\int d^4x :\langle i\vec{\chi}'^{\,\mathrm{in}}(x,\lambda\hat{n})\cdot Z^{-1/2}K(x)(\hat{n}\cdot\vec{t})\vec{\chi}(x)+2iV'^{\,\mathrm{in}}(x,\lambda\hat{n})\tilde{Z}^{-1/2}\tilde{K}(x)(\hat{n}\cdot\vec{T})V(x)\rangle_{\vec{j}',h'}:$$

$$=-\bar{v}\int d^4x\,K(x)\hat{n}\cdot Z^{-1}:\langle\vec{\chi}(x)\rangle_{\vec{j}',h'}:\,. \quad (3.32)$$

In terms of the special sources (3.10) used in this section, this condition takes the form

$$\int d^4x :\langle i\vec{j}'(x,\lambda\hat{n})\cdot(\hat{n}\cdot\vec{t})\vec{\chi}(x)+2ih'(x,\lambda\hat{n})\cdot(\hat{n}\cdot\vec{T})\cdot V(x)\rangle_{\vec{j}',h'}:=\bar{v}\int d^4x\,K(x)\hat{n}\cdot Z^{-1}:\langle\vec{\chi}(x)\rangle_{\vec{j}',h'}:$$

and is seen to be identical with the Ward-Takahashi identity (2.29) with $l(x)=\int d^4z\,j_i(z)\psi(z)=0$. The invariance of the S matrix under the transformation generated by $\vec{J}=\vec{J}^{(\chi)}+\vec{J}^{(V)}$ is therefore established.

It is worth remarking here that $\vec{J}^{(\chi)}$, $\vec{J}^{(V)}$, R_i and P_i defined by Eqs. (3.24), (3.25), and (3.30) are separately conserved and correspond to invariances of the in-field equations of motion. They *do not*, however, correspond to any *observable* symmetry, since they do not leave the S matrix unchanged, as can be seen from the calculation presented above.

Let us next examine the case when all components of $V^{\mathrm{in}}(x)$ have different masses. The equation of motion for $\vec{\chi}^{\mathrm{in}}$ remains the same, and V^{in} obeys

$$\tilde{K}(x)V^{\mathrm{in}}(x)=0, \quad (3.33)$$

with

$$\tilde{K}(x)=\Box+m^2, \quad (3.34)$$

and m^2 is an $n\times n$ diagonal matrix in a suitable basis:

$$(m^2)_{\alpha\beta}=m_\alpha{}^2\delta_{\alpha\beta},\quad m_\alpha\neq m_\beta\quad(\alpha\neq\beta). \quad (3.35)$$

The S-matrix and Heisenberg operators are still given by Eqs. (3.3), the only difference being that $\tilde{K}(x)$ in (3.4) stands for the Klein-Gordon matrix (3.34). The conditions of symmetry rearrangement are still expressed by (3.11) and (3.12), and the special sources by Eqs. (3.10), with $\tilde{K}(x)$ given in (3.34). These conditions are to be compared with the Ward-Takahashi identities (3.14) and (3.15). In the present case, however, we have

$$\int d^4z :\langle h(z)(\hat{n}\cdot\vec{T})V(z)\rangle_{\vec{j}',h'}:=0. \quad (3.36)$$

Indeed, the left-hand side is a sum of terms of the form

$$\int d^4z(\Box+m_\alpha{}^2):\langle V_\alpha^{\mathrm{in}}(z)(T_i)_{\alpha\beta}V_\beta(z)\rangle:$$

and the antisymmetry of T_i implies that V_α and V_β are different and therefore correspond to different masses and different momentum-space supports.

The identities (3.14) and (3.15) become in this case

$$i\hat{n}\cdot\vec{\tau}:\langle\phi(x)\rangle_{\vec{j}',h'}:=-i\int d^4z:\langle\phi(x),[i\vec{j}'(z,\lambda\hat{n})\cdot(\hat{n}\cdot\vec{t})\vec{\chi}(z)-\bar{v}K(z)\hat{n}\cdot Z^{-1}\vec{\chi}(z)]\rangle_{\vec{j}',h'}:, \quad (3.37)$$

$$i\hat{n}\cdot\vec{T}:\langle V(x)\rangle_{\vec{j}',h'}:=-i\int d^4z:\langle V(x),[\tfrac{1}{2}i\vec{j}'(z,\lambda\hat{n})\cdot(\hat{n}\cdot\vec{t})\vec{\chi}(z)-\tfrac{1}{2}\bar{v}\hat{n}\cdot K(z)Z^{-1}\vec{\chi}(z)]\rangle_{\vec{j}',h'}:. \quad (3.38)$$

If we now compare these identities with (3.11) and (3.12), we obtain again (3.16a) and

$$\frac{\partial h'(x,\lambda\hat{n})}{\partial\lambda}=0, \quad (3.39)$$

instead of (3.16b). Using the definition (3.10c) of h', we get then

$$\frac{\partial V'^{\text{in}}(x,\lambda\hat{n})}{\partial\lambda}=0. \tag{3.40}$$

Therefore $V^{\text{in}}(x)$ *does not transform at all under the* SU(2) *transformations of the Heisenberg fields.* All the transformation is carried by the $\vec{\chi}^{\text{in}}$ field, which obeys (3.22). The generators and their algebra can be obtained from the relations presented for the previous case, by setting $\vec{J}^{(V)}=0$ everywhere. The invariance of the S matrix is expressed by Eq. (3.32), with the $V^{\text{in}}(x)$ term absent:

$$\int d^4x :\langle i\vec{\chi}'^{\text{in}}(x,\lambda\hat{n})\cdot Z^{-1/2}K(x)(\hat{n}\cdot\vec{t})\vec{\chi}(x)\rangle_{\vec{j}',h'}: = -\bar{v}\int d^4x\,K(x)\hat{n}\cdot Z^{-1}:\langle\vec{\chi}(x)\rangle_{\vec{j}',h'}:. \tag{3.41}$$

We can summarize the results of this section in the statement that the transformation of the Goldstone field $\vec{\chi}^{\text{in}}$ is completely determined by the symmetry conditions of the model (i.e., the Ward-Takahashi identities), whereas the transformation of V^{in} is not. The latter depends crucially on the mass spectrum of the V^{in} particles, and this in turn can only be determined by dynamical calculations in the context of a specific Lagrangian.

IV. PREDICTIONS AND CONCLUSIONS

In this section we want to illustrate the type of "experimental" consequences which follow from the incomplete asymptotic symmetry discussed in the previous section. Such consequences are results of the invariance of the S matrix, which in turn is expressed by Eq. (3.32) if all the V_α^{in} masses are equal, or by Eq. (3.41) if they are all different. For the purpose of illustration, we assume the latter to be the case. If we set $\lambda=0$ in Eq. (3.41) and use (3.3b), we obtain

$$\int d^4x :\langle i\vec{\chi}^{\text{in}}(x)\cdot Z^{-1/2}t_i\,K(x)\vec{\chi}(x)e^{-\alpha}\rangle: = i\frac{\bar{v}}{Z_i}\int d^4x\,K(x)S\chi_{H,i}(x). \tag{4.1}$$

The predictions of the model are obtained by taking matrix elements of this relation in the in-particle Hilbert space. All states that appear in the following equations are understood to belong to this space; furthermore, we explicitly display in the state label the χ particles present. For example, $|a\rangle$ stands for a state with no Goldstone particles, $|a;\chi_i^{\text{in}}(\vec{k})\rangle$ is a state with one χ particle of symmetry index i and momentum $\vec{k}$, and so on; $a,b,c,\ldots$ denote collectively all other particles present.

Sandwiching (4.1) between states $|a\rangle$ and $|b\rangle$ we obtain

$$i\frac{\bar{v}}{Z_i}\int d^4x\,K(x)\langle a|S\chi_{H,i}(x)|b\rangle=0,$$

or, using the reduction formula,

$$\lim_{k\to 0}(2k)^{1/2}\langle a\,|\,S|\,b;\chi_i^{\text{in}}(\vec{k})\rangle=\lim_{k\to 0}(2k)^{1/2}\langle a;\chi_i^{\text{in}}(\vec{k})|\,S|\,b\rangle =0. \tag{4.2}$$

This result has the form of a low-energy theorem, stating that infrared Goldstone bosons do not participate in reactions of the type $a+\chi^{\text{in}}\to b$.

Similarly, if we consider states $|a\rangle$ and $|b,\chi_r^{\text{in}}(\vec{k})\rangle$, we obtain from (4.1)

$$\left(\frac{Z_i}{Z_r}\right)^{1/2}(t_i)_{rj}\langle a\,|\,S|\,b,\chi_j^{\text{in}}(\vec{k})\rangle = i\bar{v}(2\pi)^{3/2}Z_i^{1/2}\lim_{p\to 0}(2p_0)^{1/2}\langle a|\,S|\,b;\chi_r^{\text{in}}(\vec{k})\chi_i^{\text{in}}(\vec{p})\rangle. \tag{4.3}$$

Let us analyze the meaning of this relation: If the contribution of the infrared boson on the right-hand side is ignored, (4.3) becomes identical to the prediction of SU(2) symmetry with $\vec{\chi}^{\text{in}}$ forming an isotriplet, apart from the renormalization factors appearing in front. Thus the in-field transformation rotates $\vec{\chi}^{\text{in}}(x)$ as if it were a weighted isotriplet (the weight factors being the renormalization constants), and simultaneously adds an infrared Goldstone boson to the state. There is a remarkable similarity between Eq. (4.3) and current-algebra relations obtained by assuming that $\vec{\chi}^{\text{in}}(x)$ is interpolated by a partially conserved isospin current, despite the fact that the isospin current is strictly conserved in this model.

In a similar way, the statement of symmetry rearrangement (for unequal V^{in} masses),

$$\phi_H[\chi'^{\text{in}},V'^{\text{in}}]=\exp\left(i\lambda\frac{\hat{n}\cdot\vec{\tau}}{2}\right)\phi_H[\chi^{\text{in}},V^{\text{in}}],$$

$$V_H[\chi'^{\text{in}},V'^{\text{in}}]=\exp(i\lambda\hat{n}\cdot\vec{T})V_H[\chi^{\text{in}},V^{\text{in}}],$$

together with (3.17a) and (3.40) leads to

$$\left(\frac{1}{\lambda}\left\langle a\left|\left\{G\left[\exp\left(i\lambda\frac{\hat{n}\cdot\vec{\tau}}{2}\right)\phi_H, \exp(i\lambda\hat{n}\cdot\vec{\mathrm{T}})V_H\right]-G[\phi_H, V_H]\right\}\right|b\right\rangle\right)_{\lambda=0} = -\frac{\tilde{v}}{2}Z^{-1/2}\lim_{k\to 0}(2k_0)^{1/2}\langle a|\,G[\phi_H, V_H]\,|\,b;\vec{\chi}^{\,\mathrm{in}}(\vec{k})\rangle, \tag{4.4}$$

where $G[\phi_H, V_H]$ stands for any functional of the indicated Heisenberg fields and $|a\rangle$, $|b\rangle$ do not contain any χ^{in} particles. For example, if we choose $G=\overline{\phi}_H(x)\frac{1}{2}\tau_i\phi_H(x)$, we obtain

$$\epsilon_{ijk}\langle a|\,\overline{\phi}_H(x)\tfrac{1}{2}\tau_k\,\phi_H(x)|\,b\rangle = i\,\frac{\tilde{v}}{2}Z^{-1/2}\lim_{k\to 0}(2k_0)^{1/2}\langle a|\,\overline{\phi}_H(x)\tfrac{1}{2}\tau_i\,\phi_H(x)|\,b;\chi_j^{\mathrm{in}}(\vec{k})\rangle.$$

The above examples illustrate the method for obtaining phenomenological predictions from a specific form of in-field transformation.

In conclusion, the very simple model presented in this paper has several novel and interesting features, which we wish to recapitulate here.

(i) The set of in-field generators which leave the S-matrix invariant do not close to an algebra, but form part of an E(3) algebra. The extra operators needed to close the algebra are conserved, but do not lead to observable symmetry effects.

(ii) Although we started with an SU(2)-doublet Heisenberg field, we obtained among the in fields an *elementary* triplet which in many respects transforms as an isospin vector.

(iii) Despite the exact conservation of the isospin current, we obtained relations corresponding to current-algebra results which would follow from a partially conserved isocurrent hypothesis.

(iv) Comparison of the present model with the one presented in Ref. 3, where the dynamical symmetry group was again SU(2), shows the crucial role played by the boundary condition in determining the form of symmetry rearrangement. In the model of Ref. 3, the isospin group was rearranged into E(2), whereas in the present case the in-field generators do not even close to an algebra.

These results illustrate how difficult it is to infer the dynamical symmetry of the Heisenberg fields from observations at the asymptotic level (i.e., from symmetries of the S-matrix elements). On the other hand, the energence of *incomplete* phenomenological symmetries from invariance under a full group at the dynamical level opens new possibilities in the application of symmetry arguments in particle reactions. The richness of symmetry rearrangement is greater than hitherto assumed.

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