Triple-Regge region at subasymptotic energies in Reggeon field theory*

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We derive asymptotic results for the inclusive cross section in the triple-Regge region. The approach to this limit which will be detected at subasymptotic energies is especially emphasized. We discuss also the relation of the asymptotic inclusive cross section to the low-energy perturbation scheme.

The use of renormalization-group methods[1,2] in Reggeon calculus appears to be a very successful tool for understanding the high-energy behavior of scattering amplitudes. In recent publications[3] we analyze the asymptotic behavior of the elastic scattering amplitude, as well as the approach to this limit. The nonleading contributions to $\sigma_T(s)$ and $d\sigma/dt$ are crucial for the description of these quantities at subasymptotic energies and are of interest in the discussion of the transition[4] between the perturbative scheme and the onset of the critical phenomena results at high s. In discussing the asymptotic results of inclusive cross sections the knowledge of the nonleading terms is certainly of interest since for the onset of the scaling results not only a high s is needed but also large subenergies are required and the asymptotic limit will be reached at even higher s than for the total and elastic cross section. We present here an analysis of the inclusive cross section in the triple-Regge region. We discuss the relation between the asymptotic result and the bare-Pomeron perturbative scheme. The nonleading terms in the two subenergies are calculated and discussed.

Using the same notation used in Ref. 3, we discuss a Reggeon field theory with a triple-Pomeron coupling r_0, where the bare Pomeron has a linear trajectory with a slope α_0'. In order to construct the renormalization-group functions in the one-loop approximation, the diagrams of Figs. 1(a) and 1(b) should be calculated. There is, however, a difference between the Reggeon diagrams which contribute to the elastic amplitude and those which contribute to the inclusive cross section;[2] namely, we are interested in the value of the inclusive cross section at a fixed missing mass M^2. Since $\ln M^2$ is the "time" variable in our field theory, holding this variable fixed is very much reminiscent of an energy ($E=1-j$) nonconserving scattering process; that is, the scattering of a Pomeron by an external field. Since only the time coordinate, and not the space coordinate, is fixed the momentum remains conserved. Cardy, Sugar, and White and Abarbanel, Bartels, Bronzan, and Sidhu[5] have recently shown that in each Reggeon diagram which contributes to the inclusive cross section there is only one vertex which does not conserve energy.

We calculate the amputated Green's functions $\Gamma^{1,1}$ and $\Gamma^{1,2}$ from the diagrams of Fig. 1 [following Ref. 5, energy is not conserved at the darkened vertices in Fig. 1(b)]. To construct the representation of the energy nonconserving $\Gamma^{1,2}$ we use the renormalization-group integration method of Ref. 3. The parameters of the theory are defined by the following renormalization conditions[3]:

$$i\Gamma_R^{1,1}(-E_N,k_N)=-E_N-\alpha' k_N^2,\quad i\frac{\partial\Gamma_R^{1,1}(-E_N,k_N)}{\partial k^2}=-\alpha',$$
$$(2\pi)^{(D+1)/2}\Gamma_R^{1,2}(-E_N,-E_M,-E_L,\vec{k}_N,\vec{k}_M,\vec{k}_L)=r. \tag{1}$$

We define $\vec{v}_2=\vec{k}_M/|\vec{k}_N|$, $\vec{v}_3=\vec{k}_L/|\vec{k}_N|$, $l_2=E_M/E_N$, $l_3=E_L/E_N$ and the two dimensionless parameters $g=rE_N^{-\epsilon/4}\alpha'^{-D/4}$, $h=\alpha' k_N^2/E_N$ ($\epsilon=4-D$). Using Eq. (1) we have found in Ref. 3 in the one-loop approximation

$$g=g_0(1+g_0^2/g_1^2)^{-1/2},\quad \alpha'=\alpha_0'(1+g_0^2/g_1^2)^{-2\bar\zeta/\epsilon}, \tag{2a}$$
$$r=r_0(1+g_0^2/g_1^2)^{-2\bar\gamma_r/\epsilon},$$

and for the wave-function renormalization constant,

$$Z=(1+g_0^2/g_1^2)^{-2\bar\gamma/\epsilon}, \tag{2b}$$

where

$$g_1^2=\epsilon\frac{(8\pi)^{D/2}}{2\Gamma(3-D/2)}J^{-1}, \tag{3a}$$

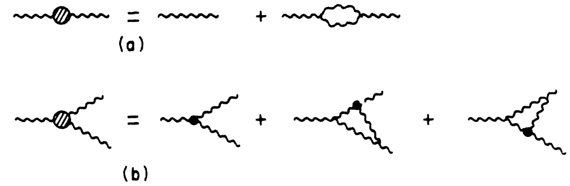

FIG. 1. The one-loop contribution to (a) $\Gamma^{1,1}$ and (b) $\Gamma^{1,2}$. Energy ($E=1-j$) is not conserved at the darkened vertices.

$$\bar{\gamma} = \frac{-\epsilon}{4J}\left[1+(x-1)\left(1+\frac{h_0}{2}\right)\right]\left(1+\frac{h_0}{2}\right)^{D/2-2}, \tag{3b}$$

$$\bar{\zeta} = -\frac{\epsilon}{4J}\left[1/2+(x-1)\left(1+\frac{h_0}{2}\right)\right]\left(1+\frac{h_0}{2}\right)^{D/2-2}, \tag{3c}$$

$$\bar{\gamma}_r = \epsilon/4 + D\bar{\zeta}/4 \quad (x^{-1} = D/2-1). \tag{3d}$$

The function $J(D, h_0, v_i^2, l_i)$ is found from the loop correction to the vertex and is given by

$$J = (D/8 - \tfrac{3}{2})\left(1+\frac{h_0}{2}\right)^{D/2-2} - \left(\frac{D-4}{D-2}\right)\left(\frac{3}{2}-\frac{D}{4}\right)\left(1+\frac{h_0}{2}\right)^{D/2-1} + 4F, \tag{3e}$$

where

$$F = \tfrac{1}{2}\int_0^1 d\zeta\{[\zeta + (1-\zeta)l_2 + h_0(v_2^2 + \zeta v_3^2 - \tfrac{1}{2}(v_2^2 - \zeta v_3^2))]^{D/2-2} + [2 \longrightarrow 3]^{D/2-2}\}, \tag{3f}$$

where g_0 and h_0 are the unrenormalized g and h.

At asymptotic energies the leading contribution to the inclusive cross section in the triple-Regge region comes from[2] the "enhanced graphs." Define[5] $a(j, j_2, j_3)$ from

$$s^2\frac{d\sigma}{dt\,dM^2} = \frac{1}{16\pi^2}\int a(j,j_2,j_3)(M^2)^j (s/M^2)^{j_2+j_3} \times \frac{dj_1\,dj_2\,dj_3}{(2\pi i)^3}. \tag{4}$$

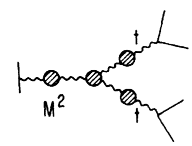

FIG. 2. The leading contribution to the inclusive cross section in the triple-Regge region.

Then the leading contribution is represented by the diagram of Fig. 2, namely

$$a(j,j_2,j_3) = N(0)N(t)^2 iG^{1,1}(E,0)(2\pi)^{(D+1)/2} \times \Gamma^{1,2}(E,E_2,E_3,0,k,-k) \times iG^{1,1}(E_2,k)iG^{1,1}(E_3,-k). \tag{5}$$

From Eq. (1) it follows that[3]

$$i\Gamma^{1,1}(E,k) = \frac{E-\alpha'(E,k)k^2}{Z(E,k)}, \tag{6}$$

where $\alpha'(E,k)$ and $Z(E,k)$ are given by Eq. (2) [take $g_0 = g_0(E) = r_0^2(-E)^{-\epsilon/2}/\alpha_0'^{D/2}$]. The interesting properties of this representation were discussed in Ref. 3. We will mention here only the fact that Eq. (6) contains the asymptotic (infrared) behavior of the Pomeron propagator in the ϵ expansion:

$$iG^{1,1} = (-E)^{-1}\{1-\alpha_0' t(-E)^{-1}[1+\kappa(-E)^{-\epsilon/2}]^{-2\bar{\zeta}/\epsilon}\}^{-1}[1+\kappa(-E)^{-\epsilon/2}]^{-2\bar{\gamma}/\epsilon}, \tag{7}$$

where $\kappa = (r_0^2/\alpha_0'^{D/2})g_1^{-2}$.

Equation (7) gives the forward diffractive peak when the Mellin transform is taken.[3] On the other hand, when Eq. (7) is expanded in powers of r_0^2 (without any ϵ expansion) one finds

$$i\Gamma^{1,1}(E,k^2) = E - \alpha_0' k^2 - \frac{1}{2}\frac{r_0^2}{\alpha_0'^{D/2}}\frac{1}{(8\pi)^{D/2}}\Gamma(1-D/2)\left(\frac{\alpha_0' k^2}{2} - E\right)^{D/2-1} + O(r_0^4), \tag{8}$$

which is the perturbative expansion of Fig. 1(a) (to order r_0^2). [In Eq. (8) $J(E_i, k_i)$ is canceled since only expressions of the form $\bar{\gamma}/g_1^2$ and $\bar{\zeta}/g_1^2$ appear.]

The above two limits can be seen also in the representation of $\Gamma^{1,2}$. From Eq. (1) we have

$$\begin{aligned} i\Gamma^{1,2}(E_i,k_i) &= \frac{r(E_i,k_i)}{(2\pi)^{(D+1)/2}} Z^{-3/2}(E_i,k_i) \\ &= \frac{r_0}{(2\pi)^{(D+1)/2}}[1+\kappa(-E)^{-\epsilon/2}]^{(2/\epsilon)[(3/2)\bar{\gamma}-\bar{\gamma}_r]} \\ &= \frac{r_0}{(2\pi)^{(D+1)/2}}[1+\kappa(-E)^{-\epsilon/2}]^{(2/\epsilon)[(3/2)\bar{\gamma}+(D/4)(1-\bar{\zeta})-1]}. \end{aligned} \tag{9}$$

$\bar{\gamma}$ and $\bar{\zeta}$ are calculated in the ϵ expansion of the expressions in Eq. (3) and the infrared behavior of $\Gamma^{1,2}$ is easily seen in Eq. (9). If, however, we expand $\Gamma^{1,2}$ in powers of r_0^2 we get (without ϵ expansion)

$$\Gamma^{1,2}(E_i, k_i) = \frac{r_0}{(2\pi)^{(D+1)/2}} \left[1 - \frac{r_0^2}{\alpha_0'^{D/2}} \frac{\Gamma(2-D/2)}{(8\pi)^{D/2}} (-E)^{-\epsilon/2} 2F + O(r_0^4)\right]. \tag{10}$$

Equation (10) gives the perturbative expansion of the energy-nonconserving $\Gamma^{1,2}$ in Fig. 1(b).

The representations in Eqs. (7) and (9) which contain, as seen above, the asymptotic scaling results of $G^{1,1}$ and $\Gamma^{1,2}$ as well as their perturbative expressions in the one-loop approximation are used now to find the asymptotic result of the inclusive cross section. We use these representations to first order in ϵ and perform the appropriate Mellin transforms[3]

$$M^2 \frac{d\sigma}{dt dM^2} = \frac{N(0)N(t)^2}{16\pi^2} r_0 \Phi(\tfrac{1}{2}, 1, -\kappa \ln M^2) \left\{ \sum_n \frac{[\alpha_0' t \ln(s/M^2)]^n}{n!} \Phi\left(-\frac{1}{6} - \frac{n}{12}, n+1, -\kappa \ln(s/M^2)\right)\right\}^2, \tag{11}$$

where $\Phi(a, b; z) \equiv {}_1F_1(a, b; z)$.

We will calculate the inclusive cross section in a consistent ϵ expansion in what follows, but one can roughly estimate Eq. (11) by using the asymptotic expression[7] of $\Phi(a, b; z)$ and neglecting terms of order ϵ. At high s, high s/M^2, and small $\alpha_0' t \ln(s/M^2)$ we have $(M^2/s = 1-x)$

$$\frac{d\sigma}{dt dx} \simeq \frac{N(0)N(t)^2}{16\pi^2} r_0 \frac{[-\kappa \ln(1-x)]^{1/3}}{1-x} \times (\kappa \ln M^2)^{-1/2} e^{2T}, \tag{12a}$$

which can be written in terms of renormalized parameters as

$$\frac{d\sigma}{dt dx} \simeq \frac{N_R(0)N_R^2(t)}{16\pi^2} r \frac{[-E_N \ln(1-x)]^{1/3}}{1-x} \times (E_N \ln M^2)^{-1/2} e^{2T}, \tag{12b}$$

where

$$T = (\alpha_0' t/\kappa)[-\kappa \ln(1-x)]^{13/12} = \frac{\alpha' t}{E_N}[-E_N \ln(1-x)]^{13/12}.$$

E_N is the energy scale introduced through the renormalization conditions. The same power behavior in $\ln M^2$ and $\ln(s/M^2)$ was found by Migdal *et al.*,[8] who calculated the leading behavior of $d\sigma/dt dx$ at $t=0$.

Another interesting aspect of the representations in Eqs. (5), (7), and (9) is their implications on the approach to the asymptotic result in Eq. (12). Note that in terms of renormalized parameters

$$\frac{r_0^2}{(\alpha_0')^{D/2}} \frac{1}{g_1^2} = \kappa = E_N^{\epsilon/2} \left(\frac{g_1^2 - g^2}{g^2}\right)^{-1} \xrightarrow[g \to g_1]{} \infty. \tag{13}$$

The expansions of Eqs. (7) and (9) in powers of κ^{-1} give the asymptotic (infrared) behavior of $\Gamma^{1,1}$ and $\Gamma^{1,2}$ multiplied by a power series in κ^{-1}. The Mellin transform of Eq. (4) and a consistent ϵ expansion [in terms of renormalized parameters—which also remove infinities caused by Eq. (13)] gives for g close to g_1

$$\begin{aligned}\frac{d\sigma}{dt dx} = &\frac{N_R(0)N_R^2(t)}{16\pi^2} r (E_N \ln M^2)^{-\epsilon/4 + (1/2)\bar\gamma - \bar\zeta} \frac{[-E_N \ln(1-x)]^{-2\bar\gamma}}{1-x} \exp\left\{\frac{2\alpha' t}{E_N}[-E_N \ln(1-x)]^{1-\bar\zeta}\right\} \\ &\times C\left[1 + \frac{1}{2}\left(\frac{g_1^2 - g^2}{g^2}\right)(E_N \ln M^2)^{-\epsilon/2} + O((g-g_1)^2 (\ln M^2)^{-\epsilon})\right] \\ &\times \left\{\phi_0(T) + \frac{1}{6}\left(\frac{g_1^2-g^2}{g^2}\right)[-E_N \ln(1-x)]^{-\epsilon/2}\phi_1(T) + O((g-g_1)^2[\ln(1-x)]^{-\epsilon})\right\}^2,\end{aligned} \tag{14a}$$

where

$$\bar\gamma = -\frac{\epsilon}{12}, \quad \bar\zeta = -\frac{\epsilon}{24}, \quad C = 1 - \frac{\epsilon}{4}\ln\gamma_e \tag{14b}$$

$$\phi_0(T) = 1 - \frac{\epsilon}{12} f(T), \tag{14c}$$

$$\phi_1(T) = 1 + \frac{T}{2} - \frac{\epsilon}{24}\left[T \frac{d}{dT} f(T) + (T-10) f(T)\right], \tag{14d}$$

$$f(T) = \left(1 + \frac{T}{2}\right)\left[-\ln\gamma_e + \int_0^T \frac{dz}{z}(1 - e^{-z})\right] + \frac{1 - e^{-T}}{2}. \tag{14e}$$

Note that at $t=0$, $N_R(0)$ is related to σ_T and if $g = g_1$ we have $r = \alpha'^{D/4} E_N^{\epsilon/4} g_1$ which determines the strength[2,6] of the inclusive cross section in terms of g_1 [$g_1 = 4\pi(\frac{2}{3}\epsilon)^{1/2}$ in the one-loop approximation].

The contribution of Eq. (12) to $\sigma_T(s)$ is found after the $d^D p_T \sim t^{D/2-1} dt$ and dM^2 integrations and gives the term[9]

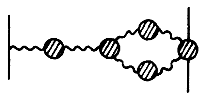

FIG. 3. The leading contribution of $\Gamma^{1,2}$ to the total cross section.

$$\sigma_T^{3P}(s) \sim (\ln s)^{-(3/2)\bar{\gamma}-(D/4)(1-\bar{\zeta})}, \tag{15}$$

which is lower than the leading $(\ln s)^{\epsilon/12}$ by a power of $-1+\epsilon/4$ in $\ln s$ (to first order in ϵ). Note, however, that this contribution of $\Gamma^{1,2}$ to σ_T is combined with other contributions which come from the renormalized two-particle two-Pomeron vertex in Fig. 3 to produce a $(\ln s)^{-1}$ term (independent of ϵ). This can be seen very easily here. The two-particle–two-Pomeron vertex ($N^{(2)}$) is given to order r_0^2 by an expression very similar to Eq. (10). [In order to describe the emission of n Pomerons from a particle line the terms $\sim\bar{g}_0^{(n)}(\Psi^\dagger)^n$ are added[2] to the Lagrangian.] Using the method of Ref. 3 we define $Z_{\bar{g}}^{(2)}$ from $\bar{g}^{(2)}=\bar{g}_0^{(2)}Z_{\bar{g}}^{(2)}$ and $\gamma_{\bar{g}}^{(2)}$ from

$$\gamma_{\bar{g}}^{(2)} = E_N \frac{d\ln Z_{\bar{g}}^{(2)}}{dE_N}\bigg|_{r_0,\alpha_0'} = \bar{\gamma}_{\bar{g}}^{(2)}\frac{g^2}{g_1^2}, \tag{16}$$

and obtain for $N^{(2)}$

$$N^{(2)} = \bar{g}_0^{(2)}(1-g^2/g_1^2)^{-(2/\epsilon)\bar{\gamma}+(2/\epsilon)\bar{\gamma}_{\bar{g}}^{(2)}}$$
$$= \bar{g}_0^{(2)}\left[1+\frac{r_0^2}{(\alpha_0')^{D/2}}(-E)^{-\epsilon/2}\right]^{-2/3} \tag{17}$$

(to first order in ϵ, $\bar{\gamma}_{\bar{g}}^{(2)}=\epsilon/4$). The renormalized $N_R^{(2)}$ can be written to this order in ϵ as

$$N_R^{(2)} = \bar{g}^{(2)}(-E/E_N)^{\bar{\gamma}_{\bar{g}}^{(2)}-\bar{\gamma}}$$
$$\times[1+(1-g^2/g_1^2)(-E/E_N)^{\epsilon/2}]^{-(2/\epsilon)(\bar{\gamma}_{\bar{g}}^{(2)}-\bar{\gamma})}. \tag{18}$$

Note that $\omega\equiv\bar{\gamma}_{\bar{g}}^{(2)}-\bar{\gamma}=\bar{\gamma}_r-\frac{3}{2}\bar{\gamma}=\epsilon/3$ and the relation[2]

$$2\omega+3(-1+\bar{\gamma})+1+\frac{D}{2}(1-\bar{\zeta})=0 \tag{19}$$

ensure that the contribution of Fig. 3 is $\sim(\ln s)^{-1}$, independent of ϵ. It is easy to see that Eq. (19) is satisfied here. It includes the critical exponents of $\Gamma^{1,2}, N^{(2)}$ [ω in Eq. (19)], and of $G^{1,1}$ [$-1-\bar{\gamma}$ in Eq. 7] and the contribution of the loop integration in Fig. 3 [$1+(D/2)(1-\bar{\zeta})$].

Our calculations were done to the first order in ϵ. Higher-order terms can be calculated to improve Eq. (14); through straightforward, these calculations are tedious. As the powers of $\ln M^2$ and $\ln(1-x)$ in the leading term, as well as $\phi_0(T)$ and $\phi_1(T)$ are changed, the nonleading terms are governed by the exponent $\lambda=(d\beta/dg)_{g=g_1}$ (this exponent was discussed in Ref. 3). The nonleading terms of Eq. (14) are then proportional to $(\ln M^2)^{-\lambda}$ and $[\ln(1-x)]^{-\lambda}$ [$\lambda=\epsilon/2$ to first order in ϵ as we discuss in Ref. 3, the method of Padé approximants suggests that $\lambda(\epsilon=2)=0.37$ to order ϵ^2].

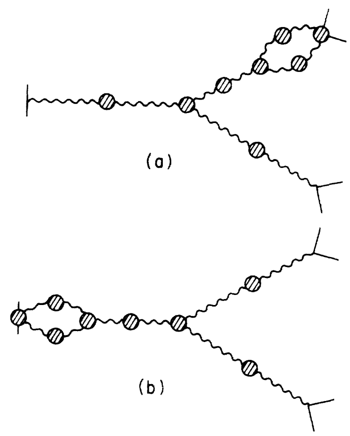

FIG. 4. Nonleading contributions to the inclusive cross section from "nonenhanced" graphs.

Although the leading contribution comes from the "enhanced" graph in Fig. 2, the nonleading terms of Fig. 2 may compete with leading terms from the "nonenhanced graphs" [e.g., Figs. (4a) and (4b)], as in the case of the elastic amplitude.[3] The contribution of Fig. 4(a) is $[\ln(1-x)]^{-1}$ as compared to $[\ln(1-x)]^{-\bar{\gamma}-\lambda}$ which comes from the nonleading term of Fig. 2 ($-\bar{\gamma}=0.38$ to order ϵ^2).[10] Figure (4b) contributes a nonleading term of the form $(\ln M^2)^{-1}$, which competes with the contribution $(\ln M^2)^{-\bar{\gamma}-\lambda}$ from the nonleading term of Fig. 2. As discussed in Ref. 3, the bad convergence properties in the ϵ expansion of λ preclude the determination of the leading corrections to the asymptotic result. A better method for the calculation of λ is needed. Another source for nonleading contributions might come from other possible interactions of the Pomeron. We believe therefore that Eq. (14) should be considered as a first trial in fitting the data at preasymptotic energies in the region of high s, high s/M^2, and small T.

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[8]Equation (6.20) in Ref. 2 contains an error: The exponent of $\xi = \ln s$ should be $\epsilon/24$ (this was corrected also in Ref. 6).

[9]We use $d^D P_T \sim t^{D/2-1} dt$ and take $D=2$ after performing the ϵ expansion of the exponents. This is done in order to have a consistent evaluation of loop integrations as presented for example in the relation of Eq. (19). If we ignore this point, taking $D=2$ and integrating with respect to dt we get $\sigma_T \sim (\ln s)^{-\epsilon/8}$ compared to $(\ln s)^{-1+\epsilon/3}$ in Eq. (15).

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