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Impact picture and the ratio of elastic to total cross section at CERN ISR energies

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We show that the impact picture of the Pomeron together with f^0 and ω exchange account for the shrinkage (antishrinkage) of the elastic forward peak for proton-proton (antiproton-proton) scattering for energies greater than 20 GeV. We predict that the ratio of the elastic to total pp cross section rises by 7% in the energy range available at the CERN ISR.

In 1970, on the basis of relativistic quantum field theory, we described the general qualitative features of the impact picture of high-energy scattering.[1] Subsequently, in 1972, we made quantitative predictions for experiments at the CERN Intersecting Storage Rings (ISR).[2] At that time we predicted

(i) that the proton-proton total cross section would increase by about 3 mb (results of experiments[3,4] in 1973 showed a significant rise in the proton-proton total cross section; in fact the rise was about 5 mb); and

(ii) that a dip would develop in the elastic proton-proton differential cross section at $|t| \simeq 1.3 (\mathrm{GeV}/c)^2$ (this also was subsequently confirmed).[5]

We have used the observed rise of 5 mb in the proton-proton total cross section to determine the value of the parameter c.[6] As the same parameter c controls all the rates of increase of hadronic total cross sections, we then made predictions for the energy dependence of $\pi^\pm$, $K^\pm$, and $\bar{p}$ total cross sections at high energy.[6]

Very recently, preliminary measurements[7] at the Fermi National Accelerator Laboratory have been shown to be in excellent agreement with all of these predictions. Thus encouraged, we are led to consider additional consequences of the impact picture which could be tested at Fermilab[8] and the ISR in the near future.

In this paper, we consider in detail the ratio of elastic to total proton-proton cross section. After taking due account of Regge background terms in the elastic amplitude, thereby obtaining a more reliable estimate of the energy dependence of the elastic cross section at low energies, we predict that the ratio of elastic to total proton-proton cross section will increase by 7% in the energy range available at the ISR, namely $500 \lesssim s \lesssim 3800\ \mathrm{GeV}^2$. Sufficiently accurate measurements appear feasible and constitute a critical test of the impact picture.

At the energies for which this prediction is mad and for the accuracy which is required, our previous formulation of the impact picture must be supplemented by the addition of some Regge concepts. We have incorporated the dominant Regge background in terms of f^0 and ω exchanges. Inclusion of these exchanges in a phenomenological manner together with our previous model of the impact picture of the Pomeron allows us to describe

(1) rapid shrinkage of the pp diffraction peak at Serpukhov energies ($s \lesssim 100\ \mathrm{GeV}^2$),

(2) slow shrinkage of the pp diffraction peak at Fermilab and ISR energies ($100 \lesssim s \lesssim 3000\ \mathrm{GeV}^2$),

(3) slow antishrinkage of the antiproton-proton diffraction peak at Serpukhov energies ($s \lesssim 100\ \mathrm{GeV}^2$),

(4) the ratio of real to imaginary parts of the proton-proton elastic amplitude as a function of energy and momentum transfer, t.

We start by assuming that elastic hadron-hadron scattering amplitudes are dominated by three terms, namely

(1) the Pomeron as described and parametrized by us previously,[6]

(2) tensor exchanges (f^0, A_2) with even signature and trajectory $\alpha(t) = 0.5 + t$ [t in $(\mathrm{GeV}/c)^2$],

(3) vector exchanges (ρ, ω) with odd signature and trajectory degenerate with $\alpha(t)$.

For simplicity, we neglect spin-flip amplitudes, whose contributions to elastic scattering are known to be small near the forward direction. In addition we neglect the isospin exchange contributions, which are known to be small from measurements on charge-exchange scattering. We express the Pomeron contribution to the amplitude as usual by

$$A_p(s,t)=\frac{is}{2\pi}\int dx_\perp e^{-i\vec{\Delta}\cdot\vec{x}_\perp}D(s,x_\perp)\ ,$$

where $\vec{\Delta}$ is the momentum transfer, $-t=\Delta^2$, $x_\perp$ is the impact parameter, and

$$D(s,x_\perp)=1-\exp[-S(s)F(x_\perp{}^2)]\ .$$

On the basis of our experience with field theory we have taken

$$S(s)=\frac{(Ee^{-i\pi/2})^c}{[\ln(Ee^{-i\pi/2})]^{c'}}\quad ,$$

where E is the incident laboratory energy in GeV, c is a real constant, and c' has been chosen as before to be 0 or 1. As the predictions we obtain are similar in the two cases, we present the $c'=0$ solution. In addition, we have chosen

$$F(x_\perp{}^2)=f\ \exp[-\lambda(x_\perp{}^2+x_0{}^2)^{1/2}]\ ,$$

where f, λ, and x_0 are parameters which have already been determined.[6]

The f^0- and ω-type exchanges are assumed in the usual way to give rise to an amplitude of the form

$$A_{f^0}(s,t)=\gamma_f(t)\left[\frac{\alpha(t)}{\alpha(0)}\right]\left(\frac{1+e^{-i\pi\alpha(t)}}{\sin\pi\,\alpha(t)}\right)s^{\alpha(t)-1}$$

and

$$A_\omega(s,t)=\gamma_\omega(t)\left(\frac{1+e^{-i\pi\alpha(t)}}{\sin\pi\,\alpha(t)}\right)s^{\alpha(t)-1}\ ,$$

where $\gamma_{f,\omega}(t)$ is the residue function and s is in units of GeV^2. The differential cross sections are given by

$$\frac{d\sigma}{dt}(pp)=1.2233\left|\frac{1}{s}A_p+A_{f^0}-A_\omega\right|^2\ \text{mb}/(\text{GeV}/c)^2$$

and

$$\frac{d\sigma}{dt}(\bar{p}p)=1.2233\left|\frac{1}{s}A_p+A_{f^0}+A_\omega\right|^2\ \text{mb}/(\text{GeV}/c)^2\ .$$

Fits by other authors[9] to pp and $\bar{p}p$ elastic scattering at less than 20 GeV have led to the parameterization

$$\gamma_\omega(t)=\frac{\bar{\gamma}_\omega e^{0.5t}(1+t/0.14)[(1+t)^2+0.16]}{1.16}\ .$$

This parameterization reproduces the well-known crossover effect in the elastic differential cross section at $t=0.14$ (GeV/c)^2 and is accurate for $|t|<0.4$ (GeV/c)^2. We take the same residue function and apply it for energies greater than 20 GeV with no new parameters.

The residue $\gamma_f(t)$ has not previously been determined without some theoretical assumption on the Pomeron contribution. For simplicity we take

$$\gamma_f(t)=-\bar{\gamma}_f\,e^{dt}$$

for the small-t region $0\le|t|\le0.4$ (GeV/c)^2, where d is a single adjustable parameter.

TABLE I. Parameters for nucleon-nucleon scattering.

c	0.082 925
c'	0
λ	0.600 71
x_0	3.8750
f	6.5812
4.893 $(\gamma_f-\bar{\gamma}_\omega)$	47.39
4.893 $(\gamma_f+\bar{\gamma}_\omega)$	101.8

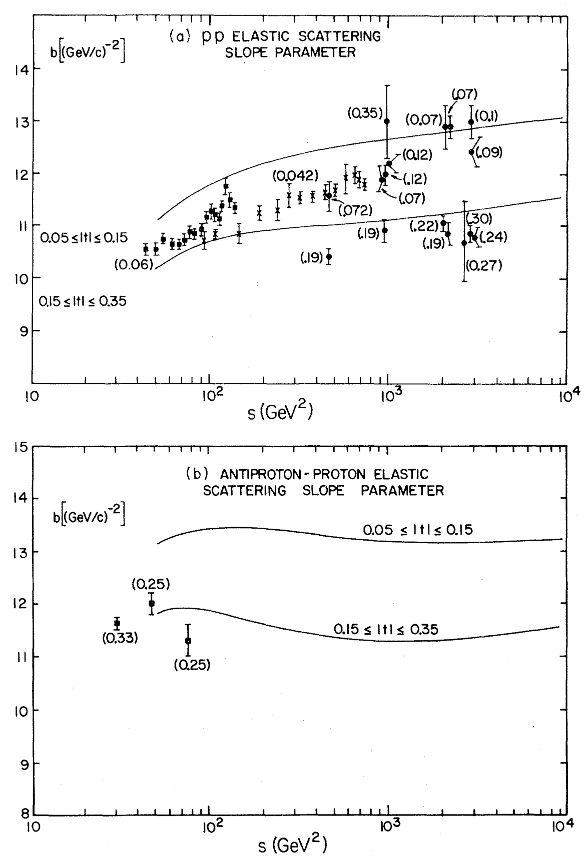

FIG. 1. The slope parameter b for proton-proton and antiproton-proton elastic scattering plotted versus the square of the center-of-mass energy s. The curves are for the two intervals of $|t|$ indicated. The average value of $|t|$ for each set of data is shown. Data from Serpukhov, Fermilab, and the ISR are shown as squares, crosses, and circles, respectively.

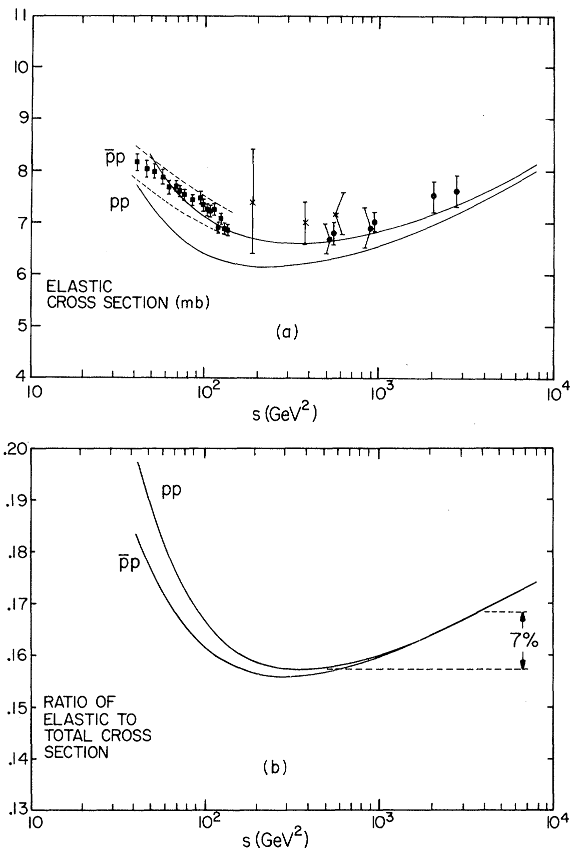

FIG. 2. The predicted ratio of the real to imaginary parts of the proton-proton and antiproton-proton elastic scattering amplitudes plotted versus the square of the center-of-mass energy s. The data shown are for $|t| = 0$ $(\mathrm{GeV}/c)^2$.

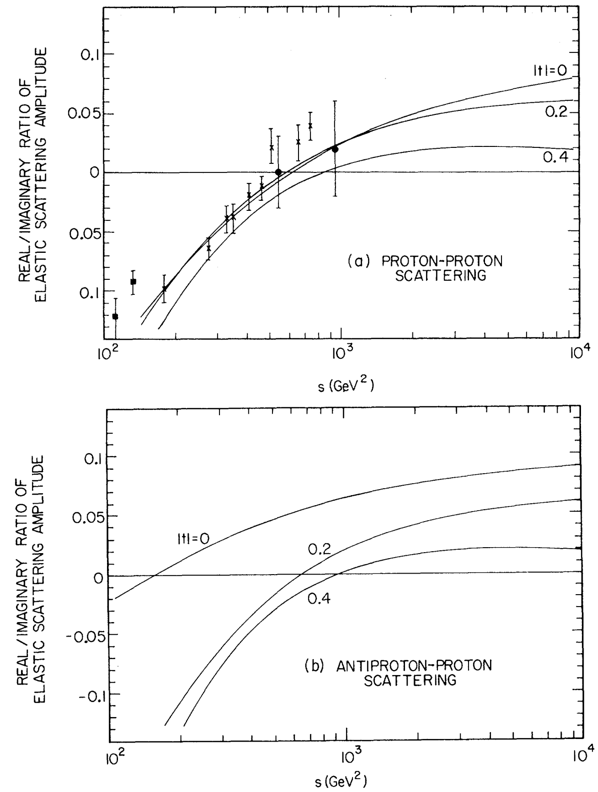

FIG. 3. (a) The predicted elastic cross section for proton-proton and antiproton-proton scattering plotted versus the square of the center-of-mass energy s (GeV^2). (b) The predicted ratio of the elastic to total cross section for proton-proton and antiproton-proton interactions plotted versus the square of the center-of-mass energy s (GeV^2).

From the optical theorem we have

$$\sigma_{\text{total}}(\bar{p}p) = 4.893\left[\frac{\mathrm{Im}A_p(s,0)}{s} + s^{-1/2}(\bar{\gamma}_f + \bar{\gamma}_\omega)\right] \text{ mb},$$

$$\sigma_{\text{total}}(pp) = 4.893\left[\frac{\mathrm{Im}A_p(s,0)}{s} + s^{-1/2}(\bar{\gamma}_f - \bar{\gamma}_\omega)\right] \text{ mb}.$$

We have previously[6] obtained the parameters given in Table I for the case of nucleon-nucleon scattering. Hence the residue normalizations are given by $\bar{\gamma}_f = 15.25$ and $\bar{\gamma}_\omega = 5.55$.[10]

Thus, as stated earlier, we are left with one free parameter, namely d, which can be adjusted for a satisfactory description of the shrinkage (antishrinkage) phenomenon of pp ($\bar{p}p$) elastic scattering. We find that a reasonable fit to the data is obtained with $d = 0.5$ $(\mathrm{GeV}/c)^{-2}$.

The comparison of this model with all available data for b, the logarithmic derivative of $d\sigma/dt$, is shown in Figs. 1(a) and 1(b). The fits tend to lie systematically slightly higher than the Serpukhov[11] and Fermilab[12] data for b in pp scattering. However, the data have a ± 0.3 systematic error which is not shown in the figures. The overall energy dependence of b in pp scattering is reproduced satisfactorily by the addition of ω and f^0 contributions to our previously parameterized Pomeron amplitude. The trend of the data suggests that in our original[6] choice of input data to determine the parameters of the Pomeron contribution, we may have chosen too large a value of b $(=11.4)$ in the range $0.14 \leq |t| \leq 0.24$ $(\mathrm{GeV}/c)^2$. However, this does not affect our prediction for the energy dependence of the ratio of elastic to total cross section in a significant way. In Fig. 1(b) it can be seen that the magnitude of the experimental values[13] of $b_{\bar{p}p}$ and the slow antishrinkage are again well reproduced by the model.

We have calculated the ratio of the real to imaginary parts of the elastic amplitudes for pp and $\bar{p}p$ scattering at various fixed t values as a function of s. These predictions are given in Figs. 2(a) and

2(b). The curves for $t=0$ are identical to those we obtained earlier.[7] The recent results of the Dubna-Fermilab-Rockefeller-Rochester[14] collaboration for pp scattering at $t=0$ are shown in Fig. 2(a). In addition, the previous data from the CERN-Rome group[15] at the ISR are also shown. There is good agreement between the curve for $t=0$ and the data. The other curves are new predictions of our model for the scattering amplitude away from the forward direction.[16] Experimental data on both the pp and $\bar{p}p$ cases will be of considerable interest.

We may conclude that the simplest Regge exchange contributions, namely the ω and f^0 exchanges, in addition to the Pomeron amplitude are sufficient to account for the preasymptotic (20 to ≈200 GeV) behavior of proton-proton and antiproton-proton scattering. The successful description of the energy and momentum dependence of these data at less than 200 GeV provides confidence in the accuracy of the Regge background contribution to the predicted energy dependence of $\sigma_{el}(s)$ above 200 GeV.

Figure 3(a) shows the integrated elastic cross section $\sigma_{el}(s)$ for both pp and $\bar{p}p$ scattering versus s. Data from Serpukhov,[17] Fermilab,[18] and the ISR[19] can be seen to follow closely the general trend of the pp curve. The curve lies a few percent below the data because of our original choice of too large a slope parameter b as mentioned earlier. Finally, Fig. 3(b) shows the predicted ratio R defined to be $\sigma_{el}(s)/\sigma_{total}(s)$, plotted versus s. It can be seen that $R(pp)$ and $R(\bar{p}p)$ both fall at low s, reach a minimum at $s \approx 300$ GeV^2, and then tend to rise at high s. In the range accessible to the ISR, $500 \leq s \leq 3800$ GeV^2, there is a 7% increase in R. The accuracy[20] of the available data is not quite sufficient to allow observation of the predicted increase in R.

In conclusion, we would like to emphasize that in Regge theories with cuts, such as those of Gribov,[21] Frautschi and Margolis,[22] and Dean,[23] it is very difficult to see how $\sigma_{el}(s)$ can increase with increasing energy. On the other hand, the impact picture[1] predicts not only that $\sigma_{el}(s)$ rises with energy, but also that it increases faster than σ_{total}. Therefore, the 7% rise in the ratio of elastic to total cross sections, if confirmed by experiment at the ISR, will represent a significant step forward in establishing the validity of the impact picture of high-energy scattering. It is strongly urged that more accurate measurements be made in the near future to confront this prediction.

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[10]The ratio of the residue functions $\gamma_f(0)/\gamma_\omega(0) = 2.75$. This value has to be compared with unity, which would be expected on the basis of exact exchange degeneracy. On the other hand, the total pp cross-section data clearly indicate that exchange degeneracy is not exact. It may be remarked that our other fit with $c' = 1$ allows a much milder (the above ratio is about 1.3) breaking of exchange degeneracy. We regard this as weak but positive evidence in favor of that choice for c'.

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[16]We do not regard these predictions on the ratio of real to imaginary parts of the elastic amplitude away from the forward direction to be as reliable as our prediction for the rise of $R = \sigma_{el}(s)/\sigma_{total}(s)$. The increase in R emerges in a natural way from the impact picture, whereas the real-to-imaginary ratios are very sensitive to the Regge background terms. However, we believe that the predictions of the real-to-imaginary ratios are interesting and hope that they will stimulate measurement.

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[19]See Refs. 3 and 15.

[20]In a measurement of $R = \sigma_{el}(s)/\sigma_{total}(s)$, errors in the luminosity need not enter if the measurements are made simultaneously at the one interaction region. In this way, it is hoped that the required accuracy may be achieved.

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