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Sum rules and ultraviolet divergences in a gauge model*

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By examining an induced neutral-current semileptonic process, ν + baryon → ν + baryon, in the SO(3) gauge model with strong interactions, we show with the aid of the scaling assumption that the nonexistence of ultraviolet divergences in the process is equivalent to the validity of sum rules dictated by CCC (continuity equations and commutation relations of the currents) of the model.

I. INTRODUCTION

In previous works on semileptonic decay in gauge models we have confirmed the following two statements.[1,2] (1) The ultraviolet divergences of a radiatively corrected semileptonic process are determined completely by weak and electromagnetic processes of the model alone, regardless of the details of strong interactions among the hadrons. (2) The continuity equations and commutation relations of currents[3] (CCC) in a model are in themselves manifestations of gauge properties of the model. For example, the gauge independence of a semileptonic decay matrix is proved by extensive use of the CCC of the model.[1] Also, the CCC make it possible to combine all of the ultraviolet-divergent contributions due to radiative corrections to the semileptonic process into a certain form that is proportional to the zeroth-order matrix, so that divergence, in sum, can be removed by rescaling coupling parameters of the model, i.e., the model is renormalizable.[1,2]

In this paper we examine an induced neutral-current semileptonic process,

$$\nu + \text{baryon} \to \nu + \text{baryon}, \tag{1}$$

in the SO(3) gauge model with strong interactions[1] [see Figs. 1(a)–1(c)]; there appears no divergence in the S matrix of the process (1) in the model without strong interactions.[4] Careful investigation will lead us to the same conclusion as described above: The CCC are indeed manifestations of the gauge properties of the model and therefore their validity guarantees that we obtain a finite description for semileptonic processes.

The important feature of the model is that it provides the following CCC among baryonic weak and electromagnetic currents[1]:

$$\begin{aligned} &i\partial^\mu J^\pm_\mu(x) = M_W S^\pm(x), \\ &[J_0^+(x), J_\mu^-(y)]\,\delta(x^0-y^0) = -i\delta^3(\vec{x}-\vec{y})J_\mu^{\text{em}}(x), \\ &[J_0^+(x), S^-(y)]\,\delta(x^0-y^0) = \delta^3(\vec{x}-\vec{y})S^0(x), \text{ etc.}, \end{aligned} \tag{2}$$

where $J^\alpha_\mu(x)$ and $S^\alpha(x)$ are the αth components of baryonic vector and scalar currents, respectively.

In order to make our discussion simpler we evaluate the S matrix of the process at zero momentum transfer ($Q=0$ in Fig. 1), and in the 't Hooft-Feynman gauge where the propagators for the vector boson $W^\pm_\mu$ and the scalar meson $\phi^\pm$ are respectively taken as

$$\begin{aligned} \Delta^W_{\mu\nu}(k) &= \frac{-ig_{\mu\nu}}{k^2-M_W{}^2}, \\ \Delta^\phi(k) &= \frac{+i}{k^2-M_W{}^2}. \end{aligned} \tag{3}$$

Contributions from diagrams in Fig. 1(b) are finite after renormalization. In fact they do not contribute at zero momentum transfer as the electric charge of the neutrino vanishes. The diagrams in Fig. 1(c) do not render any contribution at any momentum transfer, because the neutrino is massless. The contribution from the diagrams in Fig. 1(a) where the neutrino undergoes an intermediate state Y^+ is given at $Q=k+k'=0$ by

$$\begin{aligned} M(Y^+) &= +ie^4\sin^2\beta\int\frac{d^4k}{(2\pi)^4}\,(L^{\mu\nu}T_{\mu\nu}+L^\mu T_\mu+\tilde{L}^\nu\tilde{T}_\nu+LT) \\ &\quad\times\frac{1}{(k^2-M_W{}^2)^2[(k+q_i)^2-m_{Y^+}{}^2]}, \end{aligned} \tag{4}$$

$$\begin{aligned} T_{\mu\nu} &= \int d^4x\, e^{-ik(x-y)}\langle P|T(J^+_\mu(x)J^-_\nu(y))|P\rangle, \\ T_\mu &= \int d^4x\, e^{-ik(x-y)}\langle P|T(J^+_\mu(x)S^-(y))|P\rangle, \\ \tilde{T}_\nu &= \int d^4x\, e^{-ik(x-y)}\langle P|T(S^+(x)J^-_\nu(y))|P\rangle, \\ T &= \int d^4x\, e^{-ik(x-y)}\langle P|T(S^+(x)S^-(y))|P\rangle, \end{aligned} \tag{5}$$

$$\begin{aligned} L_{\mu\nu} &= \bar{u}_\nu(q_f)\gamma_R\gamma_\nu(\not{q}_i+\not{k})\gamma_\mu\gamma_L u_\nu(q_i), \\ L_\mu &= \frac{m_{Y^+}{}^2}{M_W}\bar{u}_\nu(q_f)\gamma_\mu\gamma_L u_\nu(q_i) = \tilde{L}_\mu, \\ L &= \left(\frac{m_{Y^+}}{M_W}\right)^2\bar{u}_\nu(q_f)\not{k}\gamma_L u_\nu(q_i). \end{aligned} \tag{6}$$

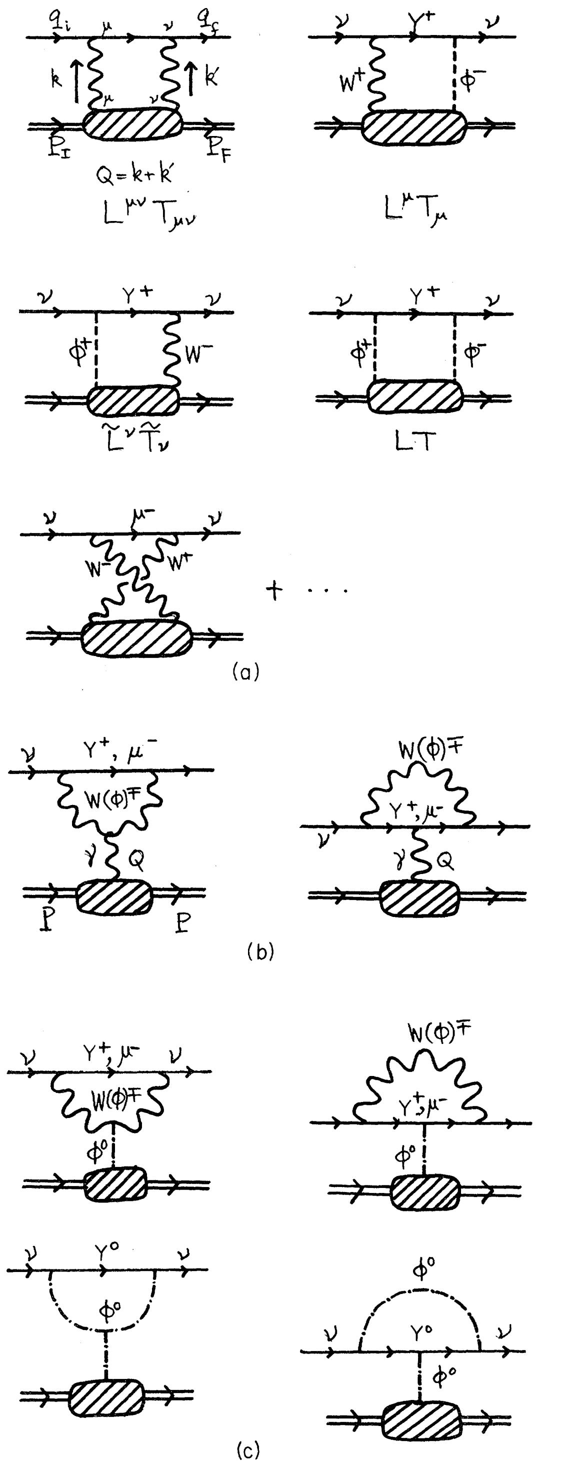

FIG. 1. An induced neutral-current semileptonic process, ν + baryon$\rightarrow\nu$ +baryon. (a) Box diagrams with momentum transfer $Q=k+k'$. The diagrams with leptonic intermediate state μ^- are not shown except a diagram with two-$W^{\pm}$-boson exchange. (b) Diagrams with radiative corrections to the $\bar{\nu}\nu A^{\mathrm{em}}$ vertex. (c) Diagrams with radiative corrections to the $\bar{\nu}\nu\phi^0$ vertex.

There are similar contributions, $M(\mu^-)$, with muon intermediate state instead of Y^+. Since we know that ultraviolet divergences contained in $M(Y^+)$ and $M(\mu^-)$ do not cancel each other out except at the symmetry limit, $m_{Y^+}=m_{\mu^-}$, we can discuss the question of ultraviolet divergence in the process considering only $M(Y^+)$. This reminds us of the cancellations of logarithmic divergences for the same process (1) without strong interactions in the U gauge where we need both leptonic intermediate states μ^- and Y^+ to eliminate the quadratic divergence of the box diagram, but the logarithmic divergence from each intermediate state is offset by another nonbox diagram with the identical intermediate state (see Fig. 2).

It is very important to recognize that the "bridge theorem"[3,5] (or power counting) alone renders $M(Y^+)$ finite in the generalized renormalizable gauge, and in particular in the 't Hooft-Feynman gauge, where all the computations are done throughout this paper. By use of the theorem, the asymptotic behavior of $T_{\mu\nu}$, T_μ, and T for large Euclidean momentum K is at most[3]

$$T_{\mu\nu},\ T_\mu,\ \text{and}\ T\sim O(K^{-1})\ \text{as}\ K\rightarrow\infty, \tag{7}$$

where $k_0=ik_4$, $k^2=-K^2$. From Eqs. (4), (6), and (7), we can already conclude that $M(Y^+)$ is indeed finite.

While the "bridge theorem" is, in general, a powerful guide in examining renormalization and ultraviolet behaviors of the Green's functions,[1,3,5] the gauge model provides us an independent and also powerful means, the CCC (2), to investigate the ultraviolet behaviors. In the following section we will demonstrate that the CCC, governed by the

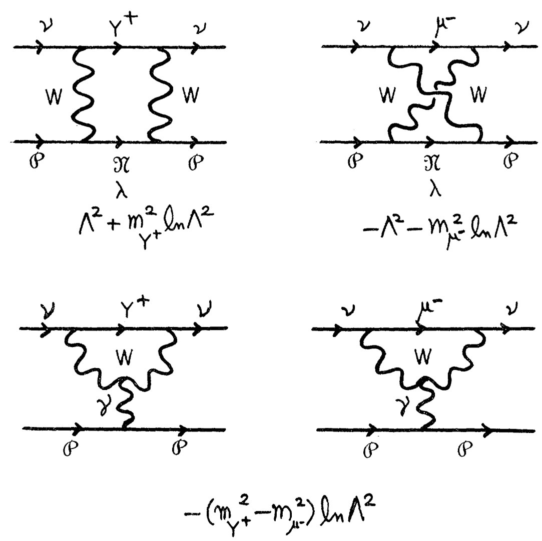

FIG. 2. Ultraviolet divergences in the U gauge of the induced neutral-current process with pointlike baryons.

gauge properties of the gauge model, independently demand that $M(Y^+)$ is indeed finite, as we have known by power-counting arguments. Our demonstration is in the following order: By rewriting $M(Y^+)$ in terms of the structure functions, $F_i(\omega, K^2)$ (definition will be given below), we will observe that the condition that $M(Y^+)$ is finite reduces to the one that $F_i(\omega, K^2)$ should satisfy certain sum rules. Then we will show that these sum rules are dictated by the CCC of the model (2).

II. SUM RULES AND FINITE $M(Y^+)$

In order to investigate the subtle connection between CCC and finiteness of $M(Y^+)$ we had better rewrite $M(Y^+)$ [Eqs. (4)–(6)] by use of the following relations among $T_{\mu\nu}$, T_μ, and T:

$$T_\mu = \frac{1}{M_W}[k^\nu T_{\mu\nu} - \langle P|J_\mu^{\rm em}(0)|P\rangle],$$
$$T = \frac{1}{M_W{}^2}[k^\mu k^\nu T_{\mu\nu} - k^\mu \langle P|J_\mu^{\rm em}(0)|P\rangle - \tfrac{1}{2}\langle P|S^0(0)|P\rangle]. \tag{8}$$

These relations are obtained through the CCC [Eq. (2)]. In turn, $M(Y^+)$ is expressed by

$$M(Y^+) = ie^4\sin^2\beta\,\frac{m_{Y^+}{}^2}{M_W{}^4}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{(k^2-M_W{}^2)^2[(k+q_i)^2-m_{Y^+}{}^2]}\,[k^\mu k^\nu T_{\mu\nu} - k^\nu\langle P|J_\nu^{\rm em}(0)|P\rangle]k_\rho \bar u_\nu(q_f)\gamma_\rho\gamma_L u_\nu(q_i) + \cdots, \tag{9}$$

where we have only identified potentially divergent terms and indicated obviously finite contributions from Eq. (9) by the dots.

The T product $T_{\mu\nu}$ can be decomposed into invariant amplitudes:

$$T_{\mu\nu} = -g_{\mu\nu}T_1 + \frac{p_\mu p_\nu}{m_N{}^2}T_2 - \frac{i\epsilon_{\mu\nu\alpha\beta}}{2m_N{}^2}p_\alpha k_\beta T_3 + \frac{k_\mu k_\nu}{m_N{}^2}T_4 + \frac{k_\mu p_\nu + k_\nu p_\mu}{2m_N{}^2}T_5, \tag{10}$$

where $T_i = T_i(\nu, k^2)$, $\nu = 2p\cdot k$, and m_N is the mass of the baryon. We assume $T_{2,3,5}$ satisfy unsubtracted dispersion relations, whereas $T_{1,4}$ each require one subtraction[6]:

$$T_1(\nu,k^2) = T_1(0,k^2) - \frac{1}{m_N}\int_0^1 d\omega'\left[\frac{F_1^-(\omega',k^2)}{\omega'-\omega} + \frac{F_1^+(\omega',k^2)}{\omega'+\omega}\right],$$
$$T_4(\nu,k^2) = T_4(0,k^2) + \frac{2m_N}{k^2}\int_0^1 d\omega'\omega'\left[\frac{F_4^-(\omega',k^2)}{\omega'-\omega} + \frac{F_4^+(\omega',k^2)}{\omega'+\omega}\right],$$
$$T_i(\nu,k^2) = \frac{2m_N\omega}{k^2}\int_0^1 d\omega'\left[\frac{F_i^-(\omega',k^2)}{\omega'-\omega} - \frac{F_i^+(\omega',k^2)}{\omega'+\omega}\right],\quad \text{for } i=2,3,5 \tag{11}$$

where

$$\omega = -k^2/2\nu,$$
$$F_1^\pm = m_N W_1^\pm$$

and

$$F_i^\pm = \frac{\nu}{m_N}W_i^\pm \quad \text{for } i=2,3,4,5 \text{ (see Ref. 7)}.$$

$W_i^\pm(\nu,k^2)$ are the hadronic structure functions of the neutrino-hadron scattering:

$$W_{\mu\nu}^\pm = \int d^4x\,e^{-ikx}\langle P|[J_\mu^\pm(x), J_\nu^\mp(0)]|P\rangle$$
$$= -g_{\mu\nu}W_1^\pm + \frac{p_\mu p_\nu}{m_N{}^2}W_2^\pm - \frac{i\epsilon_{\mu\nu\alpha\beta}}{2m_N{}^2}p^\alpha k^\beta W_3^\pm + \frac{k_\mu k_\nu}{m_N{}^2}W_4^\pm + \frac{k_\mu p_\nu + k_\nu p_\mu}{2m_N{}^2}W_5^\pm + i\frac{k_\mu p_\nu - k_\nu p_\mu}{2m_N{}^2}W_6^\pm, \tag{12}$$

where

$$W_i^\pm = W_i^\pm(\nu,k^2).$$

By applying a Cottingham rotation,

$$k_0 = ik_4,\quad k^2 = -K^2,\quad z = k_4/K,$$
$$\int\frac{d^4k}{(2\pi)^4} = \frac{1}{(2\pi)^4}\int_{-1}^1 dz(1-z^2)^{1/2}\int_0^\infty K^2dK^2\int d\Omega_{\vec k}, \tag{13}$$

we obtain the following description for $M(Y^+)$:

$$M(Y^+) \propto iG_F\alpha\,\frac{m_{Y^+}{}^2}{M_W{}^2}\bar u_\nu(q_f)\gamma_0\gamma_L u_\nu(q_i) \times\int_{-1}^1 dz\,z^2(1-z^2)^{1/2}\int\frac{K^4dK^2}{(K^2+M_W{}^2)^2(K^2+m_{Y^+}{}^2)} \times\left(\int_0^1\frac{K^2d\omega}{K^2+4m_N{}^2z^2\omega^2}\{[F_1^-(\omega,K^2)-F_1^+(\omega,K^2)] + 2\omega[F_4^-(\omega,K^2)-F_4^+(\omega,K^2)] - [F_5^-(\omega,K^2)+F_5^+(\omega,K^2)]\} - \tfrac{1}{2}\langle P|I_3|P\rangle\right) + \text{finite corrections}, \tag{14}$$

where

$$\frac{G_F}{\sqrt{2}} = \frac{e^2 \sin^2\beta}{4M_W{}^2},$$

$$\alpha = \frac{e^2}{4\pi},$$

and $\langle P|I_3|P\rangle$ is the electric charge of the baryon in the SO(3) gauge model,

$$\langle P|I_3|P\rangle = \int d^3x \langle P|J_\nu^{\text{em}}(0)|P\rangle\delta_{\nu 0}. \tag{15}$$

In the process of obtaining Eq. (14) from Eq. (9) with the aid of Eqs. (10)–(13) we have eliminated integrands which are odd with respect to z.

In order to identify divergent contributions, we furthermore assume the scaling law for $F_i^\pm(\omega, K^2)$:

$$F_i^\pm(\omega, K^2) \to F_i^\pm(\omega) \quad \text{as } K^2 \to \infty \text{ with } \omega \text{ fixed}. \tag{16}$$

Then the expression for $M(Y^+)$, Eq. (14), can be simplified as

$$\begin{aligned} M(Y^+) \propto\, & iG_F\,\alpha\,\frac{m_{Y^+}{}^2}{M_W{}^2}\ln\Lambda^2 \bar{u}_\nu(q_f)\gamma_0\gamma_L u_\nu(q_i) \\ & \times\left\{\int_0^1 d\omega[F_1^-(\omega) - F_1^+(\omega)] - \tfrac{1}{2}\langle P|I_3|P\rangle + 2\int_0^1 d\omega\,\omega[F_4^-(\omega) - F_4^+(\omega)] + \int_0^1 d\omega[F_5^+(\omega) + F_5^-(\omega)]\right\} \\ & + \text{finite contributions}, \end{aligned} \tag{17}$$

where $\ln\Lambda^2$ indicates that $M(Y^+)$ diverges logarithmically as

$$\ln\Lambda^2 = \int^\infty K^{-2}dK^2. \tag{18}$$

However, we can now prove that $M(Y^+)$ converges despite the presence of $\ln\Lambda^2$, recalling that there hold sum rules in the model satisfied by the structure functions $F^\pm_{1,4,5}(\omega)$. Taking a Bjorken-Johnson-Low (BJL) limit of $T_{0\mu}$ ($\mu = 0, 1, 2, 3$) and using the CCC of the model [Eq. (2)] we acquire the following sum rules for $F^\pm_{1,4,5}(\omega)$:

$$\begin{aligned} &\int_0^1 d\omega[F_1^-(\omega) - F_1^+(\omega)] = \tfrac{1}{2}\langle P|I_3|P\rangle, \\ &2\int_0^1 d\omega\,\omega[F_4^-(\omega) - F_4^+(\omega)] + \int_0^1 d\omega[F_5^+(\omega) + F_5^-(\omega)] = 0. \end{aligned} \tag{19}$$

This is our proof. Because of the sum rules of the model, Eqs. (19), $M(Y^+)$ has no ultraviolet divergence since the term proportional to $\ln\Lambda^2$ in Eq. (17) vanishes identically. $M(\mu^-)$ is also convergent by the same arguments.

We have observed that the S matrix for the semileptonic process (1) is finite, regardless of whether strong interactions among the hadrons are taken into consideration or not. If we treat baryons as being pointlike, the finiteness is obvious. In the presence of strong interactions, it is quite interesting to realize that the sum rules dictated by the CCC of the model force the S matrix to be convergent. The significance behind the connection between the nonexistence of ultraviolet divergences and the sum rules is that the CCC is in fact a manifestation of the gauge properties which make the model to be renormalizable (Fig. 3).

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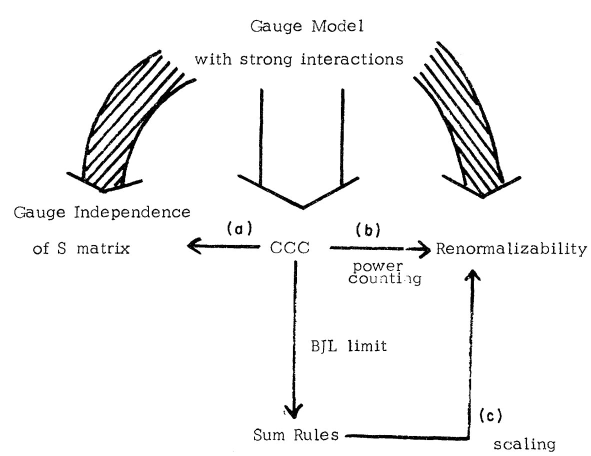

FIG. 3. Significance of CCC in a gauge model with strong interactions. The relation (a) was shown in Refs. 1 and 3, and (b) in Refs. 1, 2, and 3. In this paper we observe the relation (c).

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$$\frac{\nu^2}{m_N{}^2} W_i^{\pm}(\nu, K^2) = F_i^{\pm}(\omega, K^2) \to F_i^{\pm}(\omega) \text{ as } K^2 \to \infty$$

for $i=4,5$. Because of the nonconservation of the axial-vector current, W_4 and W_5 fall faster than W_2 and W_3 in the Bjorken limit. Then there is no divergent contribution from either F_4 or F_5 in Eqs. (14) and (17). However, we have assumed in our paper that W_4 and W_5 fall no faster than W_2 and W_3. With this assumption we could show that the S-matrix element of the process (1) is finite, regardless of the scaling behavior of the structure functions.