

Formulation of Abelian gauge theories without regulators. II

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A new method of renormalizing Abelian gauge theories without regulators is developed in this series of papers. In the present part infrared problems are avoided by modifying the Lagrangian such that all masses are nonvanishing. It is shown that conventional renormalization of the classical Lagrangian leads to anomalies of the current. A modified renormalization scheme is proposed and shown to imply the classical form of partial current conservation and the desired Ward identities. Differential vertex operations are studied which will be used in paper III of this series for carrying out the required zero-mass limits.

I. INTRODUCTION

In the first paper of this series a new approach was proposed for renormalizing gauge theories with spontaneous or explicit symmetry breaking.¹ The models considered were based on the classical Lagrangian

$$\begin{aligned} \mathcal{L}_{cl} = & z_2 (D_\mu \varphi)^* D^\mu \varphi - \mu_0^2 \varphi^* \varphi - z_1 g^2 (\varphi^* \varphi)^2 \\ & - \frac{1}{4} z_3 F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_0^2 A_\mu A^\mu \\ & - \frac{1}{2\alpha} (\partial_\mu A^\mu)^2 + \frac{\delta}{\sqrt{2}} (\varphi + \varphi^*), \end{aligned} \quad (1.1)$$

$$D_\mu = \partial_\mu - ie A_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

$v/\sqrt{2}$ denotes the vacuum expectation value $\langle \phi \rangle$ of the field ϕ . Hermitian fields ψ and χ with vanishing expectation values are introduced by

$$\varphi = \frac{1}{\sqrt{2}} (\psi + v + i\chi), \quad (1.2)$$

$$\langle \psi \rangle = \langle \chi \rangle = 0. \quad (1.3)$$

z_2 and z_3 are normalization factors of the fields.

The nonlinear part of (1.1) is invariant under gauge transformations

$$\varphi \rightarrow e^{ie\Lambda} \varphi, \quad A_\mu \rightarrow A_\mu + \partial_\mu \Lambda, \quad (\square + \alpha m_0^2) \Lambda = 0, \quad (1.4)$$

up to a total divergence. It represents the minimal gauge-invariant coupling of a neutral vector-meson field to a complex scalar field with quartic self-interaction. The gauge invariance is explicitly broken by the term proportional to $\phi + \phi^*$ and remains spontaneously broken in the limit $\delta \rightarrow 0$, provided $v \neq 0$. δ is determined by (1.3) for other given parameters of the Lagrangian. In particular, $v=0$ implies $\delta=0$. Thus v measures the strength of the symmetry breaking, whether spontaneous or explicit.

According to the discussion of paper I the models

described by the Lagrangian (1.1) may be classified as follows.

(A) Goldstone-type models, $e=0$, $g \neq 0$.

(1) Symmetric case of complex scalar field with quartic self-interaction, $v=\delta=0$.

(2) Goldstone model, $v \neq 0$, $\delta=0$.

(3) Explicitly broken Goldstone model, $v \neq 0$, $\delta \neq 0$.

(B) Higgs-type models, $e \neq 0$, $g \neq 0$.

(1) Electrodynamics of model (A1), $v=\delta=m_0=0$.

(2) Vector-meson dynamics of model (A1), $v=\delta=0$, $m_0 \neq 0$.

(3) Higgs model, $v \neq 0$, $\delta=0$, $m_0=0$.

(4) Pre-Higgs model, $v \neq 0$, $\delta=0$, $m_0 \neq 0$.

(5) Explicitly broken pre-Higgs model, $v \neq 0$, $\delta \neq 0$, $m_0 \neq 0$.

All models listed are meaningful in perturbation theory except for the explicitly broken pre-Higgs model where the S matrix is not unitary.

In this paper the new renormalization method will be presented in detail for the explicitly broken models. These models do not involve zero-mass particles. The formulation of the spontaneously broken models, which require an investigation of the infrared behavior, will be obtained in paper III by taking the appropriate limits $\delta \rightarrow 0$ and $m_0 \rightarrow 0$.^{2,3}

Two alternative methods of renormalizing gauge theories with symmetry breaking have been developed by Lee and Symanzik.^{4,5} In Symanzik's approach conventional renormalization techniques are applied to a Lagrangian which is not manifestly gauge-invariant, but with the coefficients correlated such that the desired Ward identities hold. Without using a regularization this program has been carried out by Becchi, Rouet, and Stora for the Higgs model in the 't Hooft gauge and by Piguet

for the pre-Higgs model in the Stueckelberg gauge.⁶⁻⁸

Lee's method is based on a gauge-invariant Lagrangian like (1.1). The system is first regularized and quantized in a gauge-invariant manner. It is then shown that the regularization can be removed for the renormalized Feynman amplitudes. In the treatment that follows we use Lee's method of gauge-invariant quantization but without introducing a regularization. Instead we will deal directly with the unregularized, but properly renormalized Feynman amplitudes in momentum space. To this end we modify the renormalization scheme by including subtractions with respect to a subtraction parameter s .⁹ The connection to Symanzik's approach will be discussed in Sec. VI using an equivalence theorem of the type first considered by Rouet.¹⁰

Though the symmetry may be badly broken in the mass spectrum, the theory retains gauge-invariant features to a remarkable extent. The most important examples are the Ward identities. As was discussed in paper I these identities are responsible for the unitarity of the S matrix in the Higgs and the pre-Higgs model. Other examples are some gauge-invariant linear relations among differential vertex operations which, for instance, imply a gauge-invariant form of the Callan-Symanzik equation. We will emphasize this aspect of the theory by using a manifestly gauge-invariant notation whenever possible.

Section II summarizes the free-field theory of the explicitly broken pre-Higgs model. The normal-product formalism in its conventional form is reviewed in Sec. III and applied to the Higgs-type models. It is seen that anomalies occur in the current conservation law which destroy the unitarity of the S matrix for the pre-Higgs and the Higgs model. This fact calls for a modified subtraction procedure which preserves the current conservation laws of the classical theory. The new method is briefly explained for the explicitly broken Goldstone model (Sec. IV) and then applied to the explicitly broken pre-Higgs model (Sec. V). The theory of differential vertex operations is developed in Sec. VI. Finally an equivalence theorem is proved which provides the connection to Symanzik's approach.

II. FREE-FIELD THEORY OF THE EXPLICITLY BROKEN PRE-HIGGS MODEL

The free Lagrangian of the explicitly broken pre-Higgs model is given by

$$\begin{aligned} \mathcal{L}_{\text{cl}0} = & \frac{1}{2}(\partial_\mu \psi \partial^\mu \psi + \partial_\mu \chi \partial^\mu \chi) - \frac{1}{2}M^2\psi^2 - \frac{1}{2}\mu^2\chi^2 \\ & - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2A_\mu A^\mu - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 - wA_\mu \partial^\mu \chi, \end{aligned} \quad (2.1)$$

with

$$w = ev, \quad m^2 = m_0^2 + w^2. \quad (2.2)$$

We briefly state the properties of the free fields $A_\mu^{(0)}$, $\psi^{(0)}$, $\chi^{(0)}$ as described by (2.1). $\psi^{(0)}$ is a free neutral scalar field of mass M which commutes with $A_\mu^{(0)}$ and $\chi^{(0)}$. The particles associated with $\psi^{(0)}$ are called σ mesons. The fields $A_\mu^{(0)}$ and $\chi^{(0)}$ do not commute with each other, but may be written as linear combinations of commuting fields V_μ , ρ_+ , and ρ_- ,¹¹

$$\begin{aligned} A^{(0)\mu} = & V^\mu - \frac{u}{(1-u^2)^{1/2}} \frac{\sqrt{\alpha}}{\kappa} \partial^\mu \rho_+ \\ & - \frac{1}{(1-u^2)^{1/2}} \frac{\sqrt{\alpha}}{\lambda} \partial^\mu \rho_-, \\ \chi^{(0)} = & \frac{1}{(1-u^2)^{1/2}} \frac{\kappa}{\mu} \rho_+ + \frac{u}{(1-u^2)^{1/2}} \frac{\lambda}{\mu} \rho_-. \end{aligned} \quad (2.3)$$

V^μ is a free neutral vector-meson field of mass m (in the Proca gauge). ρ_+ is a free neutral scalar field describing particles of mass κ which are called π mesons. ρ_- is a free field of neutral scalar ghosts of mass λ . The masses κ , λ , and the parameter u are given by

$$\begin{aligned} \kappa^2 = & \frac{1}{2}(\alpha m_0^2 + \mu^2) - \frac{1}{2}[(\alpha m_0^2 + \mu^2)^2 - 4\alpha\mu^2 m^2]^{1/2}, \\ \lambda^2 = & \frac{1}{2}(\alpha m_0^2 + \mu^2) + \frac{1}{2}[(\alpha m_0^2 + \mu^2)^2 - 4\alpha\mu^2 m^2]^{1/2}, \\ u = & \frac{1}{2\mu w \sqrt{\alpha}} \{ \mu^2 - \alpha m_0^2 + [(\alpha m_0^2 + \mu^2)^2 - 4\alpha\mu^2 m^2]^{1/2} \}. \end{aligned} \quad (2.4)$$

The model is only meaningful if the mass squares κ^2 , λ^2 are real and non-negative. The condition for this is

$$(\alpha m_0^2 - \mu^2)^2 \geq 4\alpha\mu^2 w^2. \quad (2.5)$$

In the limit $\mu \rightarrow 0$ the mass squares approach the values

$$\begin{aligned} m^2 = & m_0^2 + w^2 \quad (\text{vector meson}), \\ \kappa^2 = & 0 \quad (\text{Goldstone particle } \pi), \\ \lambda^2 = & \alpha m_0^2 \quad (\text{ghost particle}), \\ M^2 = & \quad (\sigma \text{ particle}). \end{aligned}$$

The propagators of Lagrangian (2.1) were given explicitly in paper I, Eq. (30).

III. FAILURE OF CONVENTIONAL QUANTIZATION OF THE PRE-HIGGS MODEL

We consider the explicitly broken pre-Higgs model. As usual an indefinite-metric formulation is employed in order to quantize the Lagrangian (1.1). In general the S matrix will not be unitary since the ghost particles of negative probabilities participate in the interaction. No physical interpretation of the model is possible then. In the

limit $\delta \rightarrow 0$, however, the ghost particles are expected to decouple from the rest of the system. The argument proceeds as in electrodynamics. The ghost particles are described by the divergence $\partial_\mu A^\mu$ of the vector potential. In the classical theory the Lagrangian (1.1) implies the field equation

$$-\partial_\nu F^{\mu\nu} + \frac{1}{\alpha} \partial^\mu \partial_\nu A^\nu + m_0^2 A^\mu = -j_{\text{cl}}^\mu \quad (3.1)$$

of the vector-meson field with

$$j_{\text{cl}}^\mu = iz_2 e [\varphi^* D^\mu \varphi - \varphi (D^\mu \varphi)^*] - (z_3 - 1) \partial_\nu F^{\mu\nu}. \quad (3.2)$$

The current j_{cl}^μ is partially conserved,

$$\partial_\mu j_{\text{cl}}^\mu = i \frac{\delta e}{\sqrt{2}} (\varphi^* - \varphi) = \delta e \chi, \quad (3.3)$$

as a consequence of the classical field equation of ϕ . (3.1) and (3.3) yield

$$-(\square + \alpha m_0^2) \partial_\mu A^\mu = \alpha \delta e \chi \quad (3.4)$$

as field equation of $\partial_\mu A^\mu$. In the limit $\delta \rightarrow 0$ the divergence $\partial_\mu A^\mu$ becomes a free field:

$$(\square + \alpha m_0^2) \partial_\mu A^\mu = 0. \quad (3.5)$$

If one can establish that in the quantized theory the divergence $\partial_\mu A^\mu$ is also a free field the ghost particles do not interact and the S matrix will be unitary.

Due to possible anomalies it is not easy to satisfy (3.3) and (3.4) in the quantized theory. In this case the S matrix cannot be expected to be unitary in the limit $\delta \rightarrow 0$. For an acceptable formulation of the model we therefore require that the current be partially conserved and represent the source of the vector-meson field. It will be seen in this section that conventional quantization fails to meet these requirements. This result will motivate the modified quantization procedure to be proposed in the following sections.

To simplify the notation we combine A_μ , ψ , and χ by a six-component vector,

$$A_0, A_1, A_2, A_3, A_4 = \psi, A_5 = \chi. \quad (3.6)$$

For raising and lowering indices we use the metric tensor

$$g_{00} = g_{44} = g_{55} = 1, \quad g_{11} = g_{22} = g_{33} = -1, \quad (3.7)$$

$$g_{jk} = 0 \text{ if } j \neq k.$$

Latin indices run from 0 to 5, while Greek indices run from 0 to 3.

Let $O_i(x)$ denote monomials of dimension d_i in the fields and their derivatives at the point x . Time-ordered functions involving fields and nor-

mal products are constructed by the renormalized Gell-Mann-Low expansions

$$\begin{aligned} & \left\langle T \prod_i N_{a_i} [O_i(x_i)] \right\rangle \\ &= \left\langle T \exp \left\{ i \int N_4 [\mathcal{L}_{\text{cl}}^{(0)}(z)] dz \right\} \prod_i N_{a_i} [O_i^{(0)}(x_i)] \right\rangle^{\text{norm}}. \end{aligned} \quad (3.8)$$

Some or all of the O_i may be field components or linear combinations thereof, in which case the symbol N has no effect. For the nonlinear monomials we take $a_i \geq d_i$. The superscript norm indicates that vacuum diagrams (disconnected closed loops) should be omitted. The interaction Lagrangian is given by

$$\mathcal{L}_{\text{cl}I} = \mathcal{L}_{\text{cl}} - \mathcal{L}_{\text{cl}0}, \quad (3.9)$$

with the free Lagrangian (2.1). The superscript (0) indicates that free fields pertaining to $\mathcal{L}_{\text{cl}0}$ should be inserted.

The right-hand side of (3.8) only involves free-field expressions and is defined in the following way. The expansion of the exponential leads to Green functions

$$\langle T N_{\delta_1} [Q_1^{(0)}(u_1)] \cdots N_{\delta_m} [Q_m^{(0)}(u_m)] A_{r_1}^{(0)}(v_1) \cdots A_{r_n}^{(0)}(v_n) \rangle, \quad (3.10)$$

where the Q_k are nonlinear and $\delta_j \geq d_j$. (3.10) represents a time-ordered Green function of the Wick products

$$: Q_k^{(0)}(u_k) :, \quad k = 1, \dots, m \quad (3.11)$$

and free-field components. While the Wightman functions involving Wick products (3.11) are unique, the time-ordered functions are only defined up to contact terms. The symbols $N_{\delta_1} \cdots N_{\delta_m}$ serve to specify a unique choice of time-ordered functions, yet to be defined. As has been emphasized by Bogoliubov the renormalization of the Gell-Mann-Low formula rests upon a proper definition of the time-ordered functions of free fields and their Wick products. For defining (3.10) one expands the formal expression with respect to Feynman diagrams. To each diagram the renormalized contribution is constructed. The number of subtractions used is specified by the degree

$$\delta(\gamma) = 4 - N_\gamma + \sum_j (\delta_j - 4), \quad (3.12)$$

which is assigned to each proper subdiagram γ of Γ . N is the number of external lines.

For formulating the equations of motion we introduce the effective Lagrangian

$$\begin{aligned}
\mathcal{L}_{\text{eff}} &= \mathcal{L}_{\text{eff}0} + \mathcal{L}_{\text{eff}I}, \\
\mathcal{L}_{\text{eff}0} &= N_4[\mathcal{L}_{\text{cl}0}], \\
\mathcal{L}_{\text{eff}I} &= N_4[\mathcal{L}_{\text{cl}I}].
\end{aligned} \tag{3.13}$$

The equations of motion may be derived from the following renormalized versions of Wick's theorem:

$$\left\langle T \frac{\hat{\partial} \mathcal{L}_{\text{eff}0}^{(0)}}{\partial A_r^{(0)}}(x) X \right\rangle = i \left\langle T \frac{\delta X}{\delta A_r^{(0)}(x)} \right\rangle, \tag{3.14}$$

$$\left\langle T A_s^{(0)}(x) \frac{\hat{\partial} \mathcal{L}_{\text{eff}0}^{(0)}}{\partial A_r^{(0)}}(x) X \right\rangle = i \left\langle T A_s^{(0)}(x) \frac{\delta X^{(0)}}{\delta A_r^{(0)}(x)} \right\rangle, \tag{3.15}$$

with the functional derivative $\delta/\delta\phi(x)$ and $\hat{\partial}/\hat{\partial}\phi$ denoting the Euler derivative

$$\frac{\hat{\partial}}{\hat{\partial}\phi} = \frac{\partial}{\partial\phi} - \partial_\nu \frac{\partial}{\partial(\partial_\nu\phi)}. \tag{3.16}$$

Moreover, the shorthand notations

$$\phi(x) N_a[M(y)] = N_{a+1}[\phi(x) M(y)], \tag{3.17}$$

$$\frac{\delta N_a[M(y)]}{\delta\phi(x)} = N_{a-1} \left[\frac{\delta M(y)}{\delta\phi(x)} \right],$$

$$\frac{\partial}{\partial\phi} N_a[M] = N_{a-1} \left[\frac{\partial M}{\partial\phi} \right], \tag{3.18}$$

$$\frac{\partial}{\partial(\partial_\nu\phi)} N_a[M] = N_{a-2} \left[\frac{\partial M}{\partial(\partial_\nu\phi)} \right]$$

were used in (3.14) and (3.15). Applying the identities (3.14) and (3.15) to the expansions (3.8) of

$$\left\langle T \frac{\hat{\partial} \mathcal{L}_{\text{eff}0}}{\partial A_r}(x) X \right\rangle$$

and

$$\left\langle T A_s(x) \frac{\hat{\partial} \mathcal{L}_{\text{eff}0}}{\partial A_r}(x) X \right\rangle$$

we obtain

$$\left\langle T \frac{\hat{\partial} \mathcal{L}_{\text{eff}}}{\partial A_r}(x) X \right\rangle = i \left\langle T \frac{\delta X}{\delta A_r(x)} \right\rangle, \tag{3.19}$$

$$\left\langle T A_s(x) \frac{\hat{\partial} \mathcal{L}_{\text{eff}}}{\partial A_r}(x) X \right\rangle = i \left\langle T A_s(x) \frac{\delta X}{\delta A_r(x)} \right\rangle, \tag{3.20}$$

or, in operator form,

$$\frac{\hat{\partial} \mathcal{L}_{\text{eff}}}{\partial A_r} = 0, \quad A_s \frac{\hat{\partial} \mathcal{L}_{\text{eff}}}{\partial A_r} = 0. \tag{3.21}$$

These relations represent renormalized forms of the equations of motion. For the field operators ϕ and A_μ the equations

$$\phi \frac{\hat{\partial} \mathcal{L}_{\text{eff}}}{\partial\phi} = 0, \quad \phi^* \frac{\hat{\partial} \mathcal{L}_{\text{eff}}}{\partial\phi^*} = 0, \quad \frac{\hat{\partial} \mathcal{L}_{\text{eff}}}{\partial A_\mu} = 0 \tag{3.22}$$

follow. Explicitly the field equation of the potential becomes

$$-\partial_\nu F^{\mu\nu} + \frac{1}{\alpha} \partial^\mu \partial_\nu A^\nu + m_0^2 A^\mu = -j^\mu, \quad j^\mu = N_3[j_{\text{cl}}^\mu] \tag{3.23}$$

with the classical current given by (3.2). According to our requirements the current—defined as the source of the vector field—should be partially conserved. One finds

$$\begin{aligned}
\partial_\mu j^\mu &= \partial_\mu N_3[j_{\text{cl}}^\mu] \\
&= N_4[\partial_\mu j_{\text{cl}}^\mu] \\
&= ie N_4[\phi^* D^\mu D_\mu \phi - \phi D^\mu * D_\mu^* \phi^*] \\
&= ie N_4 \left[\phi \frac{\hat{\partial} \mathcal{L}_{\text{cl}}}{\partial\phi} - \phi^* \frac{\hat{\partial} \mathcal{L}_{\text{cl}}}{\partial\phi^*} \right] + i \frac{\delta e}{\sqrt{2}} (\phi^* - \phi).
\end{aligned} \tag{3.24}$$

The second term in the last line is of the desired form, but the first term fails to vanish despite the field equations (3.22). Instead we get

$$\begin{aligned}
N_4 \left[\phi \frac{\hat{\partial} \mathcal{L}_{\text{cl}}}{\partial\phi} \right] &= \phi \frac{\hat{\partial} \mathcal{L}_{\text{eff}}}{\partial\phi} + \frac{v}{2} \left(N_4 \left[\frac{\partial \mathcal{L}_{\text{cl}}}{\partial\phi} \right] - N_3 \left[\frac{\partial \mathcal{L}_{\text{cl}}}{\partial\phi} \right] \right) \\
&= \frac{v}{\sqrt{2}} \left(N_4 \left[\frac{\partial \mathcal{L}_{\text{cl}}}{\partial\phi} \right] - N_3 \left[\frac{\partial \mathcal{L}_{\text{cl}}}{\partial\phi} \right] \right).
\end{aligned} \tag{3.25}$$

Substituting this and the Hermitian-conjugate expression into (3.24) we find an anomaly for the divergence of the current which destroys current conservation in the limit $\delta \rightarrow 0$.

The quantization prescription (3.8) is not unique. The interaction Lagrangian contains various terms of dimension less than four. In (3.8) the degree four was assigned to all vertices, thus causing oversubtractions for many diagrams. However, assigning minimal degree to each term of the interaction Lagrangian would not resolve the difficulty. Experimenting with various possibilities, we found the only remedy to be inclusion of terms of the type

$$N_4[\psi^2] - N_2[\psi^2], \quad N_4[\chi^2] - N_2[\chi^2], \tag{3.26}$$

with coefficients which do not vanish in zeroth order in e . With suitable degree assignments for the other terms it is indeed possible to meet all requirements. Since the coefficients of (3.26) do not vanish in zero order another problem appears, however. In every order of e one would have to sum an infinite number of Feynman integrals. In momentum space the sums involve geometric series which have to be integrated over outside their domain of convergence. This is certainly not satisfactory. In Sec. V a modified procedure will be proposed which specifies the subtractions in closed form for every order of e .

An alternative method of renormalizing models with spontaneously broken symmetry is known

from Symanzik's work on the σ model, which has been extended to the Higgs model by Lee.^{5,12} In this approach one abandons the formal gauge symmetry and includes all possible interaction terms of dimension less or equal to four permitted by the true symmetries of the theory. The coefficients of the interaction terms have to be correlated in such a way that the current is partially conserved. In Sec. VI such a formulation will be given and shown to be equivalent to the manifestly gauge-invariant treatment.

IV. GAUGE INVARIANCE QUANTIZATION OF THE EXPLICITLY BROKEN GOLDSTONE MODEL

We first explain the modified subtraction procedure for the simpler case of the Goldstone-type models with the classical Lagrangian

$$\mathcal{L}_{cl} = z_2 \partial_\mu \varphi^* \partial^\mu \varphi - \mu_0^2 \varphi^* \varphi - z_1 g^2 (\varphi^* \varphi)^2 + \frac{\delta}{\sqrt{2}} (\varphi + \varphi^*) . \quad (4.1)$$

We introduce a dependence on a new parameter s by setting

$$\mu_0^2 = \eta^2 - s^2 \omega^2, \quad \delta = \delta(s), \quad (4.2)$$

$$\varphi = \frac{1}{\sqrt{2}} (\psi + s v + i \chi), \quad (4.3)$$

with

$$\langle \psi \rangle = \langle \chi \rangle = 0 \text{ at } s=1. \quad (4.4)$$

δ is a polynomial in s with $\delta(0)=0$, of which only the value $\delta(1)$ at $s=1$ is relevant. Though eventually s will be set equal to one, we need the theory in the full interval $0 \leq s \leq 1$ for formulating the subtraction procedure. The parameter η is restricted to a permissible range, but is otherwise arbitrary. It can be proved that the Green functions of the model do not depend on the value of η . Apart from s and η the independent parameters of the model will be the coupling constant g and the masses M , μ associated with the fields ψ or χ , respectively. ω , z_1 , and z_2 are chosen independent of s . Suitable renormalization conditions at $s=1$ determine $\delta(1)$, ω , z_1 , and z_2 as power series in g with coefficients depending on M , μ , and η , but independent of s .

The free Lagrangian $\mathcal{L}_{cl0}$ is defined by the bilinear part of the full Lagrangian (4.1), with the coefficients replaced by their zero-order values,

i.e.,

$$\mathcal{L}_{cl0} = \frac{1}{2} \partial_\mu \psi \partial^\mu \psi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} M(s)^2 \psi^2 - \frac{1}{2} \mu(s)^2 \chi^2, \quad (4.5)$$

with

$$M(s)^2 = \lim_{g \rightarrow 0} (\eta^2 - s^2 \omega^2 + 3g^2 s^2 v^2), \quad (4.6)$$

$$\mu(s)^2 = \lim_{g \rightarrow 0} (\eta^2 - s^2 \omega^2 + g^2 s^2 v^2). \quad (4.7)$$

Since η , ω , g , v are independent of s we may express $M(s)$, $\mu(s)$ by their values M , μ at $s=1$:

$$M(s)^2 = \eta_{(0)}^2 + s^2 (M^2 - \eta_{(0)}^2) \\ \mu(s)^2 = \eta_{(0)}^2 + s^2 (\mu^2 - \eta_{(0)}^2). \quad (4.8)$$

$\eta_{(0)}$ denotes the zero-order value of η . Since we need the theory in the range $0 \leq s \leq 1$ we impose the consistency condition

$$M(s)^2 \geq 0, \quad \mu(s)^2 \geq 0. \quad (4.9)$$

As long as $M > 0$, $\mu > 0$ the model is not expected to suffer from infrared problems. One should therefore avoid vanishing mass values in the range $0 \leq s \leq 1$ by imposing the stronger condition

$$M(s)^2 > 0, \quad \mu(s)^2 > 0 \text{ if } M > 0, \mu > 0. \quad (4.10)$$

This restricts the permissible values of η by

$$\eta_{(0)}^2 > 0. \quad (4.11)$$

A possible choice of $\eta_{(0)}$ consistent with this is

$$\eta_{(0)} = \mu. \quad (4.12)$$

With this the s dependence (4.8) becomes

$$M(s)^2 = \mu^2 + s^2 (M^2 - \mu^2), \quad \mu(s)^2 = \mu^2. \quad (4.13)$$

Let $O_i(x)$ denote monomials of the field components

$$\xi_1 = \psi, \quad \xi_2 = \chi \quad (4.14)$$

and their derivatives at the point x . The dimension of O_i is denoted by d_i . We will now set up a perturbative expansion for the time-ordered functions

$$\left\langle T \prod_i N_{a_i} [O_i(x_i)] \right\rangle \quad (4.15)$$

of normal products $N_{a_i} [O_i(x_i)]$ with $a_i \geq d_i$. Some of the O_i may be linear, in which case the normal-product symbol has no effect. The time-ordered functions (4.15) are constructed by

$$\left\langle T \prod_i N_{a_i} [O_i(x_i)] \right\rangle = \left\langle T_s \exp \left\{ i \int N_4 [\mathcal{L}_{cl}^{(0)}(z)] dz \right\} \prod_i N_{a_i} [O_i^{(0)}(x_i)] \right\rangle^{\text{norm}}, \quad \mathcal{L}_{clI} = \mathcal{L}_{cl} - \mathcal{L}_{cl0}. \quad (4.16)$$

The symbol T_s indicates a special time ordering which we are going to define now. The expansion of (4.16) leads to expressions of the form

$$\langle T_s \zeta_{\alpha_1}^{(0)}(y_1) \cdots \zeta_{\alpha_m}^{(0)}(y_m) N_{\rho_1} [s^{\lambda_1} M_1^{(0)}(u_1)] \cdots N_{\rho_r} [s^{\lambda_r} M_r^{(0)}(u_r)] \rangle, \quad (4.17)$$

where we have distinguished the single field components from the nonlinear monomials M_j . These time-ordered functions of free fields and their products are defined by making subtractions not only for the external momenta, but also with respect to the parameter s .¹³ For each diagram contributing to (4.17) the unrenormalized Feynman integral is written in the usual way, but with the s -dependent masses $M(s)$, $\mu(s)$ used in the propagators. In the case of a primitive divergent diagram (with external lines amputated) the integrand R_Γ of the corresponding renormalized integral

$$\int dk_1 \cdots dk_n R_\Gamma(p_1 \cdots p_m, k_1 \cdots k_n) \quad (4.18)$$

is obtained from the unrenormalized integrand I_Γ by

$$R_\Gamma = (1 - t_{ps}^{d(\Gamma)}) I_\Gamma. \quad (4.19)$$

The degree $d(\Gamma)$ of Γ is given by (3.12). $t_{ps}^{d(\Gamma)}$ is the operation of taking the Taylor series to order $d(\Gamma)$ in p_j and s . In general, R_Γ is given by the forest formula

$$R_\Gamma = S_\Gamma \sum_U \sum_{\gamma \in U} (-t_{p_\gamma s_\gamma}^{d(\gamma)} S_\gamma) I_\Gamma(U), \quad (4.20)$$

where the sum is over all forests (families of non-trivial nonoverlapping, proper subdiagrams of Γ). $I_\Gamma(U)$ is the unsubtracted integrand of the Feynman integral in which for each line of $\gamma \in U$, not belonging to a subdiagram $\gamma' \in U$ of γ , the external momenta and the variable s are inserted as dummy variables $p_{j\gamma}$ and s_γ , respectively. S_γ is the substitution operator which transforms from variables $p_{j\gamma}, s_\gamma$ of a subdiagram γ' of γ to the variables appropriate to γ .

It can be shown that the renormalized integrals (4.18) constructed according to these rules are absolutely convergent for $\epsilon > 0$ and approach tempered distributions in the limit $\epsilon \rightarrow 0$.¹⁴

For s -dependent normal products we will use the convention

$$N_a[P(s, z)] = \sum_{j=0}^r s^j N_{a-j}[P_j(z)] \quad (4.21)$$

if

$$P(s, z) = \sum_{j=0}^r s^j P_j(z). \quad (4.22)$$

No change of notation is made for s -independent normal products. (4.21) satisfies the general rules

$$N_a[\alpha P] = \alpha N_a[P] \quad (\alpha \text{ s independent}), \quad (4.23)$$

$$N_a[P_1 + P_2] = N_a[P_1] + N_a[P_2], \quad (4.24)$$

$$\partial_\mu N_a[P] = N_{a+1}[\partial_\mu P], \quad (4.25)$$

$$s N_a[P] = N_{a+1}[sP]. \quad (4.26)$$

We will also use the shorthand notations (3.17) for the fields ψ and χ . With (4.23)–(4.26) similar relations follow for ϕ and ϕ^* , in particular

$$\begin{aligned} \varphi \frac{\delta N_a[P]}{\delta \varphi} &= N_a \left[\varphi \frac{\delta P}{\delta \varphi} \right], \\ \varphi^* \frac{\delta N_a[P]}{\delta \varphi^*} &= N_a \left[\varphi^* \frac{\delta P}{\delta \varphi^*} \right]. \end{aligned} \quad (4.27)$$

The equations of motion following from (4.16) will be found to be associated with the effective Lagrangian

$$\mathcal{L}_{\text{eff}}(s, z) = N_4[\mathcal{L}_{\text{cl}}(s, z)]. \quad (4.28)$$

The analog of (3.14) also holds for the modified time-ordered functions,

$$\begin{aligned} \left\langle T_s \frac{\delta \mathcal{L}_{\text{eff}}^{(0)}}{\delta \xi_i^{(0)}}(s, x) X^{(0)} \right\rangle &= i \left\langle T_s \frac{\delta X^{(0)}}{\delta \xi_i^{(0)}(x)} \right\rangle, \\ X &= \prod_j N_{a_j}[O_j(s, x_j)]. \end{aligned} \quad (4.29)$$

This implies the identity

$$\left\langle T_s E N_3 \left[\frac{\delta \mathcal{L}_{\text{cl}}^{(0)}}{\delta \xi_i^{(0)}}(x) \right] X^{(0)} \right\rangle^{\text{norm}} = i \left\langle T_s \frac{\delta X}{\delta \xi_i^{(0)}(x)} \right\rangle, \quad (4.30)$$

with the abbreviation

$$E = \exp \left\{ i \int N_4[\mathcal{L}_{\text{cl}}(s, z)] dz \right\}. \quad (4.31)$$

On the left-hand side of (4.30) some terms occur with a power of s assigned to the vertex with coordinate x . For such terms the new subtraction rules imply

$$\begin{aligned} \langle T_s E N_a[s^b M^{(0)}(x)] X^{(0)} \rangle^{\text{norm}} \\ &= s^b \langle T_s E N_{a-b}[M^{(0)}(x)] X^{(0)} \rangle^{\text{norm}} \\ &= s^b \langle T N_{a-b}[M(x)] X \rangle. \end{aligned} \quad (4.32)$$

Using definition (4.21) we find

$$\langle T_s E N_a[s^b M^{(0)}(x)] X^{(0)} \rangle^{\text{norm}} = \langle T N_a[s^b M(x)] X \rangle. \quad (4.33)$$

With this Eq. (4.30) becomes

$$\left\langle T \frac{\delta \mathcal{L}_{\text{eff}}}{\delta \xi_i}(x) X \right\rangle = i \left\langle T \frac{\delta X}{\delta \xi_i(x)} \right\rangle. \quad (4.34)$$

Similarly,

$$\left\langle T \zeta_k(x) \frac{\delta \mathcal{L}_{\text{eff}}}{\delta \xi_i}(x) X \right\rangle = i \left\langle T \zeta_k(x) \frac{\delta X}{\delta \xi_i(x)} \right\rangle. \quad (4.35)$$

In operator form the equations of motion become

$$\frac{\delta \mathcal{L}_{\text{eff}}}{\delta \zeta_k} = 0, \quad \zeta_l \frac{\delta \mathcal{L}_{\text{eff}}}{\delta \zeta_k} = 0. \quad (4.36)$$

For the original field ϕ we obtain

$$\phi \frac{\delta \mathcal{L}_{\text{eff}}}{\delta \phi} = 0, \quad \phi^* \frac{\delta \mathcal{L}_{\text{eff}}}{\delta \phi^*} = 0. \quad (4.37)$$

With these results a partial conservation law may easily be derived for the current

$$j^\mu = iz_2 N_3 [\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*]. \quad (4.38)$$

The divergence of the current becomes

$$\begin{aligned} \partial_\mu j^\mu &= iz_2 N_4 [\phi^* \square \phi - \phi \square \phi^*] \\ &= iN_4 \left[\phi \frac{\delta \mathcal{L}_{\text{cl}}}{\delta \phi} - \phi^* \frac{\delta \mathcal{L}_{\text{cl}}}{\delta \phi^*} \right] + \delta \chi. \end{aligned} \quad (4.39)$$

(4.27) and (4.37) imply

$$\begin{aligned} N_4 \left[\phi \frac{\delta \mathcal{L}_{\text{cl}}}{\delta \phi} - \phi^* \frac{\delta \mathcal{L}_{\text{cl}}}{\delta \phi^*} \right] &= \phi \frac{\delta \mathcal{L}_{\text{eff}}}{\delta \phi} - \phi^* \frac{\delta \mathcal{L}_{\text{eff}}}{\delta \phi^*} \\ &= 0. \end{aligned} \quad (4.40)$$

Hence the current is partially conserved,

$$\partial_\mu j^\mu = \delta \chi,$$

in agreement with the classical result. The corresponding Ward identities are

$$\begin{aligned} \partial_\mu \langle Tj^\mu(x)X \rangle - \delta \langle T\chi(x)X \rangle \\ = \left\langle T \left[\phi^*(x) \frac{\delta}{\delta \phi^*(x)} - \phi(x) \frac{\delta}{\delta \phi(x)} \right] X \right\rangle. \end{aligned} \quad (4.41)$$

With the modified subtraction procedure it is thus possible to reproduce the classical conservation laws. In the following section the method will be extended to the explicitly broken pre-Higgs model.

We finally discuss two equivalence theorems. For the proof we refer to Sec. VI, where the corresponding theorems will be treated for the explicitly broken pre-Higgs model.

According to the first theorem the Green functions do not depend on η , provided the same normalization conditions are imposed. Therefore, a special value of η may be used, without loss of generality. A convenient choice consistent with (4.12) is $\eta = \mu$.

The second theorem states that there exists an equivalent Lagrangian of the form

$$\mathcal{L}_{\text{cl}} = \sum_{j=1}^9 c_j M_j, \quad (4.42)$$

where the M_j denote the monomials

$$\psi^2, \psi^3, \psi^4, \chi^2, \chi^4, \psi\chi^2, \psi^2\chi^2, \partial_\mu \psi \partial^\mu \psi, \partial_\mu \chi \partial^\mu \chi. \quad (4.43)$$

This Lagrangian does not involve a subtraction parameter s . The time-ordered functions are constructed in the conventional way by taking the

N_4 product of the interaction Lagrangian in the Gell-Mann-Low expansion. This formulation represents Symanzik's method of renormalizing the explicitly broken Goldstone model.⁵ While the Lagrangian is not manifestly gauge-invariant, the coefficients are correlated in such a way that the Ward identities hold in the desired form.

V. GAUGE-INVARIANT QUANTIZATION OF THE EXPLICITLY BROKEN PRE-HIGGS MODEL

In this section the modified subtraction procedure will be extended to the explicitly broken pre-Higgs model. As for the Goldstone model, we introduce an s dependence of the Lagrangian (1.1) by making the substitutions

$$\mu_0^2 = \eta^2 - s^2 \omega^2, \quad \delta = \delta(s), \quad (5.1)$$

$$\phi = \frac{1}{\sqrt{2}} (\psi + sv + i\chi), \quad (5.2)$$

with

$$\langle \psi \rangle = \langle \chi \rangle = 0 \text{ at } s=1.$$

Instead of g , v , and ω we introduce parameters h , w , and c through

$$v = w/e, \quad g^2 = e^2 h, \quad \omega^2 = z_1 h w^2 + c e. \quad (5.3)$$

$\delta(s)$ is a polynomial in s with $\delta(0)=0$, of which only the value $\delta(1)$ is relevant. The parameter η is arbitrary within a certain range which will be determined by consistency requirements. It will be proved in Sec. VI that the Green functions do not depend on η .

The symmetry breaking parameters s and w vary within

$$0 \leq s \leq 1, \quad 0 \leq w^2 < \infty. \quad (5.4)$$

The dependence on s is needed only for the formulation of the subtraction procedure. Eventually s is set equal to one.

The charge e will be used as an expansion parameter of perturbation theory. The unperturbed Lagrangian is given by the bilinear part of the Lagrangian with the coefficients taken in zero order. At $s=1$ we obtain the free Lagrangian (2.1), with

$$\begin{aligned} m^2 &= \lim_{e \rightarrow 0} (m_0^2 + z_2 w^2), \\ \mu^2 &= \lim_{e \rightarrow 0} (\eta^2 - \omega^2 + z_1 h w^2), \\ M^2 &= \lim_{e \rightarrow 0} (\eta^2 - \omega^2 + 3z_1 h w^2). \end{aligned} \quad (5.5)$$

Apart from the coupling constant e we choose η^2 , μ^2 , m^2 , ρ , w , h , and α as independent parameters. The vector-meson mass m and the σ mass M then become

$$m^2 = m_0^2 + w^2, \quad M^2 = \mu^2 + 2hw^2, \quad (5.6)$$

using $\lim_{e \rightarrow 0} z_j = 1$ which will be implied by the normalization conditions.

The theory is still strictly invariant under the substitution

$$A_\mu \rightarrow -A_\mu, \quad \chi \rightarrow -\chi. \quad (5.7)$$

Accordingly, Green functions involving an odd number of A_μ and χ fields vanish identically.

We now state five normalization conditions which are imposed at $s=1$:

$$\begin{aligned} (1) \quad & \Pi_A(0) = 0, \\ (2) \quad & \text{Re} \Pi_A(w^2) = 0, \\ (3) \quad & \text{Re} \Pi_\psi(M^2) = 0, \\ (4) \quad & \langle \psi \rangle = 0, \\ (5) \quad & \Pi_\chi(0) = 0, \end{aligned} \quad (5.8)$$

where the functions Π are related to the corresponding vertex functions by (see paper I, Sec. III C)

$$\begin{aligned} \Gamma_{AA}^T &= -i(p^2 - m^2 - \Pi_A), \\ \Gamma_{\psi\psi} &= i(p^2 - M^2 - \Pi_\psi), \\ \Gamma_{\chi\chi} &= i(p^2 - \mu^2 - \Pi_\chi). \end{aligned} \quad (5.9)$$

These normalization conditions uniquely determine the parameters c , z_1 , z_2 , z_3 , and $\delta(1)$ as power series in e with finite coefficients, independent of s . An exact value of $\delta(1)$ will be derived from the normalization conditions (4) and (5) making use of the Ward identities.

Our normalization conditions provide the correct mass adjustments for the stable particles of the Higgs and the pre-Higgs model. There is no reason to adjust the mass parameters for $\mu \neq 0$ since the model lacks a reasonable physical interpreta-

tion. Condition (5) guarantees that the χ propagator has a pole at the vanishing π mass for $\mu \rightarrow 0$, $m_0 \neq 0$. Apart from the free ghost particle there are no other stable particles in the pre-Higgs model. In the limit $\mu = 0$, $m_0 \rightarrow 0$ the transverse photon mass is adjusted by (2), and the σ mass is adjusted by (3), provided the stability condition $M < 2w$ holds.

We next determine the s -dependent free Lagrangian $\mathcal{L}_{\text{cl}0}$. Taking zero order coefficients in the bilinear part of $\mathcal{L}_{\text{cl}}$ we find (2.1) with the s -dependent parameters

$$m(s)^2 = m_0^2 + s^2 w^2, \quad w(s) = sw, \quad (5.10)$$

and $M(s)$, $\mu(s)$ given by (4.5) and (4.6). The masses $M(s)$ and $\mu(s)$ may be expressed by (4.7) and (4.8) in terms of their values M and μ at $s=1$. We demand that for M , m , $\mu > 0$ the s -dependent masses $M(s)$, $m(s)$, $\mu(s)$, $\lambda(s)$ be positive in the entire interval $0 \leq s \leq 1$. This requirement can be met by choosing an η with

$$\eta_{(0)} = \mu \quad (5.11)$$

and making μ sufficiently small such that (2.5) is valid in $0 \leq s \leq 1$ for the s -dependent parameters. With (5.11) the s dependence of $M(s)$ and $\mu(s)$ becomes

$$\begin{aligned} M(s)^2 &= \mu^2 + s^2(M^2 - \mu^2) = \mu^2 + 2s^2hw^2, \\ \mu(s)^2 &= \mu^2. \end{aligned} \quad (5.12)$$

A possible choice for η is

$$\eta = \mu.$$

Let O_i denote monomials of dimension d_i in the field components and their derivatives. Time-ordered functions involving normal products $N_{a_i}[O_i]$ of degree $a_i \geq d_i$ are constructed by

$$\left\langle T \prod_i N_{a_i}[O_i(x_i)] \right\rangle = \left\langle T_s \exp \left\{ i \int N_4[\mathcal{L}_{\text{cl}I}^{(0)}(z)] dz \right\} \prod_i N_{a_i}[O_i^{(0)}(x_i)] \right\rangle^{\text{norm}}, \quad \mathcal{L}_{\text{cl}I} = \mathcal{L}_{\text{cl}} - \mathcal{L}_{\text{cl}0}. \quad (5.13)$$

As for the Goldstone model the free-field Green functions with the time ordering T_s are defined by making additional subtractions with respect to the parameter s which occurs in the Feynman denominators, as well as some interaction terms. The Feynman propagators and the interaction Lagrangian will be given explicitly at the end of this section.

As in the case of the Goldstone model equations of motion follow from the expansion (5.13). For the time-ordered functions we have

$$\left\langle T \frac{\delta \mathcal{L}_{\text{eff}}}{\delta A_j}(x) X \right\rangle = i \left\langle T \frac{\delta X}{\delta A_j(x)} \right\rangle, \quad (5.14)$$

$$\left\langle T A_I(x) \frac{\delta \mathcal{L}_{\text{eff}}}{\delta A_j}(x) X \right\rangle = i \left\langle T A_I(x) \frac{\delta X}{\delta A_j(x)} \right\rangle, \quad (5.15)$$

where $l, j = 0, \dots, 5$, and for the field operators

$$\frac{\delta \mathcal{L}_{\text{eff}}}{\delta A_j} = 0, \quad A_k \frac{\delta \mathcal{L}_{\text{eff}}}{\delta A_j} = 0. \quad (5.16)$$

Here the effective Lagrangian is

$$\mathcal{L}_{\text{eff}}(s, x) = N_4[\mathcal{L}_{\text{cl}}(s, x)]. \quad (5.17)$$

The equation of motion for the potential A_μ becomes

$$-\partial_\nu F^{\mu\nu} + \frac{1}{\alpha} \partial^\mu \partial_\nu A^\nu + m_0^2 A^\mu = -j^\mu, \quad (5.18)$$

with the current

$$j^\mu = iz_2 e N_3 [\varphi^* D^\mu \varphi - \varphi (D^\mu \varphi)^*] - (z_3 - 1) \partial_\nu F^{\mu\nu} . \quad (5.19)$$

The derivation (3.24) of the partial conservation law now goes through since

$$N_4 \left[\varphi \frac{\delta \mathcal{L}_{cl}}{\delta \varphi} \right] = \varphi \frac{\delta N_4 [\mathcal{L}_{cl}]}{\delta \varphi} = \varphi \frac{\delta \mathcal{L}_{eff}}{\delta \varphi} = 0 . \quad (5.20)$$

Therefore,

$$\partial_\mu j^\mu = e \delta \chi . \quad (5.21)$$

The current thus satisfies the two basic requirements that it is partially conserved and represents the source of the potential (with the uncorrected mass m_0). The divergence of (5.18) yields

$$\frac{1}{\alpha} (\square + \alpha m_0^2) \partial_\mu A^\mu + e \delta \chi = 0 . \quad (5.22)$$

Hence $\partial_\mu A^\mu$ becomes a free field in the limit $\delta \rightarrow 0$.

We next discuss the corresponding Ward identities. For the potential the explicit form of (5.14) is

$$\begin{aligned} \left\langle T \left[-\partial_\nu F^{\mu\nu}(x) + \frac{1}{\alpha} \partial^\mu \partial_\nu A^\nu(x) + m_0^2 A^\mu(x) + j^\mu(x) \right] X \right\rangle \\ = i \left\langle T \frac{\delta X}{\delta A_\mu(x)} \right\rangle . \end{aligned} \quad (5.23)$$

For the current follow the Ward identities

$$\partial_\mu \langle T j^\mu(x) X \rangle = e \delta \langle T \chi(x) X \rangle + e \langle T g(x) X \rangle , \quad (5.24)$$

where

$$g(x) = \varphi^*(x) \frac{\delta}{\delta \varphi^*(x)} - \varphi(x) \frac{\delta}{\delta \varphi(x)} . \quad (5.25)$$

Taking the divergence of (5.23) we find the Ward identities

$$\langle T F(x) X \rangle = i \langle T \mathcal{F}(x) X \rangle , \quad (5.26)$$

with

$$F = \frac{1}{\alpha} (\square + \alpha m_0^2) \partial_\mu A^\mu + e \delta \chi , \quad (5.27)$$

and

$$\mathcal{F}(x) = \partial_x^\mu \frac{\delta}{\delta A^\mu(x)} + i e \varphi^*(x) \frac{\delta}{\delta \varphi^*(x)} - i e \varphi \frac{\delta}{\delta \varphi(x)} .$$

The special case where X contains only one field A_μ or χ has important consequences. Then the Ward identities (5.26) are

$$\begin{aligned} \langle T F(x) A_\nu(y) \rangle &= i \partial_\nu \delta(x-y) , \\ \langle T F(x) \chi(y) \rangle &= -i w \delta(x-y) \end{aligned} \quad (s=1) \quad (5.28)$$

using condition (4) of (5.8). This implies

$$\begin{aligned} \Gamma_{AA}^L + i w \Gamma_{A\chi} &= -\frac{i}{\alpha} (p^2 - \alpha m_0^2) , \\ -p^2 \Gamma_{A\chi} + i w \Gamma_{\chi\chi} &= e \delta(1) \end{aligned} \quad (s=1) \quad (5.29)$$

for the vertex functions. At $p=0$ the second equation implies

$$\delta(1) = i \frac{w}{e} \Gamma_{\chi\chi}(0) . \quad (5.30)$$

Therefore,

$$\delta(1) = \mu^2 \frac{w}{e} . \quad (5.31)$$

Since the s dependence of δ does not enter the subtraction rules we may arbitrarily set

$$\delta(s) = \mu^2 s \frac{w}{e} . \quad (5.32)$$

Substituting this, (5.1)–(5.3), and (5.12) into (1.1) we obtain the explicit form of the Lagrangian which was given in paper I, Eq. (26).

VI. DIFFERENTIAL VERTEX OPERATIONS AND EQUIVALENCE THEOREMS

The Lagrangian of the explicitly broken pre-Higgs model can further be generalized by substituting

$$\frac{1}{2} m_0^2 A_\mu A^\mu - \frac{1}{2} [s^2 m_0^2 + (1-s^2)\tau^2] A_\mu A^\mu \quad (6.1)$$

for the mass term of the vector-meson field. For $\tau > 0$ the consistency condition

$$s^2 m_0^2 + (1-s^2)\tau^2 + s^2 w^2 > 0$$

is satisfied in the interval $0 \leq s \leq 1$. At $s=1$ the value m of the free vector mesons remains

$$m = (m_0^2 + w^2)^{1/2} .$$

With this generalization the Lagrangian depends on two arbitrary parameters η and τ . In this section the Green functions will be shown to be independent of η or τ using the method of differential vertex operations.¹⁵ For the purpose of studying the differential vertex operations we write the effective Lagrangian, generalized by (6.1), in the form

$$\begin{aligned} \mathcal{L}_{eff} &= (m_0^2 - \tau^2) N_4[\mathcal{G}_{00}] + c e N_4[\mathcal{G}_0] - \eta^2 N_4[\mathcal{G}_1] \\ &+ \tau^2 N_4[\mathcal{G}_2] - z_3 N_4[\mathcal{G}_3] - \alpha^{-1} N_4[\mathcal{G}_4] \\ &+ z_2 N_4[\mathcal{G}_5] - z_1 h N_4[\mathcal{G}_6] - e^{-1} \mu^2 s w \psi , \end{aligned} \quad (6.2)$$

where

$$\begin{aligned} \mathcal{G}_{00} &= \frac{1}{2} s^2 A_\mu A^\mu , \quad \mathcal{G}_0 = s^2 \varphi^* \varphi , \quad \mathcal{G}_1 = \varphi^* \varphi , \quad \mathcal{G}_2 = \frac{1}{2} A_\mu A^\mu , \\ \mathcal{G}_3 &= \frac{1}{4} F_{\mu\nu} F^{\mu\nu} , \quad \mathcal{G}_4 = \frac{1}{2} (\partial_\mu A^\mu)^2 , \quad \mathcal{G}_5 = (D_\mu \varphi)^* D^\mu \varphi , \\ \mathcal{G}_6 &= e^2 (\varphi^* \varphi)^2 - s^2 w^2 \varphi^* \varphi . \end{aligned} \quad (6.3)$$

Let ξ be a parameter on which the coefficients of the Lagrangian depend. The time-ordered functions $G = \langle TX \rangle$ may be differentiated with respect to ξ by

$$\frac{\partial G}{\partial \xi} = i \int dx \frac{\partial \mathcal{L}_{\text{eff}}(x)}{\partial \xi} G, \quad \mathcal{L}_{\text{eff}} = N_4[\mathcal{L}_{\text{cl}}]. \quad (6.4)$$

The notation on the right-hand side is symbolic, indicating that the integral

$$i \int dx \left\langle T \frac{\partial \mathcal{L}_{\text{eff}}(x)}{\partial \xi} X \right\rangle$$

should be formed with the differentiations $\partial/\partial \xi$ acting on the coefficients of the monomials in A_μ , ψ , and χ .

Other useful relations are the counting identities which follow from the equations of motion (5.15) with $j = k$,

$$\left\langle TA_j(x) \frac{\partial \mathcal{L}_{\text{eff}}}{\partial A_j}(x) X \right\rangle = i \sum_{k=1}^{N_j} \delta(x - y_{jk}) \langle TX \rangle \quad (6.5)$$

for

$$X = \prod_{j=0}^5 \prod_{k=1}^{N_j} A_j(y_{jk}). \quad (6.6)$$

Integration over x yields the counting identity

$$-N_j G = i \int dx \left[A_j(x) \frac{\partial \mathcal{L}_{\text{eff}}}{\partial A_j}(x) + \partial_\mu A_j(x) \frac{\partial \mathcal{L}_{\text{eff}}}{\partial (\partial_\mu A_j)}(x) \right] G. \quad (6.7)$$

Vertex functions may be obtained by Symanzik's functional methods. The vertex functions $\Gamma = \langle TX \rangle^{\text{prop}}$ of the field products (6.5) are determined from a generating functional $\Gamma\{K_0, \dots, K_5\}$ which is re-

lated to the generating functional $G^{\text{conn}}\{J_0, \dots, J_5\}$ of the connected time-ordered functions by the Legendre transform

$$\Gamma\{K\} = G^{\text{conn}}\{J\} - i \int dz \sum_{b=0}^5 J_b(z) K^b(z), \quad (6.8)$$

$$K_a(x) = \frac{1}{i} \frac{\delta G^{\text{conn}}\{J\}}{\delta J_a(x)}.$$

Moreover, the relation

$$\Gamma\{N_\delta[M(x)], K\} = G^{\text{conn}}\{N_\delta[M(x)], J\} \quad (6.9)$$

holds between the generating functional $\Gamma\{N_\delta[M(x)], K\}$ of $\langle TN_\delta[M(x)]X \rangle^{\text{prop}}$ and the generating functional $G^{\text{conn}}\{N_\delta[M(x)], J\}$ of the connected functions $\langle TN_\delta[M(x)]X \rangle^{\text{conn}}$. For the vertex functions the differentiation formulas (6.4) and the counting identities (6.5) take the form

$$\frac{\partial \Gamma}{\partial \xi} = i \int dx \left\{ \frac{\partial \mathcal{L}_{\text{eff}}(x)}{\partial \xi} \right\}_{\text{NL}} \Gamma, \quad (6.10)$$

$$N_j \Gamma = i \int dx \left\{ A_j(x) \frac{\partial \mathcal{L}_{\text{eff}}}{\partial A_j}(x) + \partial_\mu A_j(x) \frac{\partial \mathcal{L}_{\text{eff}}}{\partial (\partial_\mu A_j)}(x) \right\}_{\text{NL}} \Gamma. \quad (6.11)$$

The subscript NL indicates that the nonlinear part of the expression should be taken by dropping all terms linear in the fields A_μ , ψ , and χ .

The differential vertex operations are defined by

$$\Delta_j \Gamma = i \int dx N_4[\mathcal{G}_j(x)]_{\text{NL}} \Gamma, \quad (6.12)$$

with a symbolic notation on the right-hand side, as in (6.3). Explicitly we have (at $s=1$)

$$\begin{aligned} \Delta_{00} &= \frac{1}{2} i \int dx N_2[A_\mu A^\mu](x), \quad \Delta_0 = \frac{1}{2} i \int dx N_2[\psi^2 + \chi^2](x), \\ \Delta_1 &= \frac{1}{2} i \int dx N_4[\psi^2 + \chi^2](x), \quad \Delta_2 = \frac{1}{2} i \int dx N_4[A_\mu A^\mu](x), \\ \Delta_3 &= \frac{1}{4} i \int dx N_4[F_{\mu\nu} F^{\mu\nu}](x), \quad \Delta_4 = \frac{1}{2} i \int dx N_4[(\partial_\mu A^\mu)^2](x), \\ \Delta_5 &= \frac{1}{2} i \int dx \{ N_4[\partial_\mu \psi \partial^\mu \psi + \partial_\mu \chi \partial^\mu \chi + 2e A^\mu (\chi \vec{\partial}_\mu \psi) + e^2 A_\mu A^\mu (\psi^2 + \chi^2)](x) \\ &\quad + 2w N_3[e A_\mu A^\mu \psi - A_\mu \partial^\mu \chi](x) + w^2 N_2[A_\mu A^\mu](x) \}, \\ \Delta_6 &= \frac{1}{4} i \int dx \{ e^2 N_4[(\psi^2 + \chi^2)^2](x) + 4ew N_3[\psi(\psi^2 + \chi^2)](x) + 4w^2 N_2[\psi^2](x) \}. \end{aligned} \quad (6.13)$$

We next derive a gauge-invariant relation between the normal products $N_2[\phi^* \phi]$ and $N_4[\phi^* \phi]$. Relations between normal products of different degrees may easily be generalized to Lagrangians involving a subtraction parameter s . For a monomial M in the basic fields of dimension $d \leq b$ and any integer $a > b$ we have

$$\begin{aligned} \langle TN_b[M(z)]X \rangle &= \langle TN_a[M(z)]X \rangle \\ &\quad + \sum c_j \langle TN_a[M_j(s, z)]X \rangle, \end{aligned} \quad (6.14)$$

$$N_b[M(z)] = N_a[M(z)] + \sum c_j N_a[M_j(s, z)]. \quad (6.15)$$

The sum extends over all monomials M_j of s and

the basic fields with dimension d satisfying

$$b < d \leq a.$$

d is defined by assigning the dimension one to each factor s , A_j , and ∂_μ . Moreover, due to the discrete symmetries of $\mathcal{L}_{\text{eff}}$, we need consider only

$$\nu_1(M_j) = \text{number of } \chi \text{ factors} + \text{number of } A_\mu \text{ factors in } M_j,$$

$$\nu_2(M_j) = \text{number of } \psi \text{ factors} + \text{number of } s \text{ factors in } M_j,$$

with $\nu_i(M)$, $i=1,2$, defined analogously. Throughout this section the factors of X are required to be linear in the basic fields.

Let Q be a polynomial in s and the basic fields with each term of dimension $d \leq b$. As a consequence of (6.13) $N_b[Q]$ may be expressed by the normal products of degree $a > b$. Thus there exists a polynomial P of s and the basic fields with

$$\langle TN_b[Q(s, z)]X \rangle = \langle TN_a[P(s, z)]X \rangle \quad (6.16)$$

and

$$N_b[Q] = N_a[P]. \quad (6.17)$$

Each term of P has a dimension $d \leq a$. Combining (6.16) with the Ward identities (5.34) we obtain

$$\begin{aligned} 0 &= \langle TF(x) \{N_b[Q(s, z)] - N_a[P(s, z)]\}X \rangle \\ &= \langle T \{N_b[\mathcal{F}(x)Q(s, z)] - N_a[\mathcal{F}(x)P(s, z)]\}X \rangle. \end{aligned} \quad (6.18)$$

If Q is such that $\mathcal{F}(x)Q(s, z) = 0$, then by (6.17)

$$\langle TN_a[\mathcal{F}(x)P(s, z)]X \rangle = 0 \quad (6.19)$$

for any product X of basic fields. Due to the linear independence of the Green functions of $N_a[M]$ for the various field monomials, we obtain

$$\mathcal{F}(x)P(s, z) = 0. \quad (6.20)$$

The equation $\mathcal{F}(x)R(s, z) = 0$ is a necessary and sufficient condition for the invariance of a field polynomial R under the gauge transformations (1.4). Therefore, to any gauge-invariant normal product $N_b[Q]$ there is a normal product $N_a[P]$ of degree $a > b$ which is identical in the sense of (6.16) and (6.17). Applying this result to the case

$$Q = \phi^* \phi, \quad a=4, \quad b=2,$$

we find

$$P = \sum_{j=0}^6 b_j \mathcal{G}_j + b \square \mathcal{G}_1 \quad (6.21)$$

those monomials $M_j(s, z)$ satisfying

$$(-1)^{\nu_1(M_j)} = (-1)^{\nu_1(M)},$$

$$(-1)^{\nu_2(M_j)} = (-1)^{\nu_2(M)},$$

where

[for the definition of the $\mathcal{G}_j$ see (6.2)] with vanishing coefficients

$$b_2 = b_4 = 0 \quad (6.22)$$

for the noninvariant terms $\mathcal{G}_2$ and $\mathcal{G}_4$. Thus

$$\begin{aligned} \langle TN_2[\phi^* \phi(s, z)]X \rangle &= \sum_{j=0}^6 b_j \langle TN_4[\mathcal{G}_j(s, z)]X \rangle \\ &\quad + b \square_z \langle TN_4[\mathcal{G}_1(s, z)]X \rangle. \end{aligned} \quad (6.23)$$

Integrating over z and passing to the vertex functions we obtain the linear relation

$$\Delta_0 \Gamma = \sum_{j=1}^6 r_j \Delta_j \Gamma, \quad (6.24)$$

with $r_2 = r_4 = 0$ for the differential vertex operations (6.12).

We next consider the expression

$$Q = \frac{1}{2} A_\mu A^\mu, \quad (6.25)$$

which is not gauge-invariant. Nevertheless an identity similar to (6.22) can be found, since the linearity of

$$\mathcal{F}Q = \partial_\mu A^\mu \quad (6.26)$$

allows one to write (6.17) in the form

$$\begin{aligned} 0 &= \langle TF(x) \{ \frac{1}{2} N_2[A_\mu A^\mu(z)] - N_4[P(s, z)] \} X \rangle \\ &= \langle TN_4[\mathcal{F}(x) \{ \frac{1}{2} A_\mu A^\mu(z) - P(s, z) \}] X \rangle. \end{aligned} \quad (6.27)$$

This implies

$$\mathcal{F}(x) \{ P(s, z) - \frac{1}{2} A_\mu A^\mu(z) \} = 0. \quad (6.28)$$

Hence P is of the form

$$P = \frac{1}{2} A_\mu A^\mu + R, \quad (6.29)$$

with R being a gauge-invariant polynomial. We thus arrive at the linear relation

$$\langle TN_2[A_\mu A^\mu(z)]X \rangle = \sum_{j=0}^6 c_j \langle TN_4[\mathcal{G}_j(s, z)]X \rangle + c \square_z \langle TN_4[\mathcal{G}_1(s, z)]X \rangle \quad (c_2=1, \quad c_4=0). \quad (6.30)$$

For the differential vertex operation Δ_{00} we obtain the identity

$$\Delta_{00}\Gamma = \sum_{j=1}^6 t_j \Delta_j \Gamma \quad (t_2=1, t_4=0). \quad (6.31)$$

We now discuss two equivalence theorems. The first theorem states that Lagrangians with different values of η^2 and τ^2 are equivalent, provided the same normalization conditions (5.8) are imposed. Condition (5) is satisfied by (5.31). The parameters $z_0=c$, z_1, z_2, z_3 are uniquely determined as functions of η^2, τ^2 by the first four normalization conditions

$$c_j = \Gamma^{(j)}(z_0, z_1, z_2, z_3, \eta^2, \tau^2, e, \mu^2, m_0^2, w, h, \alpha) \quad (s=1), \quad (6.32)$$

where

$$\begin{aligned} \Gamma^{(0)} &= i\Gamma_{AA}^T(0), \quad \Gamma^{(1)} = \text{Re } i\Gamma_{AA}^T(w^2), \\ \Gamma^{(2)} &= \text{Re } i\Gamma_{\psi\psi}(2hw^2), \quad \Gamma^{(3)} = \langle \psi \rangle, \\ c_0 &= -m^2, \quad c_1 = -m_0^2, \quad c_2 = c_3 = 0. \end{aligned} \quad (6.33)$$

It suffices to show that the vertex functions of the fields do not depend on η^2, τ^2 . To this end we form

$$\begin{aligned} \frac{\partial \Gamma}{\partial \eta^2} &= \sum_{k=0}^3 \frac{\partial' \Gamma}{\partial z_k} \frac{\partial z_k}{\partial \eta^2} + \frac{\partial' \Gamma}{\partial \eta^2}, \\ \frac{\partial \Gamma}{\partial \tau^2} &= \sum_{k=0}^3 \frac{\partial' \Gamma}{\partial z_k} \frac{\partial z_k}{\partial \tau^2} + \frac{\partial' \Gamma}{\partial \tau^2} \end{aligned} \quad (s=1). \quad (6.34)$$

Here $\partial/\partial \xi$ denotes the derivative with respect to one of the independent variables $\eta^2, \tau^2, e, \mu^2, m_0^2, w, h, \alpha$. $\partial'/\partial \xi$ is the derivative with respect to one of the variables $z_0, z_1, z_2, z_3, \eta^2, \tau^2, \mu^2, m_0^2, w, h, \alpha$, if considered as independent parameters of the Lagrangian. Using

$$\begin{aligned} \frac{\partial' \Gamma}{\partial \eta^2} &= -\Delta_1 \Gamma, \\ \frac{\partial' \Gamma}{\partial \tau^2} &= (\Delta_2 - \Delta_{00}) \Gamma, \\ \frac{\partial' \Gamma}{\partial z_0} &= e \Delta_0 \Gamma, \\ \frac{\partial' \Gamma}{\partial z_1} &= -h \Delta_8 \Gamma, \\ \frac{\partial' \Gamma}{\partial z_2} &= \Delta_5 \Gamma, \\ \frac{\partial' \Gamma}{\partial z_3} &= -\Delta_3 \Gamma, \end{aligned} \quad (s=1) \quad (6.35)$$

and (6.23), we can eliminate $\partial' \Gamma / \partial \eta^2$ in (6.34). Therefore

$$\begin{aligned} \frac{\partial \Gamma}{\partial \eta^2} &= \sum_{k=0}^3 A_k \frac{\partial' \Gamma}{\partial z_k}, \\ \frac{\partial \Gamma}{\partial \tau^2} &= \sum_{k=0}^3 B_k \frac{\partial' \Gamma}{\partial z_k} \end{aligned} \quad (s=1). \quad (6.36)$$

The normalization conditions (6.32) imply

$$\begin{aligned} 0 &= \frac{\partial \Gamma^{(j)}}{\partial \eta^2} = \sum_{k=0}^3 A_k \frac{\partial' \Gamma^{(j)}}{\partial z_k}, \\ 0 &= \frac{\partial \Gamma^{(j)}}{\partial \tau^2} = \sum_{k=0}^3 B_k \frac{\partial' \Gamma^{(j)}}{\partial z_k} \end{aligned} \quad (s=1). \quad (6.37)$$

Since the functional determinant of (6.32) does not vanish we have $A_k = B_k = 0$ and $\partial \Gamma / \partial \eta^2 = \partial \Gamma / \partial \tau^2 = 0$ identically. This completes the proof.

The second theorem concerns the equivalence of the gauge-invariant quantization to Symanzik's approach, which was used in Lee's work on the Higgs model. For the purpose of this discussion we denote the Lagrangian (1.1), (5.1)–(5.3) of the explicitly broken pre-Higgs model by $\mathcal{L}'_{\text{cl}}$. The theorem states that a Lagrangian

$$\mathcal{L}_{\text{cl}}'' = \mathcal{L}_{\text{cl}0}'' + \mathcal{L}_{\text{cl}I}'' \quad (6.38)$$

equivalent to $\mathcal{L}'_{\text{cl}}$ exists which does not involve a subtraction parameter s . The Green functions are constructed as usual with the N_4 product applied to the interaction Lagrangian $\mathcal{L}_I''$. For the proof we construct a family of Lagrangians

$$\mathcal{L}_{\text{cl}} = \sum_j u_j M_j + \sum_{k=1}^3 s^k \sum_l t_{kl} Q_{kl}, \quad (6.39)$$

where M_j denotes the monomials in the fields A_μ, ψ, χ and derivatives of dimension less or equal to four. Q_{kl} denotes the field monomials of dimension less or equal to $4-k$. All monomials are supposed to be even in A_μ, χ . Moreover, monomials are omitted which can linearly be expressed by others up to a total divergence. The rules of the preceding sections for constructing time-ordered functions are generalized to the s -dependent Lagrangians (6.39) accordingly. It will be shown that the coefficients of (6.39) can be chosen such that the family consists of equivalent Lagrangians including the original Lagrangian $\mathcal{L}'_{\text{cl}}$ of explicitly broken pre-Higgs model and a Lagrangian,

$$\mathcal{L}_{\text{cl}}'' = \sum u_j'' M_j, \quad (6.40)$$

where all $t_{kl}=0$. The effective Lagrangian corresponding to (6.39) is

$$\mathcal{L}_{\text{eff}} = \sum u_j N_4[M_j] + \sum_{k=1}^3 s^k \sum_l t_{kl} N_{4-k}[Q_{kl}]. \quad (6.41)$$

We extend (5.17) to a complete and independent set

$$c_j = \Gamma^{(j)}(u_r, t_{kl}) \quad (6.42)$$

of normalization conditions which uniquely determine the coefficients u_j as power series in the coupling constants for any Lagrangian (6.39) with given t_{kl} . We restrict the family (6.39) by imposing

$$c'_j = \Gamma^{(j)}(u_r, t_{kl}) \quad (s=1), \quad (6.43)$$

where the c'_j denote the values of c_j for the original Lagrangian $\mathcal{L}'_{cl}$. Differentiating the vertex functions Γ with respect to t_{kl} we obtain

$$\frac{\partial \Gamma}{\partial t_{kl}} = \sum_j \frac{\partial' \Gamma}{\partial u_j} \frac{\partial u_j}{\partial t_{kl}} + \frac{\partial' \Gamma}{\partial t_{kl}}. \quad (6.44)$$

Primed derivatives refer to u_j , t_{kl} as sets of independent variables. With the identities (6.13) we get

$$\frac{\partial \Gamma}{\partial t_{kl}} = \sum_j A_{kl,j} \frac{\partial' \Gamma}{\partial u_j} \quad (6.45)$$

and

$$0 = \frac{\partial \Gamma^{(j)}}{\partial t_{kl}} = \sum_j A_{kl,j} \frac{\partial' \Gamma^{(j)}}{\partial u_j} \quad (6.46)$$

from (6.43). Since the functional determinant of (6.42) does not vanish, we have $A_{kl,j} = 0$ and $\partial \Gamma / \partial t_{kl} = 0$ identically. This completes the proof of the theorem.

We finally give formulas for the derivatives of vertex functions with respect to the independent variables e , μ^2 , m_0^2 , w , h , α , setting $\eta = \mu$, $\tau = m_0$, and $s=1$. These relations, along with the identities (6.23) and (6.31), will be used in paper III. Since the terms $N_4[\mathcal{Q}_j]$ of the effective Lagrangian (6.1) only involve coefficients depending on e , s , and w we have

$$\begin{aligned} N_A - e \frac{\partial}{\partial e} - w \frac{\partial}{\partial w} &= -e \left(c + e \frac{\partial c}{\partial e} + w \frac{\partial c}{\partial w} \right) \Delta_0 + 2m_0^2 \Delta_2 + \left(e \frac{\partial z_3}{\partial e} - 2z_3 \right) \Delta_3 - \frac{2}{\alpha} \Delta_6 - e \frac{\partial z_2}{\partial e} \Delta_5 + h \left(e \frac{\partial z_1}{\partial e} + 2z_1 \right) \Delta_6, \\ N_\psi + N_\chi - w \frac{\partial}{\partial w} &= e \left(2c + w \frac{\partial c}{\partial w} \right) \Delta_0 - 2\mu^2 \Delta_1 + 2z_2 \Delta_5 - 2hz_1 \Delta_6 \end{aligned} \quad (s=1). \quad (6.51)$$

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$$\begin{aligned} \frac{\partial \Gamma}{\partial \xi} &= \left(e \frac{\partial c}{\partial \xi} \Delta_0 - \frac{\partial \mu^2}{\partial \xi} \Delta_1 + \frac{\partial m_0^2}{\partial \xi} \Delta_2 + \frac{\partial z_3}{\partial \xi} \Delta_3 \right. \\ &\quad \left. + \frac{1}{\alpha^2} \frac{\partial \alpha}{\partial \xi} \Delta_4 + \frac{\partial z_2}{\partial \xi} \Delta_5 - \frac{\partial (z_1 h)}{\partial \xi} \Delta_6 \right) \Gamma \end{aligned} \quad (6.47)$$

for the parameters

$$\xi = \mu^2, \quad m_0^2, \quad h, \quad \alpha.$$

For the derivatives of Γ with respect to e and w we find

$$\begin{aligned} e \frac{\partial \Gamma}{\partial e} &= \left[e \left(c + e \frac{\partial c}{\partial e} \right) \Delta_0 - e \frac{\partial z_3}{\partial e} \Delta_3 + e \frac{\partial z_2}{\partial e} \Delta_5 \right. \\ &\quad \left. - h \left(2z_1 + e \frac{\partial z_1}{\partial e} \right) - v \Delta' + e \Delta'' \right] \Gamma, \end{aligned} \quad (s=1) \quad (6.48)$$

$$w \frac{\partial \Gamma}{\partial w} = \left[\left(2hw^2 z_1 + ew \frac{\partial c}{\partial w} \right) \Delta_0 + v \Delta' \right] \Gamma$$

where

$$\Delta' = \frac{i}{\sqrt{2}} \int dx \left[\frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi}(x) + \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi^*}(x) \right]_{\text{NL}} \Big|_{s=1}, \quad (6.49)$$

$$\Delta'' = z_2 \int dx N_4 [A_\mu \varphi (D^\mu \varphi)^* - A_\mu \varphi^* D^\mu \varphi]_{\text{NL}} \Big|_{s=1}.$$

Δ' and Δ'' may be eliminated by combining these relations with the counting identities for N_A $= \sum_{j=0}^3 N_j$, $N_\psi + N_\chi = N_4 + N_5$. From (6.11) we get

$$N_A \Gamma = (2m_0^2 \Delta_2 - 2z_3 \Delta_3 - 2/\alpha \Delta_4 + e \Delta'') \Gamma, \quad (6.50)$$

$$\begin{aligned} (N_\psi + N_\chi) \Gamma &= [(2ce - 2hw^2 z_1) \Delta_0 - 2\mu^2 \Delta_1 + 2z_2 \Delta_5 \\ &\quad - 2hz_1 \Delta_6 - v \Delta'] \Gamma. \end{aligned}$$

Combining these equations with (6.48) we find

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Formulation of Abelian gauge theories without regulators. III

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Parametric differential equations are derived which allow one to study the pre-Higgs and Higgs limits of the explicitly broken pre-Higgs model, as well as its short-distance behavior and gauge dependence. It is shown that all physical scattering amplitudes in the pre-Higgs and Higgs models can be expressed in terms of the Green functions of the conserved current. A gauge-invariant composite field is defined which serves as interpolating field for the heavy spin-one particles in the Higgs model. Reduction formulas are obtained which make transparent the principal features of the Higgs phenomenon.

I. INTRODUCTION

In papers I and II (Ref. 1) of the present study, an explicitly broken (Abelian) Higgs model was discussed using a modified Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ) subtraction prescription in renormalized perturbation theory. Green functions were defined on the basis of an effective Lagrangian,

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & -\frac{1}{4}z_3(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 + \frac{1}{2}m_0^2 A_\mu A^\mu \\ & + \frac{1}{2}z_2[\partial_\mu \psi \partial^\mu \psi + \partial_\mu \chi \partial^\mu \chi + 2eA_\mu(\chi \tilde{\partial}^\mu \psi) - 2usA_\mu \partial^\mu \chi + A_\mu A^\mu(e^2\psi^2 + e^2\chi^2 + 2use\psi + w^2s^2)] \\ & - \frac{1}{4}z_1h[e^2(\psi^2 + \chi^2)^2 + 4use\psi(\psi^2 + \chi^2) + 4w^2s^2\psi^2] + \frac{1}{2}cs^2[e(\psi^2 + \chi^2) + 2us\psi] - \frac{1}{2}\mu^2(\psi^2 + \chi^2). \end{aligned} \quad (1.1)$$

Here the parameter s plays a special role in the subtraction scheme which is described at length in papers I and II. After the subtractions have been made in a given Feynman integral, however, s is always set equal to unity. The finite renormalization constants z_1 , z_2 , z_3 , and c in (1.1) are formal power series in the perturbation parameter e , and are determined recursively by the "tadpole" condition,

$$\langle \psi \rangle = 0, \quad (1.2)$$

and the normalization conditions,

$$\begin{aligned} \Gamma^T(0)|_{w^2=\rho^2} &= i(m_0^2 + \rho^2), \\ \text{Re}i\Gamma^T(\rho^2)|_{w^2=\rho^2} &= -m_0^2, \\ \text{Re}i\Gamma^{(020)}(2h\rho^2)|_{w^2=\rho^2} &= 0, \end{aligned} \quad (1.3)$$

where