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Form factor in the Bethe-Salpeter model with exponential interaction

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The asymptotic behavior of the nucleon form factor is investigated, treating the nucleon as the bound state of its two constituent particles interacting via the exchange of an exponential field. It is suggested that the result that the form factor behaves as t^{-1} for $|t|\to\infty$ could be interpreted as signifying the localizability of the exponential theory.

I. INTRODUCTION

The role of form factors as typical field-theoretic constructs eminently suited to extract physical information about the structure of particles is well known. Thus, the excellent agreement between the theoretical result that the electromagnetic form factor of nucleons tends to zero as t^{-2} (t being the momentum transfer squared) and the experimental values has helped to establish a composite picture of hadrons.[1]

The precise nature of the compositeness of hadrons, it should be mentioned, plays little role in determining the qualitative features of strong interactions, the important property being the softness of the hadrons, i.e., their reluctance to absorb large momentum transfers. Even though processes characterized by such momentum transfers are rare, they are in fact of great interest because of the possibility they offer of investigating the details of the bound-state structure.[2]

With the above considerations in mind we choose to investigate in this paper the *asymptotic* behavior of the nucleon form factor in the Bethe-Salpeter model with a nonpolynomial Lagrangian, viz., $L_{\rm int}=e^{g\phi}$. We do not wish to stress here the attractive features of nonpolynomial theories, or the important role they are likely to play in particle physics; this has been done admirably by Salam in his numerous papers.[3] Rather, we note that the choice of the exponential interaction Lagrangian has been dictated by the knowledge[4] that it leads to a theory which is causal, local, analytic, unitary, and positive-definite. Further, the superpropagators for this theory are entire functions of g^2/x^2, Borcher's theorem is valid, on-shell Froissart-boundedness is ensured, and the theory is free from ambiguities. On the other hand, the rational Lagrangian, e.g., $1/(1+g\phi)$, signifies a nonlocal theory. The theory is unitary, but not positive-definite.[5] The superpropagators in this theory are represented by divergent series, and consequently the problem of ambiguities remains. It is transparent, therefore, that the exponential-Lagrangian theory appears to possess all desirable features of a good field theory. It is worthwhile then to investigate the asymptotic behavior of the form factor in the relativistic nonpolynomial theory of the exponential type.

The plan of the paper is as follows. In Sec. II we spell out the model with which we work and write down the equation for the form factor in terms of vertex functions. In Sec. III we obtain the asymptotic solutions of the Bethe-Salpeter equation for the vertex function with the exponential interaction. This enables us to investigate easily the large-t dependence of the form factor, which is found to be t^{-1} for $|t|\to\infty$. Section IV is devoted to a brief discussion of our result.

II. EQUATION FOR THE FORM FACTOR

We assume the nucleon (mass M) to be made up of two particles of mass μ each, and refer to the latter as the constituent particles. Both the nucleons and the photons are taken to be spinless. Then, assuming that the class of diagrams that contributes to the form factor is confined to the type illustrated in Fig. 1, we can write down the equation for the form factor in the ladder approximation as

$$F(t)=i\int\frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2-\mu^2}\,\frac{\Gamma^*(M^2;p^2,(p+p_2)^2)}{(p+p_2)^2-\mu^2}\times\frac{\Gamma(M^2;p^2,(p+p_1)^2)}{(p+p_1)^2-\mu^2}\,, \tag{1}$$

where

$$t=q^2$$

and

$$q=p_2-p_1\,.$$

The vertex function $\Gamma(p_2^2;p^2,(p+p_2)^2)$ describes the vertex connecting a nucleon of momentum p_2 to its two constituent particles of momenta $-p$ and $(p+p_2)$, respectively.

We note that the choice of the diagrams of Fig. 1 as the dominant dynamical processes which con-

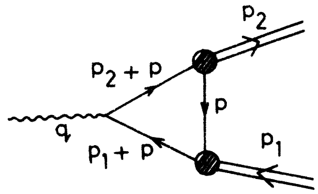

FIG. 1. The class of diagrams included in Eq. (1). The double line represents a spin-0 nucleon, the wavy line is a photon.

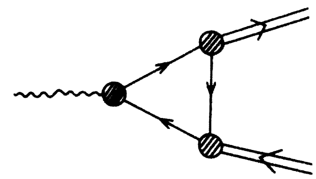

FIG. 2. A more complicated class of diagrams which could also be used. The blob on the left-hand side is supposed to include t-channel ladders.

tribute to the form factor is governed by, apart from reasons of simplicity, the knowledge that it provides an excellent agreement between theory and experiment in the case of the electromagnetic form factor in the ordinary theory.[1] That this class of one-loop graphs dominates the higher-loop graphs, insofar as the asymptotic behavior of the form factor is concerned, can also be seen by the simple artifice of power-counting. Further, in writing down Eq. (1) we have assumed a point interaction between the photon and the two constituent particles. This is justified by the observation that the use of some completely off-shell form factor $G(t; (p+p_2)^2, (p+p_1)^2)$ representing a structure in the t channel, as depicted in Fig. 2, can only make $F(t)$ converge more rapidly as $|t| \to \infty$ than it does if G is replaced by unity. For example a choice of $G \sim 1/(t-m^2)$ would make $F(t)$ fall off by at least one power of t faster.

Before we go on to discuss the equation for the vertex function, we observe that in the coordinate system where q is purely timelike we have

$$\begin{aligned} q &= (\sqrt{t}, \vec{0}), \\ p_2 &= (\tfrac{1}{2}\sqrt{t}, \vec{p}_2), \\ p_1 &= (-\tfrac{1}{2}\sqrt{t}, \vec{p}_1), \end{aligned} \tag{2}$$

with $|\vec{p}_1| = |\vec{p}_2| = (\frac{1}{4}t - M^2)^{1/2}$. Thus, as t becomes very large

$$\begin{aligned} (p+p_2)^2 &\to p^2 + M^2 + \sqrt{t}\,(p_0 - \vec{p}\cdot\hat{p}_2), \\ (p+p_1)^2 &\to p^2 + M^2 - \sqrt{t}\,(p_0 + \vec{p}\cdot\hat{p}_1). \end{aligned} \tag{3}$$

III. ASYMPTOTIC SOLUTION OF THE EQUATION FOR THE VERTEX FUNCTION WITH EXPONENTIAL INTERACTION AND BEHAVIOR OF THE FORM FACTOR

The large-t behavior of the form factor can be easily determined provided the vertex functions occurring in Eq. (1) are known. To this end we first write down the equation for the momentum-space Bethe-Salpeter amplitude for the bound state of the constituent particles interacting via the exchange of the nonpolynomial field $U(x) = e^{g\phi(x)}$ in the ladder approximation, where g is the minor coupling constant and $\phi(x)$ a massless neutral scalar field. This equation has been derived and discussed in detail by Biswas, Chaudhuri, and Malik,[6] and reads

$$\psi(q) = -\frac{2\lambda f}{i\pi^2}\frac{1}{(\frac{1}{2}P+q)^2-\mu^2}\frac{1}{(\frac{1}{2}P-q)^2-\mu^2}\int_0^\infty d\beta\,\frac{J_2(\beta)}{\beta}\int\frac{d^4k}{(q-k)^2}\exp[-i\lambda'(q-k)^2]\,\psi(k), \tag{4}$$

where

$$\begin{aligned} -i\lambda &= g^2, \\ f &= \left(\frac{G'}{4\pi}\right)^2, \\ \lambda' &= \frac{\lambda}{4\pi^2\beta^2}, \end{aligned} \tag{5}$$

and G' is the renormalized major coupling constant.

Next, by using the well-known relation[7] between the wave function and the vertex function we write down the following equation for the vertex function:

$$\Gamma(M^2; (\tfrac{1}{2}P+q)^2, (\tfrac{1}{2}P-q)^2) = -\frac{2\lambda f}{\pi^2}\int_0^\infty d\beta\,\frac{J_2(\beta)}{\beta}\int\frac{d^4k}{(q-k)^2}\,\frac{\exp[-i\lambda'(q-k)^2]\,\Gamma(M^2; (\frac{1}{2}P+k)^2, (\frac{1}{2}P-k)^2)}{[(\frac{1}{2}P+k)^2-\mu^2][(\frac{1}{2}P-k)^2-\mu^2]}, \tag{6}$$

where we have used Eq. (4) in its Wick-rotated form.

Following Ball and Zachariasen,[1] we specialize now to the c.m. system, $P=(M,0)$, so that if

$$s_1=(\tfrac{1}{2}P+q)^2,\quad s_2=(\tfrac{1}{2}P-q)^2, \tag{7}$$

then

$$q^2=\tfrac{1}{4}M^2+\tfrac{1}{2}(s_1+s_2),\quad q_0=(s_1-s_2)/4M, \tag{8}$$

and

$$|\vec{q}|^2=[M^4-2M^2(s_1+s_2)+(s_1-s_2)^2]/4M^2 .$$

It is clear that to determine the large-t behavior of $F(t)$, we must find the large-s_1 behavior of Γ. The only dependence on s_1 in the integrand of Eq. (6) is through the factor $(q-k)^2$; for large s_1, this factor tends to

$$(q-k)^2\to s_1\left(\frac{1}{2}-\frac{k_0}{M}+\frac{\vec{k}\cdot\hat{q}}{M}\right) . \tag{9}$$

Now, since the limit $s_1\to\infty$ may be interchanged with the integral over d^4k in Eq. (6), because if this is done the integral over d^4k still exists, we may write

$$\Gamma(M^2;s_1,s_2)\underset{s_1\to\infty}{\sim}\text{const}\times\frac{1}{s_1}\int_0^\infty d\beta\,\frac{J_2(\beta)}{\beta}\,e^{-i\lambda' s_1}\quad (\lambda'=\lambda/4\pi^2\beta^2). \tag{10}$$

The problematic β integration in Eq. (10) can be performed by a method given by Okubo.[8] Letting

$$I(s_1)=\int_0^\infty d\beta\,\frac{J_2(\beta)}{\beta}\exp\left(\frac{-i\lambda s_1}{4\pi^2\beta^2}\right)=\frac{\phi(u)}{u}, \tag{11}$$

where

$$u=-\lambda s_1/4\pi^2$$

and

$$\phi(u)=u\int_0^\infty d\beta\,\frac{J_2(\beta)}{\beta}\exp\left(\frac{iu}{\beta^2}\right),$$

one can show that $\phi(u)$ satisfies the following third-order differential equation:

$$\frac{d^3\phi}{du^3}-\frac{\phi}{4iu^2}=0 . \tag{12}$$

Equation (12) may be transformed to

$$\frac{d^3\phi}{dy^3}-\frac{\phi}{y^2}=0\quad (y=-\tfrac{1}{4}iu),$$

of which the most dominant solution as $y\to\infty$ is given essentially by

$$\phi(y)=\sum_{n=0}^\infty\frac{1}{(n+2)!(n+1)!n!}\,y^{n+2} . \tag{13}$$

Thus,

$$\Gamma(M^2;s_1,s_2)\underset{s_1\to\infty}{\sim}\text{const}\times\sum_{n=0}^\infty\frac{1}{(n+2)!(n+1)!n!}\times\left(\frac{i\lambda s_1}{16\pi^2}\right)^n , \tag{14}$$

which is absolutely convergent for all values of $-g^2s_1/16\pi^2$.

Noting that it is permissible to interchange the limit $t\to\infty$ with the integral over d^4p in Eq. (1), we can now substitute the solutions given by Eq. (14) for Γ^* and Γ in the former equation to obtain

$$F(t)=\text{const}\times\int\frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2-\mu^2}\,\frac{1}{(p+p_2)^2-\mu^2}\,\frac{1}{(p+p_1)^2-\mu^2}\sum_{m=0}^\infty\sum_{n=0}^\infty\frac{1}{(m+2)!(m+1)!m!(n+2)!(n+1)!n!}\times[(p+p_1)^2]^m[(p+p_2)^2]^n . \tag{15}$$

The physical requirement that $F(t)\to 0$ as $|t|\to\infty$ dictates that the d^4p integral in the above equation be convergent, which implies that

$$m+n<1$$

or

$$m=n=0 .$$

The p integration is now trivial, and we find that

$$F(t)\underset{t\to\infty}{\sim}\text{const}\times\frac{1}{t}, \tag{16}$$

which implies that there is a contact interaction between the photon and the nucleon.[1]

IV. DISCUSSION

It is interesting to observe that the result that $F(t)\sim t^{-1}$ when $|t|\to\infty$ in the model studied can be looked upon from two different points of view. Thus, if one takes it as known that the exponential-Lagrangian theory is local,[9] then the result obtained signifies that the model we employed is a "good" one. This does not, of course, imply that there may not exist other good models. On the other hand, if one believes the composite model of nucleons used in this investigation to be a good model, then our result can be interpreted as implying the locality of the exponential-Lagrangian theory.

ACKNOWLEDGMENT

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