determination from the K^0-$\bar{K}^0$ decay parameters. The values of this quantity for various models are given in Table I.

In conclusion, we make the following remarks. The original Veneziano-Lovelace model for π-π scattering, although a good first approximation, has many undesirable features. Additional terms, which are theoretically permissible, can remove some of these. Several difficulties do, however, remain. The problem of negative-width ghosts has been already mentioned. Another problem is the position of some of the resonances. As a consequence of the exchange degeneracy, the mass of f^0, for $\alpha_0=0.56$, turns out to be 1384 instead of 1260 MeV. The mass of the second recurrence ($J^P=3^-$) becomes 1810 MeV. If we identify this with the g meson (1650 MeV) as quoted in the compilation by the Particle Data Group,[17] then again there is a discrepancy. Of course, as we go to higher energies, effects of unitarization will be more and more important, and this may shift the mass values in the right direction. Finally, we cannot ascribe any fundamental significance to the seven-term formula at the present time. However, it can be used as a reasonable model amplitude consistent with *all* the generally accepted low-energy properties of the π-π system and with the high-energy phenomenology.

[17] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

ACKNOWLEDGMENTS

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Orbital-Angular-Momentum Structure of the A_1 Meson*

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The orbital angular momentum of the A_1 meson produced from $\pi^-p \to \pi^-\pi^-\pi^+p$ at 16 GeV/c was studied by analyzing the decay angular distributions in the $\rho^0\pi^-$ state. A completely Bose-symmetrized formalism was used to measure the ratio g_1/g_0 of the helicity states of the ρ from the A_1 decay. The value obtained, $|g_1/g_0|=0.48\pm0.13$, indicates a substantial d-wave contribution in the A_1 decay.

I. INTRODUCTION

A NUMBER of theoretical calculations[1] have suggested that the axial-vector mesons, in particular the A_1(1080 MeV) and the B(1210 MeV), do not decay via a single orbital-angular-momentum wave but rather through a mixture of two different waves (s and d). Experimental evidence for this idea was recently presented by Ballam *et al.*[2] for the A_1 and by Ascoli *et al.*[3] for the B meson. In both cases, analysis of the decay distributions indicated that the vector meson from the decay (ρ for the A_1 and ω for the B) was polarized in a significantly differently manner than expected for an axial-vector resonance decaying by s wave.

The possibility of a more complicated orbital-angular-momentum structure in the A_1 is also interesting in that it could indicate that the intuitive idea that the lowest partial wave is dominant is not the best approach to use in spin-parity analyses of multibody final states. This problem is a common occurrence for baryon states and may well occur for meson resonances other than the A_1, such as the recently discovered $Q(K^*\pi)$ and $A_3(\pi f)$, both of which could be of the same spin-parity series as the A_1.

In order to study the orbital-angular-momentum structure of the A_1, we have extended the usual spin-parity techniques to the case of several partial waves and analyzed the 16-GeV/c π^-p data of Ballam *et al.* We have studied all of the independent variables associated with the A_1 decay using a completely Bose-symmetrized formalism and a one-pion-exchange (OPE) model for the background. We conclude that there is positive evidence for the presence of both orbital-angular-momentum states in the A_1. Although it is difficult to discriminate conclusively between $J^P=1^+$ or 2^- for the A_1, a 1^+ assignment would indicate that the A_1 has a large d-wave component.

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[1] F. J. Gilman and H. Harari, Phys. Rev. **165**, 1803 (1968); H. J. Lipkin, *ibid.* **176**, 1709 (1968); H. J. Schnitzer and S. Weinberg, *ibid.* **164**, 1828 (1967); S. G. Brown and G. B. West, MIT and Harvard University Report, 1969 (unpublished).

[2] J. Ballam *et al.*, Phys. Rev. Letters **21**, 934 (1968).

[3] G. Ascoli *et al.*, Phys. Rev. Letters **20**, 1411 (1968).

In the following section, we discuss the coordinate frames and angles used to study the A_1 decay. The results of our analysis are presented in Sec. III and we compare them with theoretical predictions in Sec. IV. The formalism of helicity coupling constants is briefly outlined in the Appendix.

II. EULER ANGLES AND DECAY SYSTEMS

In order to study the A_1 decay properties, we use the formalism developed by Zemach[4] to calculate the expected theoretical distributions for various spin-parity (J^P) values.

As pointed out by Zemach[4] and by Berman and Jacob,[5] a three-particle system is completely specified by five independent variables. Two of these are usually taken to be the variables describing a Dalitz plot and the remaining three are chosen as the Euler angles that specify the orientation of the three-particle system.

The Euler angles are defined explicitly by constructing orthogonal bases of unit vectors from the three-momenta of the reaction particles associated with the production and subsequent decay of the A_1. In the *production system*, we choose the Z axis along the incident beam $\hat{B}$ and the Y axis along the production plane normal $\hat{Q}$. In the *decay system*, the z axis can be taken along the direction of one of the three decay pions, such as the "bachelor" pion or along some linear combination of pion momenta, with the y axis also being defined in a corresponding manner. All of the angular correlations for the A_1 decay can then be expressed in terms of the three independent Euler angles θ, ϕ, and ψ, which relate the production basis (X,Y,Z) to the decay basis (x,y,z) (see Fig. 1). *A priori*, there is no reason to prefer any particular choice of decay axes over another. Two commonly used decay systems for studying the A_1 are those associated with the "Jackson angle"[6] and "π-π scattering" angle, where the respective z axes are chosen to be the bachelor-pion momentum and the direction of $t'=p_1'-p_2'$, the difference in the decay pion momenta from the ρ-meson decay as seen in the ρ rest frame. Each of the above choices is "natural" for the three-pion system from the viewpoint of some particular theoretical model.

In order to measure the relative amounts of the two orbital-angular-momentum states in the A_1 decay, we are motivated to use a decay system that is equally sensitive in some sense to both orbital states. An appropriate choice is to use the Jackson frame and parametrize the decay amplitudes in terms of the helicity states of the intermediate ρ meson rather than using the orbital-angular-momentum amplitudes directly. The advantage of this choice is that the two helicity spin tensors are "orthogonal," so that we are dealing with distributions of distinctly different functional form, such as $\sin^2\theta$ *or* $\cos^2\theta$, rather than some linear combination.[7]

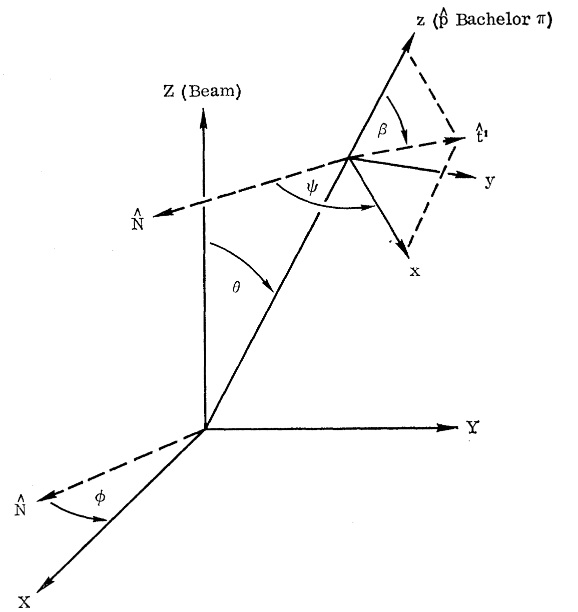

FIG. 1. Euler angles θ, ϕ, and ψ relating the production system (X,Y,Z) and the decay system (x,y,z) for the A_1 decay. β is the helicity angle for the ρ-meson decay.

The production and decay bases for the Jackson frame are defined as

$$\hat{B}=\hat{Z}, \quad \hat{Q}=\hat{Y}, \tag{1a}$$

$$\hat{p}=\hat{z}, \quad \hat{t}'=\hat{x}\sin\beta+\hat{z}\cos\beta, \tag{1b}$$

where $\mathbf{p}$ is the bachelor-pion momentum and $\beta=\cos^{-1}(\hat{p}\cdot\hat{t}')$ is the ρ-meson helicity angle. The helicity angle β replaces one of the two-pion invariant masses used to describe the Dalitz plot. The Euler angle θ is now the "Jackson angle," ϕ is the conventional Treiman-Yang angle for the A_1, and ψ describes an azimuthal rotation about the ρ-meson line of flight.

The helicity decay tensors can be easily obtained from the orbital state tensors as given by Zemach[4] and by Diebold.[8] Only the "unnatural" spin-parity cases 1^+, 2^-, etc., have two helicity states. From (A4) in the Appendix, the relation between the orbital and helicity coupling constants, we obtain the helicity tensors

$$J^P=1^+: \quad M_0=\hat{p}\cos\beta, \tag{2a}$$

$$M_1=\hat{t}'-\hat{p}\cos\beta, \tag{2b}$$

$$J^P=2^-: \quad M_0=\cos\beta(\hat{p}\hat{p}-\tfrac{1}{3}\delta_{ij}), \tag{3a}$$

$$M_1=\tfrac{1}{2}(\hat{p}\hat{t}'+\hat{t}'\hat{p})-\cos\beta\hat{p}\hat{p}, \tag{3b}$$

where the subscript on the tensor M indicates the ρ-meson helicity.

[4] C. Zemach, Phys. Rev. **133**, B1201 (1964); **139**, B97 (1965); **140**, B109 (1965).

[5] S. Berman and M. Jacob, Phys. Rev. **139**, B1023 (1965).

[6] K. Gottried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).

[7] We are indebted to Dr. F. Gilman for bringing this point to our attention.

[8] R. Diebold, CERN Report No./TC/PROG. 64-25, 1964 (unpublished).

If the A_1 is assumed to be diffraction-produced,[9] we can calculate the expected angular distribution terms

$$J^P=1^+:\quad I_0=\cos^2\beta\cos^2\theta\,, \tag{4a}$$

$$I_1=\sin^2\beta\sin^2\theta\cos^2\psi\,, \tag{4b}$$

$$I_{0,1}=\cos\beta\sin\beta\cos\theta\sin\theta\cos\psi\,, \tag{4c}$$

$$J^P=2^-:\quad I_0=\cos^2\beta(\cos^2\theta-\tfrac{1}{3})^2\,, \tag{5a}$$

$$I_1=\sin^2\beta\cos^2\theta\sin^2\theta\cos^2\psi\,, \tag{5b}$$

$$I_{0,1}=\cos\beta\sin\beta(\cos^2\theta-\tfrac{1}{3})\cos\theta\sin\theta\cos\psi\,, \tag{5c}$$

where $I_{0,1}$ is the cross term and vanishes upon integration over any one of the angles. The distributions are independent of the A_1 Treiman-Yang angle and thus predict an isotropic distribution in ϕ.

For actually fitting the experimental angular distributions, we cannot use (4) and (5) but we must Bose-symmetrize the helicity tensors (2) and (3), and include the ρ meson and A_1 propagators. The resulting Bose-symmetrized angular distributions cannot be calculated analytically and were determined numerically by computer. The effect of Bose symmetrization is to remove the complete orthogonality of the helicity tensors and introduce an additional interference term between M_0 and M_1. This term can be roughly thought of as arising from the fact that the helicity-zero tensor of one ρ meson is not orthogonal to the helicity-1 tensor of the other possible ρ meson and vice versa. Unlike the cross terms (4c) and (5c), this additional term has the same type of angular dependence as I_0 and I_1 and does not vanish upon integrating over any one of the Euler angles. Thus, the distribution for any one of the angles β, θ, ψ has the general form

$$g_0^2|A_0|^2+g_1^2|A_1|^2+2(g_1g_0)\,\mathrm{Re}(A_0^*A_1)\,, \tag{6}$$

where A_λ is the Bose-symmetric amplitude and g_0 and g_1 are helicity coupling constants that indicate the relative amounts of the two ρ-meson helicity states (see Appendix). The interference term in (6) and the cross terms (4c) and (5c) are sensitive to the sign of g_0g_1, so that they may in principle give some information about the relative phase of the two helicity states. The value of the phase is necessary for determining the relative amounts of the orbital-angular-momentum states in the A_1. For spin-parity 1^+, the ratio of d to s wave is given by the relation

$$\frac{g_d}{g_s}=2\frac{g_0/g_1-1}{g_0/g_1+2}\,, \tag{7}$$

which requires $g_0/g_1=1$ for pure s wave and $g_0/g_1=-2$ for pure d wave (see Appendix).

III. COMPARISON WITH DATA

The experimental details and data analysis have been described in Ref. 2. We have maintained the same mass selection intervals[10] for the ρ^0 and A_1 events with the $\Delta^{++}(1236)$ removed. The effects of narrower cuts on both the ρ^0 and A_1, as well as a selection on t, the momentum transfer to the recoil, were investigated. The results were consistent with the wider cuts, which were used in the data fitting to achieve the best statistics. We note that since the t distribution of the nonresonant background, as deduced from an OPE calculation, is not too different from that expected from a resonance, a cut on t should not be expected to affect our results appreciably.[11]

In order to represent the background contribution to the A_1 angular distributions, an OPE calculation by Wolf[12] and a completely isotropic background were both tried, where the absolute normalization was determined from a fit to the $\pi^-\rho^0$ mass distribution. As emphasized by Ballam *et al.*,[2] the OPE calculation provides good agreement with the detailed features of the data for those events near the A_1 region but not directly associated with the A_1, so that we can use the OPE calculation to account for some of the apparently nonresonant features in the data, such as asymmetries in the angular distributions, thereby improving the fits. The completely isotropic background was used to test qualitatively the sensitivity of the fit parameters to the form of the background.

TABLE I. Fit summary for helicity angle $\cos\beta$.

J^P	Background % OPE	Background % Isotropic	χ^2/d.f.
0^-	63±4		34.8/8
		61±4	54.2/8
1^-	64±4		56.5/8
		64±4	75.8/8
2^+	69±4		68.8/8
		70±4	91.3/8
1^+ (s)	49±5		15.5/8
		48±5	15.1/8
1^+ (d)	51±5		10.9/8
		50±5	14.6/8
2^- (p)	48±5		11.0/8
		47±5	10.7/8
2^- (f)	48±5		7.9/8
		47±5	10.4/8
1^+ fit	50±5 $\lvert g_1/g_0\rvert=0.47\pm0.14$ $\Phi=-0.9\pm1.1$		5.6/6
		49±5 $\lvert g_1/g_0\rvert=0.49\pm0.16$ $\Phi=0.14\pm0.14$	3.5/6
2^- fit	49±5 $\lvert g_1/g_0\rvert=0.60\pm0.15$ $\Phi=-0.3\pm1.9$		5.2/6
	(Φ in radians)	48±5 $\lvert g_1/g_0\rvert=0.62\pm0.14$ $\Phi=0.8\pm1.1$	4.3/6

[9] Diffraction production of the A_1 is strongly suggested by the constant cross section over a large range of incident beam momentum. See, e.g., D. R. O. Morrison, Phys. Letters **25B**, 238 (1967).

[10] The mass intervals used were 1.0–1.16 BeV/c^2 for the A_1, 0.66–0.90 BeV/c^2 for the ρ, and 1.1–1.36 BeV/c^2 for the Δ^{++}.

[11] The similarity between background and resonant t distributions for several quasi-two-body final states has been pointed out previously. See, e.g., J. Bartsch *et al.*, Phys. Letters **27B**, 336 (1968).

[12] G. Wolf, Phys. Rev. **182**, 1538 (1969).

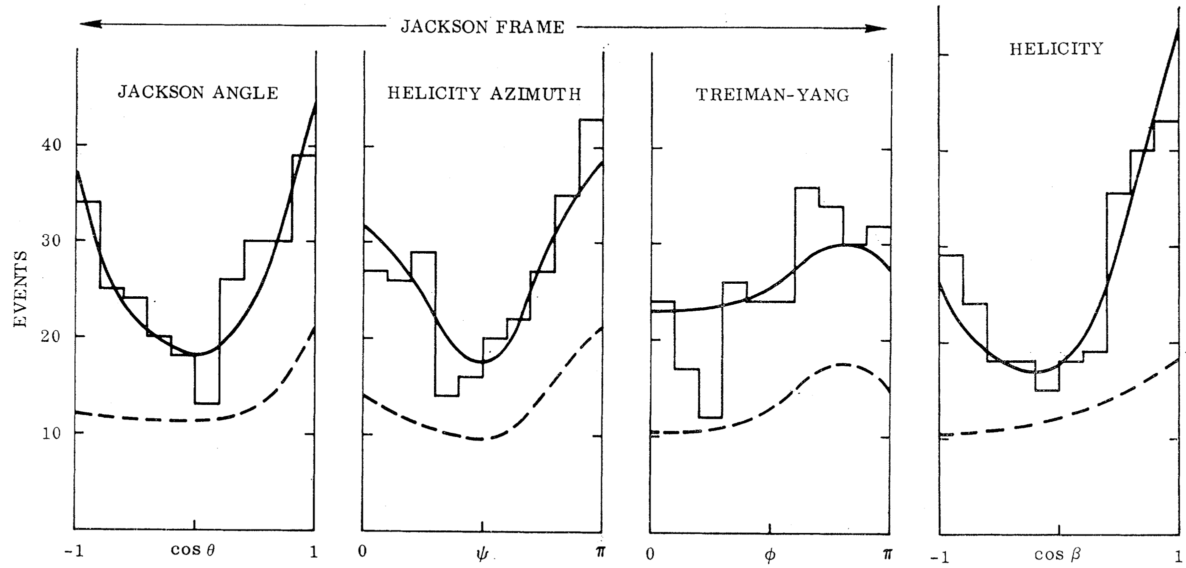

FIG. 2. Decay angular distributions for the A_1 in the Jackson frame. The solid curves are the fit for 1^+ plus OPE; the dashed curves are the OPE contribution alone.

Since only the helicity angle β is independent of the production mechanism of the A_1, the experimental $\cos\beta$ distribution was first fitted alone with the expected theoretical curves for each spin-parity J^P plus the background term described above. The background level was allowed to vary according to a Gaussian distribution about the central value $(49\pm5)\%$, obtained in Ref. 2. Interference between the theoretical A_1 amplitude and the background was neglected.

The 1^+ and 2^- spin-parities were fitted with the ratio

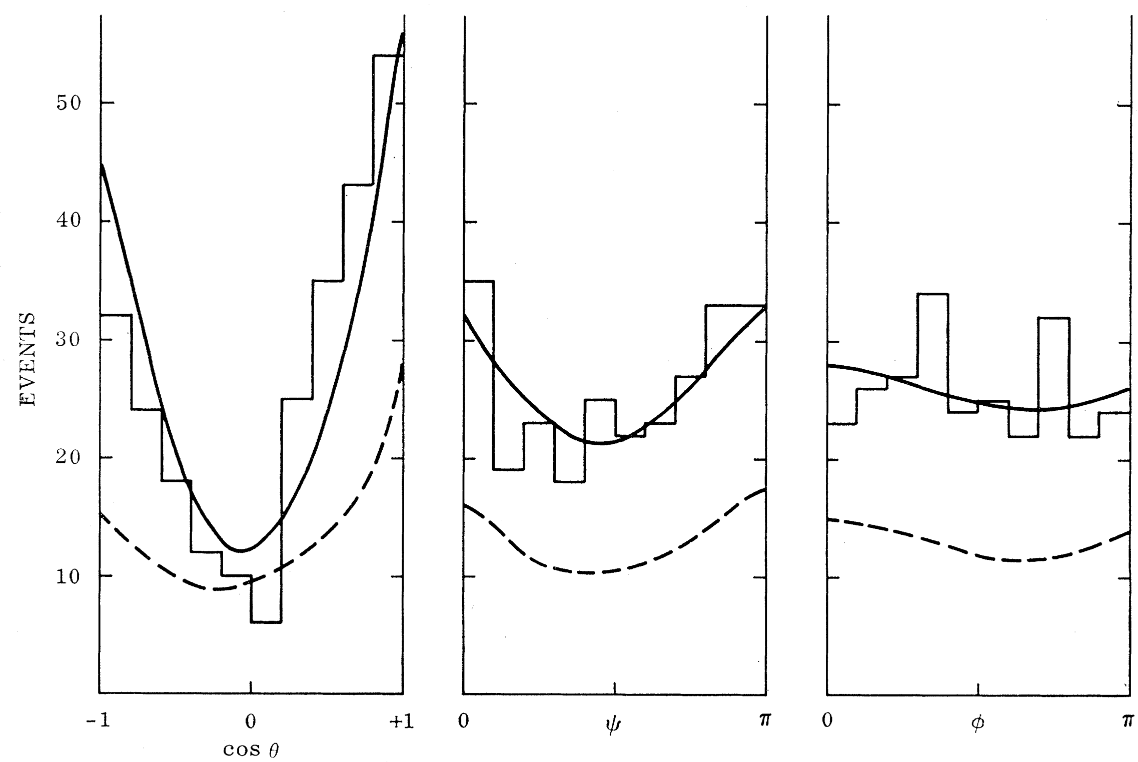

FIG. 3. Decay angular distributions for the A_1 in the π-π scattering frame. The solid curves are the fit for 1^+ plus OPE; the dashed curves are the OPE contribution alone.

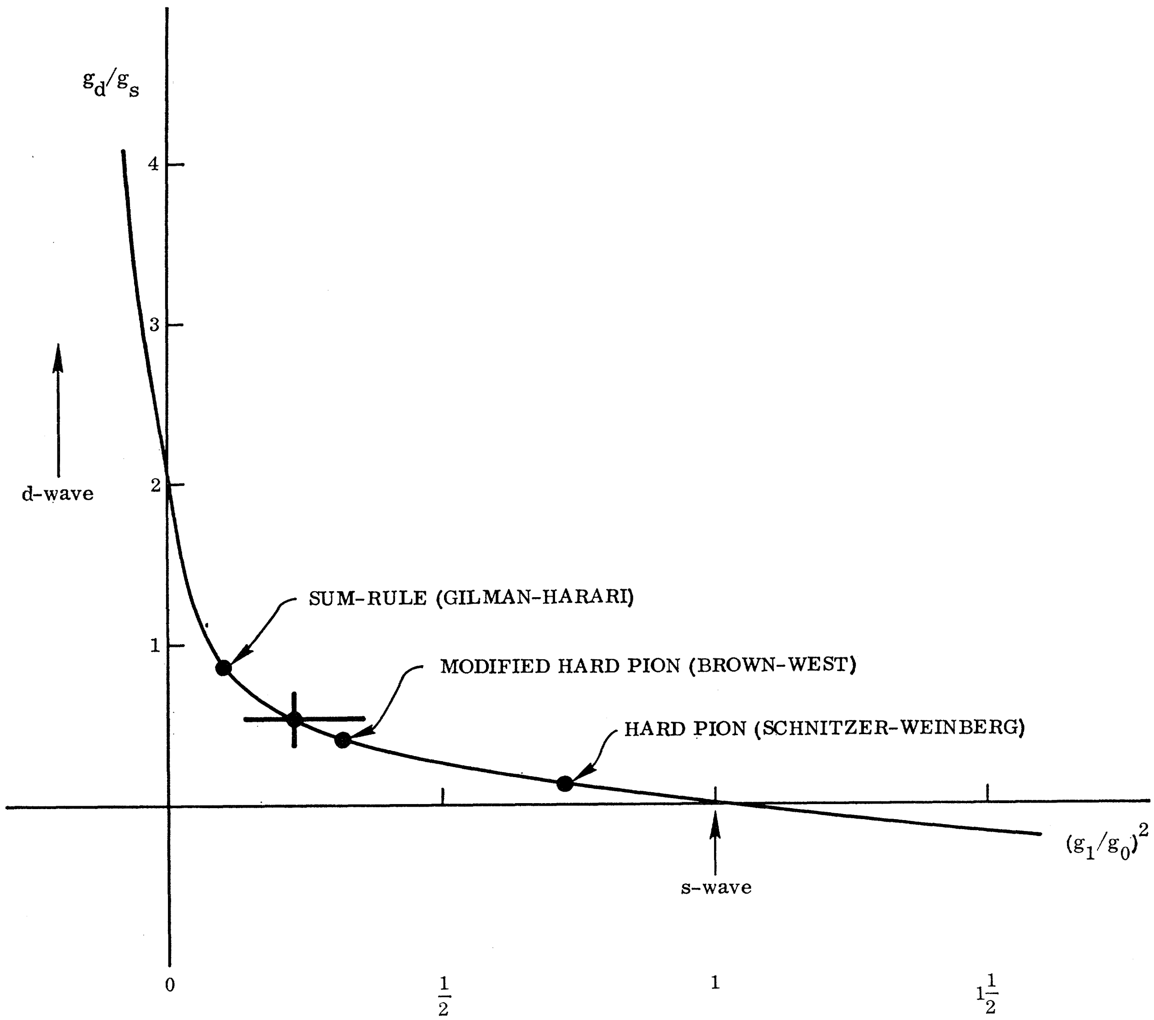

FIG. 4. Comparison of the fitted value of $(g_1/g_0)^2$ with theoretical predictions. The curve is given by Eq. (7) in the text for $J^P=1^+$.

$|g_1/g_0|$ and the relative phase Φ of g_1, g_0 as free parameters in the expression

$$W(\cos\beta)\propto|A_0|^2+|g_1/g_0|^2|A_1|^2+2|g_1/g_0| \times[\cos\Phi\,\mathrm{Re}(A_0{}^*A_1)+\sin\Phi\,\mathrm{Im}(A_0{}^*A_1)]. \quad (8)$$

The results are shown in Table I, where we have also included 1^+ and 2^- with single orbital states for comparison. The OPE background improves the fits for 0^-, 1^-, and 2^+ over that for the isotropic background, but not enough to be acceptable ($<0.5\%$). The 1^+ and 2^- fits are essentially insensitive to the difference between the OPE and isotropic backgrounds and indicate that the higher orbital state is preferred in both cases. The fitted value of $|g_1/g_0|$ is stable with respect to the two different background forms, but the phase is uncertain because of the large errors. This is consistent with the observation that the interference terms that are sensitive to Φ contribute approximately 25% to (8), which is about the accuracy to which $|g_1/g_0|$ is determined by the first two diagonal terms.

A simultaneous fit to $\cos\beta$ and the three Jackson-frame Euler angles was performed for 1^+ and 2^-, using a distribution of the form (8) for each angle. The results of the fit shown in Fig. 2 indicate that the OPE background improves the fit substantially, in contrast to the fit for $\cos\beta$ alone. This can presumably be explained by the greater "fine structure" in the Euler-angle OPE background curves, which increases the sensitivity of the fit, and also to the obviously more demanding requirement of fitting four independent angles rather than just one. The fitted value $|g_1/g_0|=0.48\pm0.11$ is essentially the same as the values in Table I.

To check the consistency of our result, we also compared the expected theoretical distributions with the Euler angles in the π-π scattering frame. We do not gain any new information about the helicity coupling constants since the π-π scattering frame is equivalent to the Jackson frame; they are simply related by a rotation of angle β (the helicity angle) about the three-pion normal direction. The π-π scattering frame angles are also expected to be less sensitive to the value of the coupling constants than the corresponding Jackson-frame angles, since the helicity spin tensors for the π-π scattering frame are not orthogonal. A four-angle fit to the π-π scattering frame Euler angles plus the helicity angle gave a value of $|g_1/g_0|=0.45\pm0.13$ with χ^2/degrees of freedom$=40.8/36$. The fitted distributions shown in Fig. 3 are compatible within statistics with the Jackson-frame results.

The error given for $|g_1/g_0|$ represents only the effects of statistics and the fitting procedure used. Both the value of $|g_1/g_0|$ and the error can be affected by the assumptions used in background calculation as well as in the theoretical model for the resonance. Also, the OPE calculation predicts that the $\rho\pi$ background is predominantly 1^+ s-wave, so that there is the possibility of interference between background and resonance which could affect our results. Although we cannot determine the precise amount of such an interference, we feel that it is not a major factor in our data for the following reasons:

(a) The $\cos^2$ contribution seen in the helicity angle distribution does not diminish with increasingly narrower cuts on the ρ and $\pi\rho$ masses around the A_1, as might be expected if it were associated only with the background or an interference term.

(b) The region between the A_1 and A_2 is consistent with pure OPE background with no visible d-wave contribution.

(c) The forward-backward asymmetry observed in the π-π scattering angle, which is an indication of some interference since it cannot be due to a resonance, can be almost completely explained (to within 1 standard deviation) by the OPE background alone.

None of the above-mentioned points completely excludes some background interference, but they strongly suggest that it is not a serious effect and most probably is within the range of our quoted error for $|g_1/g_0|$.

IV. COMPARISON WITH THEORY

In Fig. 4, we have plotted our result for $|g_1/g_0|^2$ with some representative theoretical predictions for comparison. Our result appears to be consistent with both the superconvergent sum-rule calculation of Gilman and Harari[1] and the modified hard-pion result of Brown and West,[1] so that we cannot conclusively distinguish between the two types of theories. However, the various theories which predict (a) predominantly s-wave decay appear to be excluded by several standard deviations. Also, the quark-model calculation of Lipkin,[1] which predicts purely transverse ρ mesons in the A_1 decay as well as longitudinal ω^0 polarization for the B decay, disagrees with our result and with the experimental results of Ascoli *et al.*,[3] for the B meson. Lipkin[1] has pointed out, however, that the usual quark-model picture of a 1^+ $q\bar{q}$ pair decaying by pion emission into a 1^- $q\bar{q}$ pair may not be correct. If, instead, the vector meson is emitted, in analogy with the photon in electrodynamics, the transition would be electric-dipole-like and predict a purely transverse helicity state for the ω^0 in B decay and a mixture of helicities for the ρ in the A_1 decay, in reasonable agreement with experiment.

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APPENDIX: HELICITY TENSORS AND COUPLING CONSTANTS

Helicity decay tensors can be obtained from the orbital-angular-momentum tensors in Table III of Ref. 9 by using the appropriate Clebsch-Gordan coefficients, which relate the helicity and angular-momentum bases. The formalism has been given by Gilman[13] and we briefly outline it for convenience.

The decay process $c\rightarrow a+b$ can be described by a T matrix

$$\begin{aligned}T_{fi}&=\langle S_c\lambda_c|T|\theta\phi;\lambda_a\lambda_b\rangle\\&\equiv g_{\lambda_a\lambda_b}D_{\lambda_c\lambda}{}^{S_c}(\phi,\theta,-\phi),\end{aligned}\tag{A1}$$

where λ is helicity and $g_{\lambda_a\lambda_b}$ is the *helicity coupling constant* defined by

$$g_{\lambda_a\lambda_b}=(2S_c+1/4\pi)^{1/2}\langle S_c\lambda_c|T|S_c\lambda_c;\lambda_a\lambda_b\rangle.\tag{A2}$$

Using Eq. (B5) from Jacob and Wick,[14]

$$\begin{aligned}\langle JMLS|JM\lambda_a\lambda_b\rangle&=(2L+1/2J+1)^{1/2}\\&\quad\times(LS0\lambda|S\lambda)(S_aS_b\lambda_a-\lambda_b|S\lambda),\end{aligned}\tag{A3}$$

for a given orbital state, the ratio of the coupling constants for different helicity states is then

$$\frac{g_{\lambda_a\lambda_b}}{g_{\lambda_a'\lambda_b'}}=\frac{(LSO\lambda c|S_c\lambda_c)}{(LSO\lambda_c'|S_c\lambda_c')}\frac{(S_aS_b\lambda_a-\lambda_b|S_c\lambda_c)}{(S_aS_b\lambda_a'-\lambda_b'|S_c\lambda_c')}.\tag{A4}$$

The orbital-angular-momentum tensors transform like the states $\langle JMLS|$, so that the corresponding helicity tensors can be calculated by simply taking linear combinations of the orbital tensors according to (A4). The resulting helicity tensors for $J^P=1^+$ and 2^- are given by Eqs. (2) and (3) in Sec. II. The relation for the d- to s-wave ratio, Eq. (7), is given directly by (A4).

[13] F. Gilman, Stanford Linear Accelerator Center, Stanford University, Stanford, Calif. (private communication).

[14] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).