Radiative X^0 Decays in Broken $SU(3)$

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The radiative decays of the X^0 meson are analyzed in the framework of an effective Lagrangian with vector-meson dominance, using vector gauge fields with current mixing and assuming X^0 to be a unitary singlet. The underlying VVP strong interaction obeys octet-broken $SU(3)$ symmetry. The model is shown to predict branching ratios different from those obtained in the η-X^0 mixing model. An analysis of the experimental situation is presented, and the relevance of the $X^0\to 2\gamma$, $X^0\to\omega\gamma$ partial decay modes for distinguishing between different theoretical models is pointed out.

RECENTLY, Brown, Munczek, and Singer[1] have presented a description of radiative meson processes by using an effective Lagrangian, with $SU(3)$ octet breaking, for vector and pseudoscalar mesons. The nine vector mesons were represented by gauge vector fields with singlet-octet mixing in the vector-mixing model, and the vector-dominance scheme was used for introducing the electromagnetic interaction. The PVV strong-interaction term responsible for the processes studied was taken to have the most general form allowed by octet-broken $SU(3)$.

In Ref. 1 the analysis covered the radiative decays of the nine-vector mesons and of the pseudoscalar octet. In this note, we extend their work by considering within a similar framework the radiative decays of a pseudoscalar singlet, which we identify with the X^0 meson of mass 958.3 ± 0.8 MeV/c^2. As in Ref. 1, we neglect possible η-X^0 mixing. This point will be further discussed at the end of this paper.

In order to account for the X^0 radiative decays, we supplement the PVV part of the Lagrangian of Ref. 1 with a term

$$\mathcal{L}_{P^0VV}=\tfrac{1}{4}\epsilon_{\mu\nu\sigma\tau}h_0D^{ab}V_{\mu\nu}{}^aV_{\sigma\tau}{}^bP^0\,, \tag{1}$$

where P^0 is an $SU(3)$-singlet pseudoscalar meson, $V_\mu{}^a$ is a vector-meson octet, a, $b=1$, $\ldots$, 8, and $V_{\mu\nu}{}^a=\partial_\mu V_\nu{}^a-\partial_\nu V_\mu{}^a-gf^{abc}V_\mu{}^bV_\nu{}^c$. The general form[2] of D^{ab} in octet-broken $SU(3)$ is

$$D^{ab}=\delta^{ab}+\sqrt{3}\beta d^{ab8}\,. \tag{2}$$

For the vector mesons, the formalism expressed by Eqs. (4)–(6) of Ref. 1 is being used, and likewise we take for the effective electromagnetic interaction

$$\mathcal{L}_{\rm em}=(em^2/g)[K_\rho{}^{-1/2}\rho_\mu{}^3+(3K_\omega)^{-1/2}\omega_\mu\sin\theta-(3K_\varphi)^{-1/2}\varphi_\mu\cos\theta]A_\mu\,, \tag{3}$$

where

$$K_i=m^2/m_i{}^2\qquad(i=\rho,K^*,\omega,\varphi)\,,\qquad m=847\text{ MeV},\quad \theta=27.5°\,. \tag{4}$$

We consider the following radiative processes[3–5] involving X^0 mesons: (a) $X^0\to\rho\gamma$, (b) $X^0\to\omega\gamma$, (c) $X^0\to\gamma\gamma$, and (d) $\phi\to X^0\gamma$. The invariant amplitudes for these two-body decays have the general form

$$F\epsilon_{\mu\nu\sigma\tau}\epsilon_\mu{}^{(V_1)}k_\nu{}^{(V_1)}\epsilon_\sigma{}^{(V_2)}k_\tau{}^{(V_2)}\,, \tag{5}$$

where $\epsilon_\mu{}^{(V_i)}$ are the polarization vectors of a γ or vector meson and $k_\mu{}^{(V_i)}$ are the respective four-momenta. The coefficients F calculated with the aid of (1)–(4) as well as (4)–(6) of Ref. 1 have the following form:

$$X^0\to\rho\gamma:\quad (2h_0e/g)\{(1+\beta)/\sqrt{K_\rho}\}\,, \tag{6a}$$

$$X^0\to\omega\gamma:\quad (2h_0e/3g)\{(1-\beta)\sqrt{3}\sin\theta/\sqrt{K_\omega}\}\,, \tag{6b}$$

$$X^0\to 2\gamma:\quad (8h_0e^2/3g^2)\{\tfrac{1}{2}(2+\beta)\}\,, \tag{6c}$$

$$\varphi\to X^0\gamma:\quad -(2\sqrt{2}h_0e/3g)\{(1-\beta)(\sqrt{\tfrac{3}{2}})\cos\theta/\sqrt{K_\varphi}\}\,. \tag{6d}$$

These amplitudes differ from those previously considered in the literature by symmetry-breaking factors displayed in curly brackets. In the limit of equal masses for the nonet of vector mesons and a nonet mixing angle[6] of $\sin\theta=1/\sqrt{3}$, the expressions in curly brackets approach unity.

With the amplitudes (5) and (6), the partial decay widths are

$$\Gamma(X^0\to\rho\gamma)=\frac{(m_\pi{}^2h_0{}^2/4\pi)\alpha(M_X{}^2-M_\rho{}^2)^3(1+\beta)^2}{2M_X{}^3m_\pi{}^2K_\rho(g^2/4\pi)}\,, \tag{7a}$$

$$\Gamma(X^0\to\omega\gamma)=\frac{(m_\pi{}^2h_0{}^2/4\pi)\alpha(M_X{}^2-M_\omega{}^2)^3(1-\beta)^2\sin^2\theta}{6M_X{}^3m_\pi{}^2K_\omega(g^2/4\pi)}\,, \tag{7b}$$

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[1] L. M. Brown, H. Munczek, and P. Singer, Phys. Rev. Letters **21**, 707 (1968).

[2] As far as the strong interaction is concerned, a P^0VV interaction consistent with our symmetry assumptions contains two more terms of the form $\epsilon_{\mu\nu\sigma\tau}(f_1V_{\mu\nu}{}^8V_{\sigma\tau}{}^0P^0+f_2V_{\mu\nu}{}^0V_{\sigma\tau}{}^0P^0)$, where, $V_\mu{}^0$ is an $SU(3)$-singlet vector field. These terms are not relevant to X^0 radiative decays since we take the electromagnetic current to have the $SU(3)$ transformation properties $J_\mu{}^{\rm em}=e(J_\mu{}^3+\sqrt{\tfrac{1}{3}}J_\mu{}^8)$, i.e., it contains no $SU(3)$-singlet piece and thus singlet $\to$ singlet radiative transitions are forbidden.

[3] Most of these processes were previously discussed in Ref. 4 without considering $SU(3)$ breaking and in Ref. 5, where η-X^0 mixing is assumed for the $SU(3)$-breaking mechanism.

[4] L. M. Brown and H. Faier, Phys. Rev. Letters **13**, 73 (1964).

[5] R. H. Dalitz and D. G. Sutherland, Nuovo Cimento **37**, 1777 (1965); **38**, 1945(E) (1965).

$$\Gamma(X^0\to 2\gamma)=\frac{4(m_\pi^2h_0^2/4\pi)\alpha^2M_X^3}{9m_\pi^2(g^2/4\pi)^2}(1+\tfrac{1}{2}\beta)^2, \tag{7c}$$

$$\Gamma(\varphi\to X^0\gamma)=\frac{(m_\pi^2h_0^2/4\pi)\alpha(M_\varphi^2-M_X^2)^3(1-\beta)^2\cos^2\theta}{18m_\pi^2M_\varphi^3K_\varphi(g^2/4\pi)}, \tag{7d}$$

where α is the fine-structure constant.

Our model for X^0 radiative decays has two parameters, the strong coupling h_0 and the $SU(3)$-breaking parameter β.[7] The measurement of two of the decays (6) allows the prediction of the partial decay widths for the rest. The measurement of a ratio of two of these decay modes would determine the parameter β. An exception to these statements are the decays $X^0\to\omega\gamma$ and $\phi\to X^0\gamma$, whose ratio is independent of β.

The η-X^0 mixing model of Ref. 5, with vector-meson dominance, is essentially a two-parameter model for the radiative decays involving all vector and pseudoscalar mesons (this in addition to using the strong-coupling constant determined from $\rho\to\pi\pi$ decay, the mixing η-X^0 angle from the mass formula, and using $SU(6)$ for treating the singlet vector-meson couplings). Determination of the parameters from the measured π^0 and η decays leads then to predictions for the $X^0\to 2\gamma$ and $X^0\to\rho\gamma$ decay widths. One obtains[8] $\Gamma(X^0\to 2\gamma)\simeq 50$ keV and $\Gamma(X^0\to\rho\gamma)\simeq 1$ MeV, which is hardly compatible with the experimental upper limit $\Gamma(X^0\to \text{all})\leq 4$ MeV and $\Gamma(X^0\to\rho\gamma)/\Gamma(X^0\to\text{all})=0.22$.[9] However, as the uncertainties involved in obtaining these estimates are at least of the order of 30%, the η-X^0 mixing model in conjunction with vector dominance is not yet ruled out.

In the following, we shall discuss characteristic features of our model and its experimental implications.

(1) From (7a) and (7b) we obtain the following ratio for these partial decay modes:

$$\Gamma(X^0\to\omega\gamma)/\Gamma(X^0\to\rho\gamma)=5.7\times10^{-2}(1-\beta)^2/(1+\beta)^2. \tag{8}$$

While in our model this ratio depends on the $SU(3)$-breaking parameter β, it is not affected by η-X^0 mixing and is predicted to be 0.084 in the model of Dalitz and Sutherland.[10] Hence, this ratio has particular significance and its measurement could help to distinguish between theories with different types of $SU(3)$ breaking.

(2) From (6a) and (6c), one has

$$\Gamma(X^0\to 2\gamma)/\Gamma(X^0\to\rho\gamma)=4.9\times10^{-2}(1+\tfrac{1}{2}\beta)^2/(1+\beta)^2. \tag{9}$$

It is interesting to note that the way this ratio is affected by $SU(3)$ breaking is formally similar in our model and in the η-X^0 mixing model. While in the latter the breaking is related to the decays of the pseudoscalar octet, it is a free parameter in our scheme. Once fixed, however, from one measured ratio, it predicts all the others.

(3) From (7a) and (7d), we find

$$\Gamma(\varphi\to X^0\gamma)/\Gamma(X^0\to\rho\gamma)=6.2\times10^{-3}(1-\beta)^2/(1+\beta)^2. \tag{10}$$

If $\beta\approx 0$, from the upper limit $\Gamma(X^0\to\rho\gamma)\lesssim 0.88$ MeV, one can estimate $\Gamma(\phi^0\to X\gamma)\lesssim 5.5$ keV, and hence this partial decay is expected to be of the order of 10^{-3} of the total ϕ width, except if large $SU(3)$ breaking occurs.

(4) The ratio of $\phi\to X^0\gamma$ to $X^0\to\omega\gamma$ is independent of the β parameter and is predicted in our model to have the value

$$\Gamma(\varphi\to X^0\gamma)/\Gamma(X^0\to\omega\gamma)=0.11. \tag{11}$$

In the η-X^0 mixing model, this ratio is affected by the mixing parameter.[11]

(5) From the experimental upper width of $X^0\to\rho\gamma$, one can obtain an upper limit for the couplings involved, namely, $m_\pi^2h_0^2(1+\beta)^2/4\pi<0.42$. It is instructive to compare this value with that of the appropriate combination determined in Ref. 1 for the coupling of the octet of pseudoscalar mesons, $m_\pi^2h^2(1+\epsilon_1)^2/4\pi=0.1$. If one speculates that the two quantities should be approximately equal, it results into a total width for X^0 of approximately 1 MeV. This is consistent with previous estimates.[4,5]

The same combination $h_0^2(1+\beta)^2$ appears also in the strong partial decay width for $X^0\to 4\pi$, which proceeds through two ρ's. Because of phase-space inhibition, this mode is expected to be very small. One should remark that in our model it is directly related to the strength of the $X^0\to\rho\gamma$ mode.

(6) A recent preliminary measurement[12] of the $X^0\to 2\gamma$ decay gives for this mode a branching ratio of $(5.5_{-3.0}^{+3.6})\%$. This implies a ratio $(X^0\to\rho\gamma)/(X^0\to 2\gamma)$ of $4_{+4.8}^{-1.6}$, in disagreement with the η-X^0 mixing + vector-dominance model which predicts[8] a ratio of $\simeq 20$. Although our model can accommodate the reported high ratio, the parameter β determined from it then gives what seems to be an improbable amount of $X^0\to\omega\gamma$. Thus, for 5.5% of $X^0\to 2\gamma$ we obtain from (9) $\beta=-0.71$, which when used in (8) gives $X^0\to\omega\gamma$ to

[6] S. Okubo, Phys. Letters 5, 165 (1963).

[7] The value of $g^2/4\pi$ has already been determined in Ref. 1 as 3.35 from the decays $\omega\to 3\pi$, $\omega\to\pi\gamma$, and $\pi^0\to 2\gamma$.

[8] A. Baracca and A. Bramon, Nuovo Cimento 51A, 873 (1967); M. Jacob, CERN Report No. TH-846, 1967 (unpublished).

[9] Particle Data Group, Rev. Mod. Phys. 41, 109 (1969).

[10] The value obtained in Ref. 5 is somewhat higher than our unbroken value ($\beta=0$) since they use $\sin\theta=\sqrt{\tfrac{1}{3}}$, while we take the mixing angle (4) of the vector-mixing model.

[11] With the model and notation of Ref. 5, this ratio becomes proportional to $[1-(2M/A)\tan\alpha]^2/[1+(M/A)\tan\alpha]^2$.

[12] D. Bollini, A. Buhler-Broglin, P. Dalpiaz, T. Massam, F. Navach, F. L. Navarra, M. A. Schneegans, and A. Zichichi, Nuovo Cimento 58, 289 (1968).

be twice as frequent as $X^0 \to \rho\gamma$. If one takes 2.5% for the $X^0 \to 2\gamma$ branching ratio, one determines $\beta = -0.5$, which then gives $(X^0 \to \omega\gamma)/(X^0 \to \rho\gamma) = 0.6$.[13] There are no experimental values reported so far for this ratio.

We conclude by stressing that the measurement of the $X^0 \to 2\gamma$ and $X^0 \to \omega\gamma$ decay modes is highly desirable. This will hopefully allow us to differentiate between the η-X^0 mixing model and the model proposed here for the radiative decays of X^0. However, if none of these models succeed in accounting for the experimental data, one might have to use a combination of them or to give up vector dominance and possibly use a mixing angle from the linear mass formula.[14]

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[13] In solving for β from (9) one gets two solutions; in both cases analyzed we used the β value which gives a lower $(X^0 \to \omega\gamma)/(X^0 \to \rho\gamma)$ ratio.

[14] F. Guerra, F. Vanoli, G. De Franceschi, and V. Silvestrini, Phys. Rev. **166**, 1587 (1968); R. S. Willey, *ibid.* **183**, 1397 (1969).

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Veneziano Model with Secondary Terms for Pion-Pion Scattering

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We show that the Veneziano-Lovelace one-term formula for pion-pion scattering, although a good first approximation, has some undesirable features. With the help of a few secondary terms, a model amplitude, which is consistent with the known properties of the pion-pion system up to the f^0 mass region and the high-energy phenomenology, is constructed.

I. INTRODUCTION

Sometime ago, Veneziano[1] proposed a very attractive model for scattering amplitudes of strongly interacting particles, incorporating analyticity, resonance-pole structure, crossing symmetry, and Regge behavior in a somewhat natural way. The original amplitude satisfies all the superconvergence relations and the finite-energy sum rules (FESR). Lovelace[2] took the next step by considering one-term Veneziano amplitude for the π-π scattering, and imposed Adler's self-consistency condition. This condition requires the π-π amplitude to vanish at $s=t=u=m_\pi^2$ (s, t, and u being the usual Mandelstam variables). This led to a condition on the ρ-trajectory parameter $\alpha_\rho(m_\pi^2)=\frac{1}{2}$ and the resulting π-π scattering lengths were in approximate agreement with those obtained from the current algebra. The Veneziano-Lovelace formula for the π-π scattering may be a good first approximation.[3] However, it has some undesirable features. First of all, $\alpha_\rho(m_\pi^2)$, from high-energy phenomenological fitting, is found to be somewhat larger $[\alpha_\rho(m_\pi^2)\approx 0.58]$ and the amplitude contains a term like $\Gamma(1-\alpha_\rho(t)-\alpha_\rho(u))$ which is extremely sensitive to the value of $\alpha_\rho(m_\pi^2)$ near $t=u=m_\pi^2$. If we take $\alpha_\rho(m_\pi^2)\approx 0.58$ in the one-term formula, the S-wave $I=0$ and $I=2$ π-π scattering lengths have the same sign and the $I=0$ scattering length comes out to be very large ($\sim 1/m_\pi$), in contrast to the current-algebra predictions.[4] Furthermore, the one-term formula gives the ratio of the width of the σ meson to that of the ρ meson to be about 5, which is unusually large. Since the Veneziano formula is based on a zero- (or very-narrow-) width resonance approximation, one feels uncomfortable about predictions of such large widths. Also, it is known that, in principle, one can have any number of secondary terms having essentially the same properties as the primary term.[5] In fact, it has been shown that for cases like π-ρ scattering, secondary terms are necessary to avoid parity doubling.[5] (In the case of the π-π system, Bose statistics rules out the parity doubling automatically.) Also, it appears that secondary terms may be necessary in the general case to assure factorization.[5]

In principle, one can add an infinite number of terms to the first term. However, in such a case the Regge behavior will not be necessarily obtained. Moreover, in practice one would like to have a limited number of terms to describe the physical amplitude. One can even think of determining the coefficients of the various terms by some bootstrap requirements.[6] However, in the

[1] G. Veneziano, Nuovo Cimento **57A**, 190 (1968).

[2] C. Lovelace, Phys. Letters **28B**, 264 (1968).

[3] K. Kawarabayashi, S. Kitakado, and H. Yabuki, Phys. Letters **28B**, 432 (1969). Also J. Yellin, J. Shapiro, and collaborators have considered extensively the implications of a one-term Veneziano model for π-π scattering: University of California Lawrence Radiation Laboratory Reports (unpublished).

[4] This is also noticed by K. Kang (report of work prior to publication).

[5] See, for example, Ref. 1; G. Altarelli and H. R. Rubinstein, Phys. Rev. **178**, 2165 (1969); J. E. Mandula (Cal-Tech report of work prior to publication); P. G. O. Freund and E. Schonberg, and also P. G. O. Freund (University of Chicago reports of work prior to publication).

[6] S. Mandelstam, Phys. Rev. Letters **21**, 1724 (1968).